ASYMPTOTES for Rational Functions

Dr. Robert J. Rapalje

I. Vertical: When equation is of form $y = \frac{P(x)}{Q(x)}$, Set denominator equal to zero.

II. Horizontal or Oblique: A rational function in form $y = \frac{P(x)}{Q(x)}$ will have exactly one horizontal or oblique asymptote (never both) depending upon the following cases:

$$y = \frac{a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \dots}{b_0x^n + b_1x^{n-1} + b_2x^{n-2} + \dots}$$

CASE A. If $m<n$, (degree of denom. is larger) then $y=0$ (or the x axis!) is <u>the</u> horizontal asymptote.

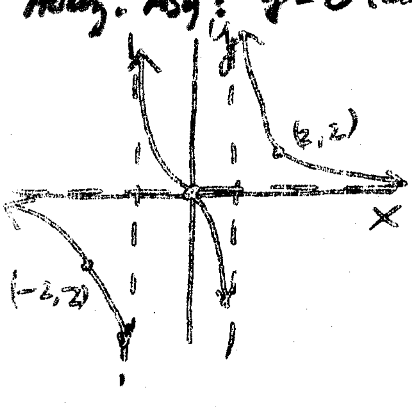
CASE B. If $m=n$, (degrees the same) then $y = \frac{a_0}{b_0}$ is the horizontal asymptote.

CASE C. If $m>n$, (degree of numerator is one larger) then there will be one oblique asymptote. Use polynomial division

1. $y = \frac{3x}{x^2-1}$

Vert. Asy: $x = \pm 1$

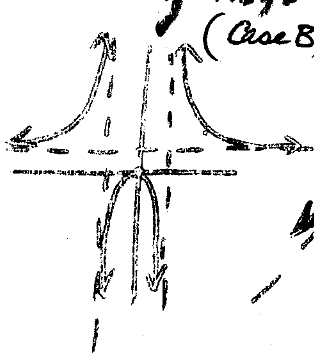
Horiz. Asy: $y=0$ (Case A)



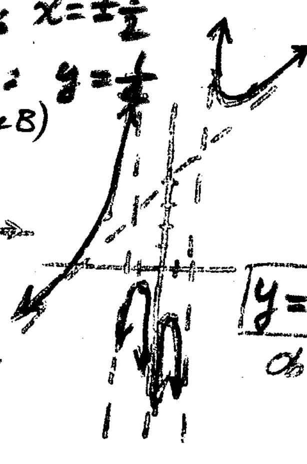
2. $y = \frac{x^2}{4x^2-1}$

Vert. Asy: $x = \pm\frac{1}{2}$

Horiz. Asy: $y = \frac{1}{4}$ (Case B)



3. $y = \frac{x^4+4x^3+5}{x(x^2-4)}$



Vert. Asy: $x = 0, 2, -2$

oblique: Case C

$$\begin{array}{r} x+4 \quad r.\ x^2+16x+5 \\ x^3-4x \,\big)\, \overline{x^4+4x^3+5} \\ \underline{x^4 \quad -4x^2} \\ 4x^3+4x^2+5 \\ \underline{4x^3 \quad -16x} \\ 4x^2+16x+5 \end{array}$$

$\boxed{y = x+4}$

Oblig. Asym.

x	1	-1	3	-3
y	[illegible]	$-\frac{7}{3}$	[illegible]	[illegible]