4.01 *Exponential Functions/ Definition of Logarithms*

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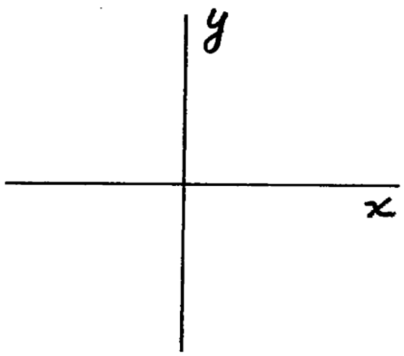
In a previous chapter, you graphed polynomial functions such as $Y = X^2$ or $Y = X^3$ in which the variable **X** is the base number and the power is a constant. In this section, you will be dealing with **exponential functions**, such as $Y = 2^X$ or $Y = 3^X$. These are called **exponential functions** (or **power functions**) because the variable is in the exponent (or power) of the equation. Begin by graphing the following equations by simple point plotting.

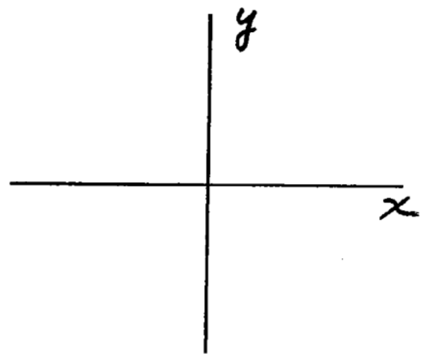
1. $Y = 2^X$

X	Y
0	
1	
2	
3	
−1	
−2	
−3	



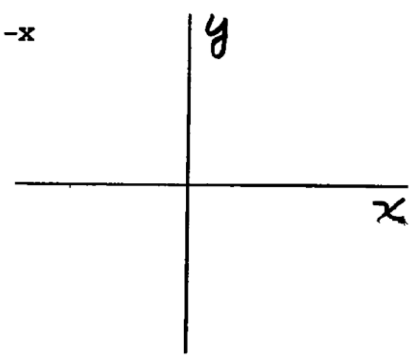
2. $Y = 3^X$

X	Y
0	
1	
2	
−1	
−2	



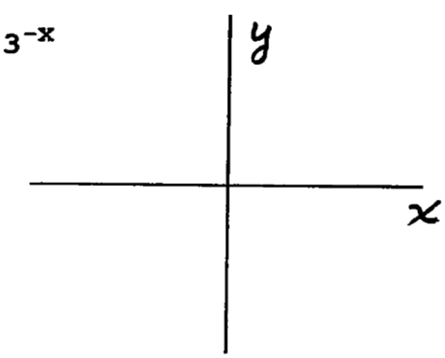
3. $Y = 2^{-X}$

X	Y
0	
1	
2	
3	
−1	
−2	
−3	



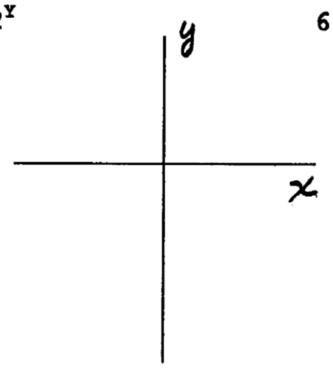
4. $Y = 3^{-X}$

X	Y
0	
1	
2	
−1	
−2	



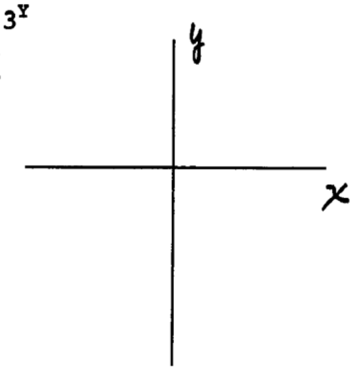
5. $X = 2^Y$

X	Y
	0
	1
	2
	3
	-1
	-2
	-3



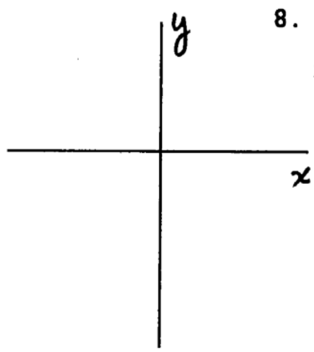
6. $X = 3^Y$

X	Y
	0
	1
	2
	-1
	-2



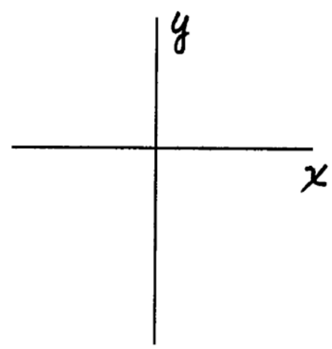
7. $X = 2^{-Y}$

X	Y
	0
	1
	2
	3
	-1
	-2
	-3



8. $X = 3^{-Y}$

X	Y
	0
	1
	2
	-1
	-2



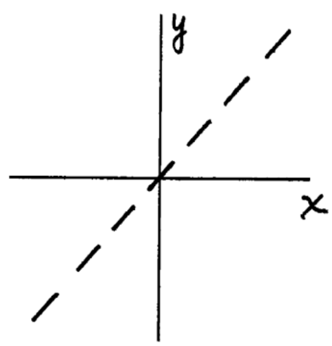
9. What did you notice about the equations $Y = 2^X$ and $X = 2^Y$? Graph both on the same graph, and explain how the graphs are related. Use the dotted line $Y=X$ to illustrate symmetry.

$Y = 2^X$

X	Y

$X = 2^Y$

X	Y



In the previous exercise, you noticed that the equation **$X = 2^Y$** is the inverse of **$Y = 2^X$** . Moreover, **$X = 2^Y$** is a function. Unfortunately, since **Y** is in the exponent of **$X = 2^Y$** , it is not possible to solve for **Y** directly in order to express it in the form **$Y = f(X)$**. Therefore, it is convenient to make the definition that **$X = 2^Y$** is equivalent to **$Y = \log_2 X$** . This, then is the definition of a logarithm:

DEFINITION OF LOGARITHM

$$Y = \log_2 X \quad \Leftrightarrow \quad 2^Y = X$$

$$Y = \log_b X \quad \Leftrightarrow \quad b^Y = X$$

assuming that **$b>0$, $b \neq 1$**, and **$X>0$**.

Where **b,** a positive number not equal to 1, is called the **base** of the logarithm, and **X** is a positive number, the value of **$\log_b X$** is the exponent to which **b** must be raised in order to obtain the value of **X**. So, to answer the question, **"What is a logarithm?"**, a logarithm is like an exponent. In the next section, you will discover some special properties of logarithms, some properties that resemble (and in fact are proven by) the laws of exponents.

You need to practice this definition (converting from **logarithmic** to **exponential form**) until it becomes a part of you:

LOGARITHMIC FORM		EXPONENTIAL FORM
$Y = \log_2 X$	$\Leftrightarrow$	$2^Y = X$
$Y = \log_b X$	$\Leftrightarrow$	$b^Y = X$
$\log_b X = Y$	$\Leftrightarrow$	$b^Y = X$

Fill in the blanks:

LOGARITHMIC FORM		**EXPONENTIAL FORM**
$Y = \log_b X$	$\Leftrightarrow$	______________
$Y = \log_b X$	$\Leftrightarrow$	______________
$\log_b X = Y$	$\Leftrightarrow$	______________
$\log_b X = Y$	$\Leftrightarrow$	______________

In the beginning exercises, first convert the problem from **logarithmic form** to **exponential form** and solve for the unknown, usually by inspection. By the way, be sure you are familiar with the **perfect powers.**

In 1 - 72, solve for the unknown.

1. $\log_2 8 = X$

   $2^x = 8$

   $x =$ ____

2. $\log_3 9 = X$
3. $\log_3 3 = X$
4. $\log_3 1 = X$
5. $\log_3 1/3 = X$
6. $\log_3 1/9 = X$
7. $\log_4 16 = X$
8. $\log_2 1 = X$
9. $\log_2 32 = X$

10. $\log_2 1/2 = X$

11. $\log_2 1/8 = X$

12. $\log_7 49 =$

13. $\log_4 64 =$

14. $\log_5 125 =$

15. $\log_5 \sqrt{5} = X$

$5^x = \sqrt{5}$

$5^x = 5^{1/2}$

$x =$ ______

16. $\log_2 \sqrt{2} =$

17. $\log_2 \sqrt[3]{2} =$

18. $\log_3 \sqrt[4]{3} =$

19. $\log_2 2\sqrt{2} = X$

$2^x = 2\sqrt{2}$

$2^x = 2^1 \cdot 2^{1/2}$

$2^x = 2^{3/2}$

$x =$ ______

20. $\log_2 2\sqrt[3]{2}$

21. $\log_3 9\sqrt{3}$

22. $\log_8 4 = X$

$8^x = 4$

[Notice that both 8 and 4 are powers of 2: $2^3 = 8$ and $2^2 = 4$]

$(2^3)^x = 2^2$

$2^{3x} = 2^2$ → $3x =$ ____

$x =$ ____

23. $\log_4 8 = X$

24. $\log_{27} 9 =$

25. $\log_2 X = 3$

$2^3 = x$

$x =$ ____

26. $\log_4 X = 2$

27. $\log_3 X = -1$

28. $\log_5 X = -2$

29. $\log_2 X = -3$

30. $\log_4 X = -3$

31. $\log_4 X = 1/2$

32. $\log_9 X = 3/2$

33. $\log_8 X = -2/3$

34. $\log_9 X = -1/2$

35. $\log_8 X = 0$

36. $\log_{10} X = 0$

37. $\log_b 8 = 3$

$b^3 = 8$

$b =$ ____

38. $\log_b 64 = 2$

$b^2 = 64$

$b =$ ____

[Remember, $b > 0$]

39. $\log_b 64 = 3$

40. $\log_b 27 = 3$

41. $\log_b 8 = -3$

$b^{-3} = 8$

$\frac{1}{b^3} = \frac{8}{1}$

$8b^3 = 1$

$b^3 =$ ____

$b =$ ____

42. $\log_b 16 = -2$

43. $\log_b 1/16 = -2$

44. $\log_b 1/27 = -3$

45. $\log_b 3 = 2$

46. $\log_b 2 = 3$

47. $\log_b 9 = 2/3$

48. $\log_b 64 = 3/2$

49. $\log_3 X = -4$

50. $\log_9 81 = X$

51. $\log_8 4 =$

52. $\log_8 X = -2/3$

53. $\log_{10} X = 2$

54. $\log_4 2 = X$

55. $\log_b 3 = 3$

56. $\log_{10} 1000 = X$

57. $\log_{64} X = 2/3$

58. $\log_{64} X = -2/3$

59. $\log_b 1/8 = -3$

60. $\log_b 9 = -2/3$

61. $\log_8 1 = X$

62. $\log_8 X = -1$

63. $\log_b 8 = 1$

64. $\log_8 X = 0$

65. $\log_{25} X = -1$

66. $\log_{25} X = 1/2$

67. $\log_{10} 0.1 = X$

68. $\log_{10} 0.01 = X$

69. $\log_{10} 0.001 = X$

70. $\log_{10} X = 0$

71. $\log_{10} X = 1$

72. $\log_b 1000 = -3$

CHAPTER FOUR

4.01 p. 493–495:

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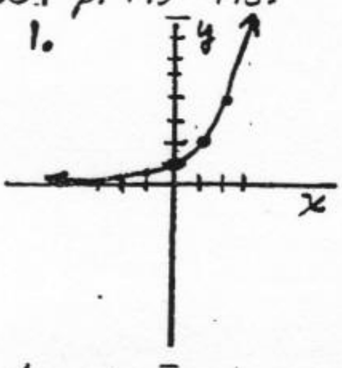

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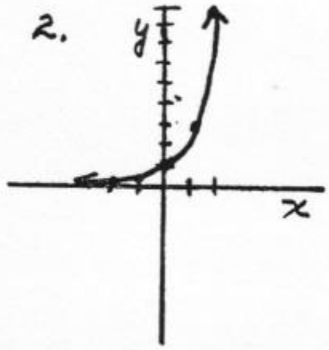

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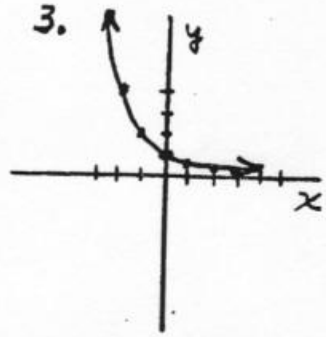

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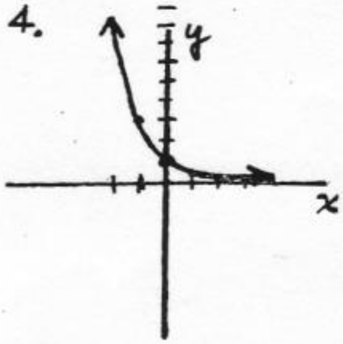

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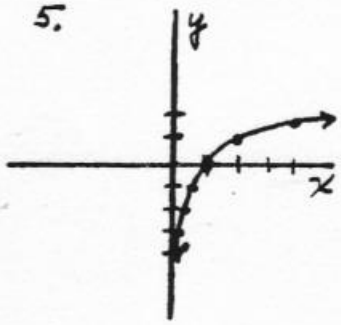

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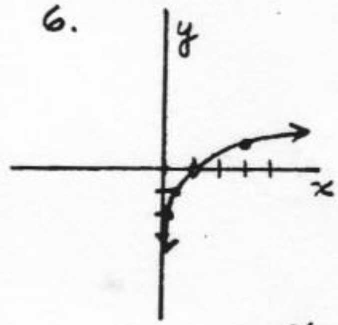

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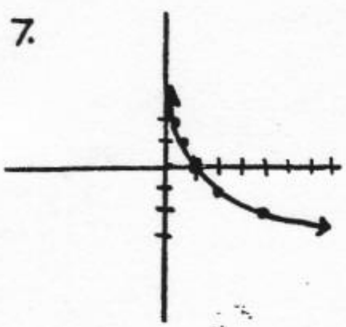

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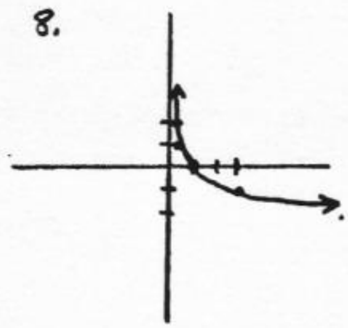

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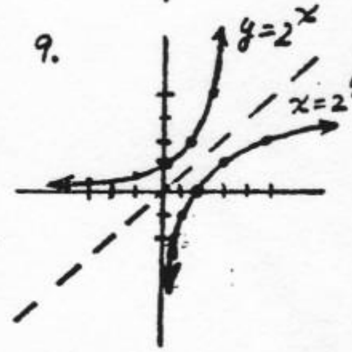

4.01 p. 496-501:

1. 3 2. 2 3. 1 4. 0 5. −1 6. −2 7. 2 8. 0 9. 5
10. −1 11. −3 12. 2 13. 3 14. 3 15. $\frac{1}{2}$ 16. $\frac{1}{2}$ 17. $\frac{1}{3}$ 18. $\frac{1}{4}$
19. $\frac{3}{2}$ 20. $\frac{4}{3}$ 21. $\frac{5}{2}$ 22. $\frac{2}{3}$ 23. $\frac{3}{2}$ 24. $\frac{2}{3}$ 25. 8 26. 16 27. $\frac{1}{3}$

4.01 p. 496-501:

28. $\frac{1}{25}$ 29. $\frac{1}{8}$ 30. $\frac{1}{64}$ 31. 2 32. 27 33. $\frac{1}{4}$ 34. $\frac{1}{3}$

35. 1 36. 1 37. 2 38. 8 39. 4 40. 3 41. $\frac{1}{2}$ 42. $\frac{1}{4}$

43. 4 44. 3 45. $\sqrt{3}$ 46. $\sqrt[3]{2}$ 47. 27 48. 16 49. $\frac{1}{81}$

50. 2 51. $\frac{2}{3}$ 52. $\frac{1}{4}$ 53. 100 54. $\frac{1}{2}$ 55. $\sqrt[3]{3}$ 56. 3 57. 16

58. $\frac{1}{16}$ 59. 2 60. $\frac{1}{27}$ 61. 0 62. $\frac{1}{8}$ 63. 8 64. 1 65. $\frac{1}{25}$

66. 5 67. -1 68. -2 69. -3 70. 1 71. 10 72. $\frac{1}{10}$

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