

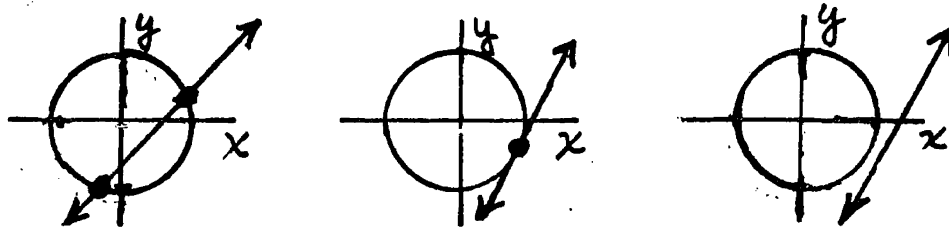
3.10 Non-Linear Systems of Equations

Dr. Robert J. Rapalje

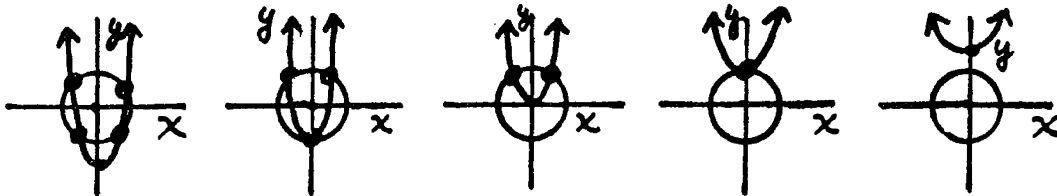
More FREE help available from my website at www.mathinlivingcolor.com

ANSWERS TO ALL EXERCISES ARE INCLUDED AT THE END OF THIS PAGE

A **non-linear system of equations** is a system of equations in which one or both of the equations does not represent a straight line. Whereas linear systems in 2 variables usually intersect in **one point** (or parallel or same lines), non-linear systems (curves) may intersect in **more than one point**. For example, a straight line may intersect a circle in two points, in only one point, or in the third case, the line may completely miss the circle. In this last case the intersection is the empty set.



As a second example, a circle and a parabola could intersect in 4 points, 3 points, 2 points, 1 point, or have no intersection at all.



Because of the diversity in types of curves, many different techniques are useful in solving different types of systems. As one calculus author stated, solving these systems often requires "trickery and guesswork." Some non-linear systems can be solved by the **elimination (addition) method** (eliminate one of the variables from the equations). More often, non-linear systems are best solved by the **substitution method** (solve for one variable in one

equation and substitute into the other equation producing an equation in only one variable). Sometimes the **elimination method** is used in conjunction with the **substitution method**.

The following exercises will demonstrate some problem-solving techniques. Remember that, in spite of all the hints and suggestions, there is not just one way to solve a problem, and the best way to solve the problem is the way you would be most likely to think of. Always look for better ways to solve the exercises. The exercises here are designed to provide more than enough practice for you. It may be helpful to do the odd exercises (which contain all necessary skills) and save the even exercises for extra practice or for review for the exam.

In exercises 1-64, solve the system of equations:

(Substitution method!)

1. $Y = X^2$
 $3X + Y = 10$
 $3X + () = 10$
 $X^2 + 3X - 10 = 0$
 $() () = 0$
 $X = \underline{\quad}$ $X = \underline{\quad}$
 $y = X^2$ $y = X^2$
 $y = \underline{\quad}$ $y = \underline{\quad}$
 $(,)$ $(,)$

2. $Y = X^2$
 $X - Y = -20$
 $X - () = -20$

3. $Y = X^2 - 4$
 $Y = 2X + 4$
 $() = ()$

4. $Y = X^2 + 4X$
 $Y = 3X + 20$

$() = ()$

5. $Y = X^2 - 6X$
 $2X - Y = 12$

$2X - () = 12$

6. $Y = X^2 - 3X + 2$
 $Y = 2X + 8$

$$\begin{aligned} 7. \quad Y &= X^2 - 6X \\ 2X - Y &= 16 \end{aligned}$$

$$\begin{aligned} 8. \quad Y &= X^2 + 2X \\ 4X - Y &= 1 \end{aligned}$$

$$\begin{aligned} 9. \quad Y &= X^2 + 4 \\ Y &= 2X - 2 \end{aligned}$$

$$\begin{aligned} 10. \quad Y &= X^2 + 2X \\ Y &= 2X - 4 \end{aligned}$$

$$\begin{aligned} 11. \quad Y &= 3X + 1 \\ Y &= X^2 - 2X - 5 \end{aligned}$$

$$\begin{aligned} 12. \quad Y &= -2X + 10 \\ Y &= X^2 + 3X - 4 \end{aligned}$$

$$\begin{aligned} 13. \quad Y &= 3X + 5 \\ Y &= X^2 + 6X - 5 \end{aligned}$$

$$\begin{aligned} 14. \quad Y &= 3X + 10 \\ Y &= X^2 + 6X \end{aligned}$$

$$\begin{aligned} 15. \quad Y &= X^2 \\ Y &= X^2 + 3X - 6 \end{aligned}$$

$$\begin{aligned} 16. \quad Y &= X^2 + 4 \\ Y &= X^2 - 2X \end{aligned}$$

$$\begin{aligned} 17. \quad Y &= X^2 - 4X \\ Y &= 16 - X^2 \end{aligned}$$

$$\begin{aligned} 18. \quad Y &= X^2 + 4X \\ Y &= 12 + 2X - X^2 \end{aligned}$$

$$19. \quad Y = X^2 + 4 \\ Y = X^2 - 4$$

$$20. \quad Y = X^2 + 4 \\ Y = -X^2$$

$$21. \quad XY = -12 \\ X + Y = 4 \rightarrow y = 4 - X \\ X(4 - X) = -12$$

$$22. \quad XY = 5 \\ Y = 2X - 3$$

$$23. \quad XY = 21 \\ Y = 3X - 2$$

$$24. \quad XY = 28 \\ Y = 3X - 5$$

$$\begin{aligned} 25. \quad XY &= 30 \\ 2X - Y &= 7 \end{aligned}$$

$$\begin{aligned} 26. \quad XY &= -12 \\ Y &= 2X + 11 \end{aligned}$$

$$\begin{aligned} 27. \quad X^2 + Y^2 &= 16 \\ X - Y &= 4 \\ X &= Y + 4 \\ (Y + 4)^2 + Y^2 &= 16 \end{aligned}$$

$$\begin{aligned} 28. \quad X^2 + Y^2 &= 25 \\ X - Y &= 1 \end{aligned}$$

$$\begin{aligned} 29. \quad X^2 + Y^2 &= 10 \\ Y &= 2X - 5 \end{aligned}$$

$$\begin{aligned} 30. \quad X^2 + Y^2 &= 50 \\ Y &= 2X + 5 \end{aligned}$$

$$\begin{aligned} 31. \quad X^2 + Y^2 &= 185 \\ 2X + Y &= 5 \end{aligned}$$

$$\begin{aligned} 32. \quad X^2 + Y^2 &= 305 \\ Y &= 3X - 5 \end{aligned}$$

$$\begin{aligned} 33. \quad X^2 - Y^2 &= 16 \\ X &= Y + 2 \end{aligned}$$

$$\begin{aligned} 34. \quad X^2 - Y^2 &= 32 \\ X - Y &= 8 \end{aligned}$$

$$\begin{aligned} 35. \quad X^2 - Y^2 &= 24 \\ X - Y &= 2 \end{aligned}$$

$$\begin{aligned} 36. \quad 4X^2 - Y^2 &= 20 \\ 3X - Y &= 5 \end{aligned}$$

$$37. \quad 4X^2 - Y^2 = -60$$
$$Y = 3X - 5$$

$$38. \quad Y^2 - 4X^2 = 105$$
$$Y = 3X - 5$$

$$39. \quad 4X^2 - 3Y^2 = 4$$
$$Y = X - 4$$

$$40. \quad 4X^2 - 9Y^2 = 448$$
$$X = 2Y + 7$$

$$41. \quad 3X^2 - 4Y^2 = 11$$
$$X = 2Y + 3$$

$$42. \quad 4X^2 - 9Y^2 = 175$$
$$Y = X - 5$$

In 43-50, use the elimination method.

$$\begin{array}{l} 43. \quad \begin{array}{l} X^2 - Y^2 = 16 \\ X^2 + Y^2 = 34 \end{array} \\ \hline \end{array} \left. \vphantom{\begin{array}{l} X^2 - Y^2 = 16 \\ X^2 + Y^2 = 34 \end{array}} \right\} \text{Add these!}$$

$$\begin{array}{l} 44. \quad \begin{array}{l} X^2 - Y^2 = 4 \\ 3X^2 + Y^2 = 32 \end{array} \end{array}$$

$$\begin{array}{l} 45. \quad \begin{array}{l} 2(5X^2 + 3Y^2 = 9) \\ -3(3X^2 + 2Y^2 = 6) \end{array} \end{array}$$

$$\begin{array}{l} 46. \quad \begin{array}{l} X^2 - Y^2 = 32 \\ 3X^2 + Y^2 = 4 \end{array} \end{array}$$

$$\begin{array}{l} 47. \quad \begin{array}{l} X^2 - Y^2 = 16 \\ X^2 + 2Y^2 = 16 \end{array} \end{array}$$

$$\begin{array}{l} 48. \quad \begin{array}{l} 5X^2 + 3Y^2 = 12 \\ 3X^2 - Y^2 = -4 \end{array} \end{array}$$

$$49. \quad 5X^2 - 3Y^2 = 12$$

$$3X^2 - 2Y^2 = -8$$

$$50. \quad 4X^2 - 3Y^2 = 16$$

$$2X^2 + Y^2 = 18$$

$$51. \quad X^2 - 2XY + Y^2 = 49$$

$$Y = 3X - 5$$

$$X^2 - 2X(\quad) + (\quad)^2 = 49$$

$$52. \quad X^2 - 2XY + Y^2 = 25$$

$$Y = 2X + 1$$

$$53. \quad X^2 + 2XY + Y^2 = 9$$

$$2X - Y = 3$$

$$\rightarrow Y = 2X - 3$$

$$54. \quad X^2 + 6XY + 9Y^2 = 25$$

$$X - 2Y = -10$$

$$55. \quad \begin{aligned} X^2 + XY + Y^2 &= 21 \\ 2X - Y &= 7 \end{aligned}$$

$$56. \quad \begin{aligned} X^2 - XY + Y^2 &= 3 \\ 2X - Y &= 3 \end{aligned}$$

$$57. \quad \begin{aligned} X^2 + 2XY + Y^2 &= 36 \\ X^2 + Y^2 &= 50 \end{aligned}$$

$\rightarrow (X+Y)^2 = 36$
 $\rightarrow X+Y = \pm 6$

$X+Y = 6$ $Y = 6 - X$	$X+Y = -6$ $Y = -X - 6$
$X^2 + (\downarrow)^2 = 50$	$X^2 + (\downarrow)^2 = 50$

$$58. \quad \begin{aligned} X^2 + 2XY + Y^2 &= 64 \\ X^2 + Y^2 &= 50 \end{aligned}$$

$$59. \quad \begin{aligned} X^2 + 2XY + Y^2 &= 25 \\ 2X^2 + 2Y^2 &= 169 \end{aligned}$$

$$60. \quad \begin{aligned} X^2 + 2XY + Y^2 &= 144 \\ 2X^2 + 2Y^2 &= 169 \end{aligned}$$

$$\begin{aligned} 61. \quad X^2Y &= -36 \\ Y &= X^2 - 13 \end{aligned}$$

$$\begin{aligned} 62. \quad X^2Y &= 36 \\ Y &= X^2 - 5 \end{aligned}$$

$$\begin{aligned} 63. \quad X^2Y &= -4 \\ Y &= X^2 - 5 \end{aligned}$$

$$\begin{aligned} 64. \quad X^2Y &= 36 \\ Y &= X^2 + 5 \end{aligned}$$

ANSWERS 3.10

- p. 473-483:
1. $(-5, 25)$, $(2, 4)$;
 2. $(5, 25)$, $(-4, 16)$;
 3. $(4, 12)$, $(-2, 0)$;
 4. $(-5, 5)$, $(4, 32)$;
 5. $(6, 0)$, $(2, -8)$;
 6. $(6, 20)$, $(-1, 6)$;
 7. $(4, -8)$;
 8. $(1, 3)$;
 9. No Solution;
 10. No Solution;
 11. $(6, 19)$, $(-1, -2)$;
 12. $(-7, 24)$, $(2, 6)$;
 13. $(-5, -10)$, $(2, 11)$;
 14. $(-5, -5)$, $(2, 16)$;
 15. $(2, 4)$;
 16. $(-2, 8)$;
 17. $(4, 0)$, $(-2, 12)$;
 18. $(-3, -3)$, $(2, 12)$;
 19. No Solution;
 20. No Solution;
 21. $(6, -2)$, $(-2, 6)$;
 22. $(5/2, 2)$, $(-1, -5)$;
 23. $(-7/3, -9)$, $(3, 7)$;
 24. $(-7/3, -12)$, $(4, 7)$;
 25. $(-5/2, -12)$, $(6, 5)$;
 26. $(-3/2, 8)$, $(-4, 3)$;
 27. $(0, -4)$, $(4, 0)$;
 28. $(4, 3)$, $(-3, -4)$;
 29. $(3, 1)$, $(1, -3)$;
 30. $(-5, -5)$, $(1, 7)$;
 31. $(8, -11)$, $(-4, 13)$;
 32. $(-4, -17)$, $(7, 16)$;
 33. $(5, 3)$;
 34. $(6, -2)$;
 35. $(7, 5)$;
 36. $(3, 4)$;
 37. $(7, 16)$, $(-1, -8)$;
 38. $(8, 19)$, $(-2, -11)$;
 39. $(-26, -30)$, $(2, -2)$;
 40. $(-29, -18)$, $(11, 2)$;
 41. $(2, -1/2)$, $(-5, -4)$;
 42. $(8, 3)$, $(10, 5)$;
 43. $(5, 3)$, $(5, -3)$, $(-5, 3)$, $(-5, -3)$;
 44. $(3, \sqrt{5})$, $(3, -\sqrt{5})$, $(-3, \sqrt{5})$, $(-3, -\sqrt{5})$;
 45. $(0, \sqrt{3})$, $(0, -\sqrt{3})$;
 46. No Solution;
 47. $(4, 0)$, $(-4, 0)$;
 48. $(0, 2)$, $(0, -2)$;
 49. $(4\sqrt{3}, 2\sqrt{19})$, $(4\sqrt{3}, -2\sqrt{19})$, $(-4\sqrt{3}, 2\sqrt{19})$, $(-4\sqrt{3}, -2\sqrt{19})$;
 50. $(\sqrt{7}, 2)$, $(\sqrt{7}, -2)$, $(-\sqrt{7}, 2)$, $(-\sqrt{7}, -2)$;
 51. $(-1, -8)$, $(6, 13)$;
 52. $(-6, -11)$;
 53. $(4, 9)$;
 53. $(2, 1)$, $(0, -3)$;
 54. $(-4, 3)$;
 54. $(-8, 1)$;
 55. $(1, -5)$, $(4, 1)$;
 56. $(2, 1)$, $(1, -1)$;
 57. $(7, -1)$, $(-1, 7)$, $(-7, 1)$, $(1, -7)$;
 58. $(7, 1)$, $(1, 7)$, $(-7, -1)$, $(-1, -7)$;
 59. $(-7/2, 17/2)$, $(17/2, -7/2)$, $(-17/2, 7/2)$, $(7/2, -17/2)$;
 60. $(7/2, 17/2)$, $(17/2, 7/2)$, $(-17/2, -7/2)$, $(-7/2, -17/2)$;
 61. $(2, -9)$, $(-2, -9)$, $(3, -4)$, $(-3, -4)$;
 62. $(3, 4)$;
 63. $(-3, 4)$;
 63. $(1, -4)$, $(-1, -4)$, $(2, -1)$, $(-2, -1)$;
 64. $(2, 9)$, $(-2, 9)$.

Dr. Robert J. Rapalje

More FREE help available from my website at www.mathinlivingcolor.com

ANSWERS TO ALL EXERCISES ARE INCLUDED AT THE END OF THIS PAGE