Math in Living C O L O R !!

1.04 Linear Equations

Intermediate Algebra: One Step at a Time, **Page 41: #3, 5, 6, 7, 8.**

Dr. Robert J. Rapalje, Retired
Central Florida, USA

See Section 1.04 with explanations, examples, and exercises, coming soon!

Solving Linear Equations

Conditional Equations

When solving an equation for x, you must get all the x terms on one side of the equation, all the non-x terms on the other side of the equation. What usually happens is that you get x = ___ some value or values that are solutions for the equation. When this happens, the equation is called a CONDITIONAL EQUATION.

Identities and Contradictions

However, sometimes in solving for x, all the variables subtract out. When this happens, there are no variables left in the equation, so you can't solve for x. However, an equation with no variables must be either TRUE or FALSE.

If the equation is true, then it is an IDENTITY, and the solution is "All REAL values of x."

If the equation is false, then the equation is a CONTRADICTION, and there is NO SOLUTION.

3. Solve for x: $6(x+3)=3(6-2x)+4x$.

Solution: First, remove parentheses by the distributive property.

$$6x+18=18-6x+4x$$

Next, combine like terms on the right side.

$$6x+18=18-2x$$

Get all the x terms on the left side by adding $+2x$ to each side.

$$\begin{array}{r} 6x+18=18-2x \\ \underline{+2x \qquad\qquad +2x} \\ 8x+18=18 \end{array}$$

Next, subtract 18 from each side:

$$\begin{array}{r} 8x+18 = 18 \\ \underline{-18 \quad -18} \\ 8x=0 \end{array}$$

Divide by 8: $x=0$

Since you can solve for x and get at least one solution, this is called a CONDITIONAL EQUATION, and the solution is $x=0$.

5. Solve for x: $6(x+3)-3(6-2x)=12x$.

Solution: First, remove parentheses by the distributive property.

$$6x+18-18+6x=12x$$

Next, combine like terms on the left side.

$$6x\cancel{+18}\cancel{-18}+6x=12x$$

$$12x=12x$$

This equation is ALWAYS TRUE, since $12x$ always equals $12x$, so this is an IDENTITY! The answer is therefore "All values of x," or "All REAL values of x."

NOTE: You could have continued one more step by subtracting $12x$ from each side. This would leave the equation $0=0$. Since there are NO variables left in the equation, it will be either always true or always false.

6. Solve for x: $x(x-6)=4-x(2-x)$.

Solution: First, remove parentheses by the distributive property.

$$x^2-6x=4-2x+x^2$$

You can subtract x^2 from each side, eliminating all the x^2 terms.

$$\begin{array}{r} x^2-6x=4-2x+x^2 \\ \underline{-x^2 \qquad\qquad\quad -x^2} \\ -6x=4-2x \end{array}$$

Add $+2x$ to each side:

$$\begin{array}{l} -6x=4-2x \\ \underline{+2x \qquad +2x} \\ -4x=4 \end{array}$$

Divide by -4: $x=-1$

Since you can solve for x and get at least one solution, this is called a CONDITIONAL EQUATION, and the solution is $x=-1$.

7. Solve for x: $x(x-2)=4-x(2-x)$.

Solution: First, remove parentheses by the distributive property.

$$x^2-2x=4-2x+x^2$$

You can subtract x^2 and add $+2x$ to each side,.

$$\begin{array}{r} x^2-2x=4-2x+x^2 \\ \underline{-x^2+2x \qquad +2x-x^2} \end{array}$$

All the variables subtract out, leaving:

$$0=4$$

This equation is FALSE! Since the equation is false, the equation is a CONTRADICTION, and there is NO SOLUTION.

8. Solve for x: $x(3x-8)=12x-3x(4-x)$.

Solution: First, remove parentheses by the distributive property.

$$3x^2-8x=12x-12x+3x^2$$

You can subtract $3x^2$ from each side, eliminating all the $3x^2$ terms.

$$\begin{array}{r} 3x^2-8x=\cancel{12x}\;\cancel{-12x}+3x^2 \\ \underline{-3x^2 \qquad\qquad\qquad\qquad -3x^2} \\ -8x=0 \end{array}$$

Divide by -8:

$$\frac{-8x}{-8}=\frac{0}{-8}$$

$$x=0$$

Since you can solve for x and get at least one solution, this is called a **CONDITIONAL EQUATION**, and the solution is $x=0$.

P. 46: 9. | 2 x – 3 | = | x + 6 |

When solving an absolute value equal to another absolute value, remember that there are actually four possible equations to solve. Fortunately, two of these four equations are the same as the other two, so you actually only need to solve two equations.

First Solution: Positive = Positive

2 x – 3 = x + 6
2 x – x = 6 + 3
x = 9

Second Solution: Positive = Negative

2 x – 3 = – (x + 6)
2 x – 3 = – x – 6
2 x + x = – 6 + 3
3x = – 3
x = – 1

P. 46: #9 continued | 2 x – 3 | = | x + 6 |

Check solutions: $|2x-3| = |x+6|$

$x = 9$ $\quad |2(9)-3| = |(9)+6|$

$|(18)-3| = |(9)+6|$

$|15| = |15|$

$x = -1$ $\quad |2(-1)-3| = |(-1)+6|$

$|(-2)-3| = |(-1)+6|$

$|-5| = |5|$

P.48. #9. Solve for x: $a(x+b)=c(x+d)$.

Solution: First, remove parentheses by the distributive property.

$$ax+ab=cx+cd$$

Next, get all the x terms on the left side by subtracting cx from each side. At the same time, subtract ab to each side to get all the non-x terms on the right side of the equation

$$\begin{array}{rcl} ax\cancel{+ab} & = & \cancel{cx}+cd \\ -cx\cancel{-ab} & & \cancel{-cx}-ab \\ \hline ax-cx & = & cd-ab \end{array}$$

Now, factor the common factor of x:

$$x(a-c) = cd - ab$$

Finally, since the x has been multiplied by $(a-c)$, you must divide both sides of the equation by $(a-c)$.

$$\frac{x \bullet \cancel{(a-c)}}{\cancel{(a-c)}} = \frac{cd-ab}{(a-c)}$$

$$x = \frac{cd-ab}{a-c}$$

NOTE: Don't be tempted to divide out the a or the c! These are "terms"! Never divide out TERMS--only FACTORS!!

P.48. #10. Solve for x: $a(x-b)=c(d-x)$.

Solution: First, remove parentheses by the distributive property.

$$ax-ab=cd-cx$$

Next, get all the x terms on the left side by adding cx from each side. At the same time, add ab to each side to get all the non-x terms on the right side of the equation

$$\begin{array}{rcl} ax-ab & = & cd-cx \\ +cx\cancel{+ab} & & +ab\cancel{+cx} \\ \hline ax+cx & = & cd+ab \end{array}$$

Now, factor the common factor of x:

$$x(a+c) = cd + ab$$

Finally, since the x has been multiplied by $(a+c)$, you must divide both sides of the equation by $(a+c)$.

$$\frac{x\bullet\cancel{(a+c)}}{\cancel{(a+c)}} = \frac{ab+cd}{(a+c)}$$

$$x = \frac{ab+cd}{a+c}$$

NOTE: Don't be tempted to divide out the a or the c! These are "terms"! Never divide out TERMS--only FACTORS!!

P.49. #12. Solve for x: $Y - a = m(x - b)$.

Solution: First, remove parentheses by the distributive property.

$$Y - a = mx - mb$$

Next, notice that there is only one x term, which is on the right side of the equation. Therefore, you must get the non- x terms all on the left side by adding mb from each side.

$$\begin{array}{rcl} Y - a & = & mx - mb \\ +mb & & \cancel{+mb} \\ \hline Y - a + mb & = & mx \end{array}$$

Finally, in order to solve for x,

$$Y - a + mb = mx$$

you must divide both sides of the equation by m.

$$\frac{Y - a + mb}{m} = \frac{\cancel{m}x}{\cancel{m}}$$

$$x = \frac{Y - a + mb}{m}$$

NOTE: Don't be tempted to divide out the m! The m in the numerator is a "term"! Never divide out TERMS--only FACTORS!!

P. 50. #22. $C = 2\pi r$, solve for r.

Solution: Since you are solving for r, and the r has been multiplied by 2π, you must "undo" the multiplication, by dividing both sides by 2π:

$$\frac{C}{2\pi} = \frac{2\pi r}{2\pi}$$

$$\frac{C}{2\pi} = \frac{\cancel{2\pi}r}{\cancel{2\pi}}$$

$$r = \frac{C}{2\pi}$$

P. 50. #27. $A = \frac{1}{2}bh$, solve for h.

Solution: Since there is a denominator of 2, multiply both sides by 2 to clear the fraction!

$$2 \bullet A = \cancel{2} \bullet \frac{1}{\cancel{2}} bh$$

$$2A = bh$$

Next, remember that you are solving for h, and the h has been multiplied by b. In order to "undo" the multiplication, you must divide both sides by b:

$$\frac{2A}{b} = \frac{\cancel{b}h}{\cancel{b}}$$

$$h = \frac{2A}{b}$$

p. 50. #29. $V = \frac{1}{3}\pi r^2 h$, solve for h.

Solution: Since there is a denominator of 3, multiply both sides by 3 to clear the fraction!

$$3 \bullet V = 3 \bullet \frac{1}{3}\pi r^2 h$$

$$3V = \pi r^2 h$$

Next, remember that you are solving for h, and the h has been multiplied by π and r^2. In order to "undo" the multiplication, you must divide both sides by π and r^2:

$$\frac{3V}{\pi r^2} = \frac{\cancel{\pi}\,\cancel{r^2}\,h}{\cancel{\pi}\,\cancel{r^2}}$$

$$\frac{3V}{\pi r^2} = h$$

$$h = \frac{3V}{\pi r^2}$$

Extra Problem (from Chris).

Solve for x: $a(x-b)=cx+ab$.

Solution: First, remove parentheses by the distributive property.

$$ax-ab=cx+ab$$

Next, get all the x terms on the left side by subtracting cx from each side. At the same time, add $+ab$ to each side to get all the non-x terms on the right side of the equation

$$\begin{array}{rcl} ax-ab & = & cx+ab \\ -cx+ab & & -cx+ab \\ \hline ax-cx & = & 2ab \end{array}$$

Now, factor the common factor of x:

$$x(a-c) = 2ab$$

Finally, since the x has been multiplied by $(a-c)$, you must divide both sides of the equation by $(a-c)$.

$$\frac{x \bullet \cancel{(a-c)}}{\cancel{(a-c)}} = \frac{2ab}{(a-c)}$$

$$x = \frac{2ab}{a-c}$$

Extra Problem

Solve for x: $1 - 3xy = 7(5xz + y)$.

Solution: First, remove parentheses by the distributive property.

$$1 - 3xy = 35xz + 7y$$

Next, get all the x terms on the right side by adding $3xy$ from each side. At the same time, subtract $7y$ from each side to get all the non-x terms on the left side of the equation

$$\begin{array}{rrcrr} 1 & \cancel{-3xy} & = & 35xz & \cancel{+7y} \\ -7y & \cancel{+3xy} & & +3xy & \cancel{-7y} \\ \hline 1-7y & & = & 35xz & +3xy \end{array}$$

Now, factor the common factor of x:

$$\begin{aligned} 1-7y &= 35xz + 3xy \\ 1-7y &= x(35z + 3y) \end{aligned}$$

Finally, since the x has been multiplied by $(35z + 3y)$, you must divide both sides of the equation by $(35z + 3y)$.

$$\frac{1-7y}{(35z + 3y)} = \frac{x\cancel{(35z + 3y)}}{\cancel{(35z + 3y)}}$$

$$x = \frac{1-7y}{35z + 3y}$$

NOTE: Don't be tempted to divide out the y! These are "terms"! Never divide out TERMS--only FACTORS!!