Math in Living C O L O R !!

1.05 Word Problems

Intermediate Algebra: One Step at a Time

Pages 51 - 76: #3,4,7,8,17,18,21,25,27,29,30,31,32,35,36,40,45,47,50,Extras

Dr. Robert J. Rapalje, Retired
Central Florida, USA

See Section 1.05 with explanations, examples, and exercises, coming soon!

P. 55:

3. Three numbers are such that the second number is 4 more than three times the first, and the third number is 12 less than the sum of the first two numbers. The sum of the three numbers is 44. Find the numbers.

Solution:

STEP 1: Let x =_____.

Let x = First number (since it is at the end of the first phrase!)

$3x + 4$ = Second number

$x + 3x + 4 - 12$ = Third number (sum of the first two numbers, less 12)

$4x - 8$ = Third number

STEP 2: Write the equation.

The sum of the three numbers is 44.

$x + 3x + 4 + 4x - 8 = 44$

STEP 3: Solve the equation.

$8x - 4 = 44$

$8x = 48$

$x = 6$

STEP 4: Answer the question.

$x = 6$ First number

$3x + 4 = 3(6) + 4 = 22$ Second number

$4x - 8 = 4(6) - 8 = 16$ Third number

STEP 5: Check.

Check: The sum of the numbers is 44.

$6 + 22 + 16 = 44$ It checks!!

P. 55:

4. Three numbers are such that the first number is 10 less than twice the second, and the second number is 4 more than three times the third. Twice the second number is equal to the sum of the first and third numbers. Find the numbers.

Solution:

STEP 1: Let x =_____.

Let x = Third number (since it is at the end of the first sentence!)
$3x + 4$ = Second number
$2(3x + 4) - 10$ = First number
$6x + 8 - 10$ = First number
$6x - 2$ = First number

STEP 2: Write the equation.

Twice the second number is equal to the sum of the first and third.
$2(3x + 4) = 6x - 2 + x$

STEP 3: Solve the equation.

$$6x + 8 = 6x - 2 + x$$
$$6x + 8 = 7x - 2$$
$$10 = x$$

STEP 4: Answer the question.

$x = 10$ Third number
$3x + 4 = 3(10) + 4 = 34$ Second number
$6x - 2 = 6(10) - 2 = 58$ First number

STEP 5: Check.

Check: Twice the second equals sum of the first and third.
$2(34) = 58 + 10$
$68 = 68$
It checks!!

P. 56:

7. Find two consecutive odd integers such that twice the second plus the first is 121.

Solution:

STEP 1: Let x =_____.

Let x = First number
$x + 2$ = Second number

STEP 2: Write the equation.

Twice the second number plus the first is 121.
$2(x + 2) + x = 121$

STEP 3: Solve the equation.

$$2x + 4 + x = 121$$
$$3x + 4 = 121$$
$$3x = 117$$
$$x = 39$$

STEP 4: Answer the question.

$x = 39$ First odd number
$x + 2 = 41$ Second odd number

STEP 5: Check.

Check: Twice the second plus the first is 121.
$2(41) + 39 = 121$
$82 + 39 = 121$
It checks!!

P. 56:

8. Three consecutive integers are such that the first, plus twice the second, plus three times the third is equal to 200. Find the numbers.

SOLUTION:

STEP 1: Let x =_____.

Let x = first number
x+1 = second number (since these are consecutive integers!)
x+2 = third number

STEP 2: Write the equation.

First + 2 * Second + 3 * Third = 200
(x) + 2 * (x+1) + 3 * (x+2) = 200

STEP 3: Solve the equation.

$$x + 2x + 2 + 3x + 6 = 200$$
$$6x + 8 = 200$$

Subtract 8 from each side:

$$6x + 8 - 8 = 200 - 8$$
$$6x = 192$$

Divide both sides by 6:

$$x = 192/6 = 32$$

STEP 4: Answer the question.

$x = 32$ First number
$x + 1 = 33$ Second number
$x + 2 = 34$ Third number

STEP 5: Check.

Check: (x) + 2 * (x+1) + 3 * (x+2) = 200
32 + 2 * 33 + 3 * 34
32 + 66 + 102 = 200 It checks!!

P. 59:

17. The length of a rectangle is three less than five times the width. The perimeter is ten times the width. Find the dimensions and perimeter of the rectangle.

SOLUTION:

STEP 1: Let x =_____.

Let x = width of the rectangle
5x-3 = length of the rectangle
10x = Perimeter

STEP 2: Write the equation.

2( ) + 2( ) = Perimeter
2(x) + 2(5x - 3) = 10x

STEP 3: Solve the equation.

$$2x + 10x - 6 = 10x$$
$$12x - 6 = 10x$$

Subtract 12x from each side:

$$12x - 12x - 6 = 10x - 12x$$
$$-6 = -2x$$

Divide both sides by -2:

$$x = 3$$

STEP 4: Answer the question.

$x = 3$ Width
$5x - 3 = 5(3) - 3 = 12$ Length
$10x = 30$ Perimeter

STEP 5: Check.

Check: $2W + 2L = P$
$2(3) + 2(12) = 30$
$6 + 24 = 30$ It checks!!

P. 59:

18. The perimeter of a rectangle is 46. Twice the length is 4 more than five times the width. Find the length and width of the rectangle.

SOLUTION:

STEP 1: Let x =_____.

Let x = width of the rectangle
5x+4 = two lengths of the rectangle

STEP 2: Write the equation.

2(W) + 2(L) = Perimeter
2(x) + 5x+4 = 46

STEP 3: Solve the equation.

$$2x + 5x + 4 = 46$$
$$7x + 4 = 46$$

Subtract 4 from each side:

$$7x + 4 - 4 = 46 - 4$$
$$7x = 42$$

Divide both sides by 7:

$$x = 6$$

STEP 4: Answer the question.

$x = 6$ Width
$5x + 4 = 5(6) + 4 = 34 = 2 \cdot$ Length
$17 =$ Length

STEP 5: Check.

Check: $2W + 2L = P$
$2(6) + 2(17) = 46$
$12 + 34 = 46$ It checks!!

P. 61:

21. A certain number of quarters, four times as many pennies as quarters, and 6 more dimes than pennies are worth $3.36. How many of each coin are there?

SOLUTION:

STEP 1: Let x =_____.

Let x = number of quarters
4x = number of pennies
4x+6 = number of dimes

Type	No Coins	Ea	Values
Q	x	25	25(x)
P	4x	1	1(4x)
D	4x+6	10	10(4x+6)
			336¢

STEP 2: Write the equation from the last column of the chart above.

$$25(x) + 1(4x) + 10(4x+6) = 336$$

STEP 3: Solve the equation.

$$25x + 4x + 40x + 60 = 336$$
$$69x + 60 = 336$$
$$69x = 276$$
$$x = 4$$

STEP 4: Answer the question.

x = 4 Quarters
4x = 4(4) = 16 Pennies
4x+6 = 16+6 = 22 Dimes

STEP 5: Check.

Check:

4	Quarters	$1.00
16	Pennies	.16
22	Dimes	2.20
	TOTAL:	$3.36 It checks!!

P. 62:

25. A box contains $6.60 in nickels, dimes, and quarters. There are three times as many nickels as quarters, and the number of dimes is 4 less than the number of nickels. How many of each coin are there?

SOLUTION:

STEP 1: Let x =_____.
Let x = number of quarters
3x = number of nickels
3x-4 = number of dimes

Type	No Coins	Ea	Values
Q	x	25	25(x)
N	3x	5	5(3x)
D	3x-4	10	10(3x-4)
			660¢

STEP 2: Write the equation from the last column of the chart above.
25(x) + 5(3x) + 10(3x-4) = 660

STEP 3: Solve the equation.
25 x + 15x + 30x - 40 = 660
70x - 40 = 660
70x = 700
x = 10

STEP 4: Answer the question.
x = 10 Quarters
3x =3(10) = 30 Nickels
3x-4 = 30-4= 26 Dimes

STEP 5: Check.
Check: 10 Quarters $2.50
30 Nickels 1.50
26 Dimes 2.60
TOTAL: $6.60 It checks!!

P. 63:

27. A certain number of pennies, four times as many dimes as pennies, and a number of quarters which is 16 less than twice the number of dimes, are worth $24.92. How many of each coin are there?

SOLUTION:

STEP 1: Let x =_____.

Let x = number of pennies

4x = number of dimes

2(4x)-16 = number of quarters

8x - 16 = number of quarters

Type	No Coins	Ea	Values
P	x	1	1(x)
D	4x	10	10(4x)
Q	8x-16	25	25(8x-16)
			2492¢

STEP 2: Write the equation from the last column of the chart above.

$1(x) + 10(4x) + 25(8x-16) = 2492$

STEP 3: Solve the equation.

$1x + 40x + 200x - 400 = 2492$

$241x - 400 = 2492$

$$\frac{241x}{241} = \frac{2892}{241}$$

$x = 12$

STEP 4: Answer the question.

x = 12 Pennies

4x =4(12) = 48 Dimes

8x-16 = 96-16 = 80 Quarters

STEP 5: Check.

Check:			
	12	Pennies	$ 0.12
	48	Dimes	4.80
	80	Quarters	20.00
		TOTAL:	$24.92 It checks!!

P. 64:

29. A box contains nickels, dimes, and quarters worth $12.60. The number of dimes is 2 less than three times the number of nickels, and the number of quarters is 4 less than twice the number of dimes. How many of each coin are there?

SOLUTION:

STEP 1: Let x =_____.

Let x = number of nickels

3x – 2 = number of dimes

2(3x– 2) –4 = number of quarters

6x – 4 – 4 or 6x – 8 = number of quarters

Type	No Coins	Ea	Values
N	x	5	5(x)
D	3x – 2	10	10(3x – 2)
Q	6x – 8	25	25(6x – 8)
			1260¢

STEP 2: Write the equation from the last column of the chart above.

$5(x) + 10(3x - 2) + 25(6x - 8) = 1260$

STEP 3: Solve the equation.

$5x + 30x - 20 + 150x - 200 = 1260$

$185x - 220 = 1260$

$185x = 1480$

$$\frac{185x}{185} = \frac{1480}{185}$$

$x = 8$

STEP 4: Answer the question.

x = 8 Nickels

3x – 2 = 3(8) – 2 = 22 Dimes

6x – 8 = 6(8) – 8 = 40 Quarters

STEP 5: Check.

Check:

8	Nickels	$ 0.40
22	Dimes	2.20
40	Quarters	10.00
	TOTAL:	$12.60 It checks!!

P. 64:

30. A box contains nickels, dimes, and quarters worth $69.50. The number of nickels is 10 more than twice the number of dimes. There are as many quarters as nickels and dimes combined. How many of each coin are there?

SOLUTION:

STEP 1: Let x =_____.

Let x = number of dimes

2x+10 = number of nickels

x + 2x+10 = number of quarters

3x + 10 = number of quarters

Type	No Coins	Ea	Values
D	x	10	10(x)
N	2x+10	5	5(2x+10)
Q	3x+10	25	25(3x+10)
			6950¢

STEP 2: Write the equation from the last column of the chart above.

$10(x) + 5(2x+10) + 25(3x+10) = 6950$

STEP 3: Solve the equation.

$$10x + 10x + 50 + 75x + 250 = 6950$$
$$95x + 300 = 6950$$
$$95x = 6650$$
$$\frac{95x}{95} = \frac{6650}{95}$$
$$x = 70$$

STEP 4: Answer the question.

x = 70 Dimes

2x+10 = 2(70) +10 = 150 Nickels

x+2x+10 = 70 + 150 = 220 Quarters

STEP 5: Check.

Check:	70	Dimes	$ 7.00
	150	Nickels	7.50
	220	Quarters	55.00
		TOTAL:	$69.50 It checks!!

P. 65:

31. A sum of money was invested at 8% simple interest, and three times this much at 10%. The total interest earned for the year was $190. How much was invested at each rate.

SOLUTION:

STEP 1: Let x =_____.

Let x = principle invested 8%
3x = principle invested at 10%

	Principle	Rate	Interest
8%	x	.08	.08(x)
10%	3x	.10	.10(3x)
			$190

STEP 2: Write the equation from the last column of the chart above.

$.08(x) + .10(3x) = 190$

STEP 3: Solve the equation.

$$.08x + .30x = 190$$
$$.38x = 190$$
$$\frac{.38x}{.38} = \frac{190}{.38}$$
$$x = \$500$$

STEP 4: Answer the question.

x = $ 500 at 8%
3x = $1500 at 10%

STEP 5: Check.

Check: $ 500 @ 8% = $ 40
$1500 @ 10% = $150
TOTAL: $190 It checks!!

P. 66:

32. A sum of money was invested at 12% simple interest, and $1000 less than this at 10%. The total interest earned for the year was $1000. How much was invested at each rate.

SOLUTION:

STEP 1: Let x =_____.

Let x = principle invested 12%
x−1000 = principle invested at 10%

	Principle	Rate	Interest
12%	x	.12	.12(x)
10%	x−1000	.10	.10(x-1000)
			$1000

STEP 2: Write the equation from the last column of the chart above.

$.12(x) + .10(x-1000) = 1000$

STEP 3: Solve the equation.

$$.12x + .10x - 100 = 1000$$
$$.22x - 100 = 1000$$
$$.22x = 1100$$
$$\frac{.22x}{.22} = \frac{1100}{.22}$$
$$x = \$5000$$

STEP 4: Answer the question.

x = $5000 at 12%
x−1000 = $5000-$1000 = $4000 at 10%

STEP 5: Check.

Check: $5000 @ 12% = $ 600
$4000 @ 10% = $ 400
TOTAL: $1000 It checks!!

P. 67:

35. A man has $10,000 to invest, some in a relatively safe account earning 5% interest per year, and the rest in more speculative investments earning 12% per year. If the total interest earned for the year was $955, how much was invested at each rate?

SOLUTION:

STEP 1: Let x =_____.

Let x = principle invested 5%
$10000 - x$ = principle invested at 12%

	Principle	Rate	Interest
5%	x	.05	.05(x)
12%	10000−x	.12	.12(10000−x)
			$955

STEP 2: Write the equation from the last column of the chart above.

$$.05(x) + .12(10000-x) = 955$$

STEP 3: Solve the equation.

$$.05x + 1200 - .12x = 955$$
$$-.07x + 1200 = 955$$
$$-.07x = -245$$
$$\frac{-.07x}{-.07} = \frac{-245}{-.07}$$
$$x = \$3500$$

STEP 4: Answer the question.

x = $3500 at 5%
10000−x = 10000-3500 = $6500 at 12%

STEP 5: Check.

Check: $3500 @ 5% = $ 175
$6500 @ 12% = $ 780
TOTAL: $ 955 It checks!!

P. 67:

36. A sum of money was invested at 5% annual interest, and $500 less than twice this amount was invested at 12%. If the total interest earned for the year was $375, how much was invested at each rate?

SOLUTION:

STEP 1: Let x =_____.

Let x = principle invested 5%
2x−500 = principle invested at 12%

	Principle	Rate	Interest
5%	x	.05	.05(x)
12%	2x−500	.12	.12(2x-500)
			$375

STEP 2: Write the equation from the last column of the chart above.

$.05(x) + .12(2x-500) = 375$

STEP 3: Solve the equation.

$$.05x + .24x - 60 = 375$$
$$.29x - 60 = 375$$
$$.29x = 435$$
$$\frac{.29x}{.29} = \frac{435}{.29}$$
$$x = \$1500$$

STEP 4: Answer the question.

x = $1500 at 5%
2x−500 = $3000-$500 = $2500 at 12%

STEP 5: Check.

Check: $1500 @ 5% = $ 75
$2500 @ 12% = $ 300
TOTAL: $ 375 It checks!!

P. 71:

40. How much water must be added to 50% solution to obtain 100 liters of 10% solution?

SOLUTION:

STEP 1: Let x =_____.

Let x = number of liters of water (which is 0% alcohol!)

Type	No Liters	Ea	Pure Stuff
Water	x	0	0(x)
50%	100-x	0.50	.50(100-x)
10%	100	0.10	.10 (100)

STEP 2: Write the equation from the last column of the chart above.

$0 + .50(100-x) = .10\,(100)$

STEP 3: Solve the equation.

$$50 - .50x = 10$$
$$-.50x = -40$$
$$\frac{-.50x}{-.50} = \frac{-40}{-.50}$$
$$x = \frac{400}{5} = 80.$$

STEP 4: Answer the question.

x = 80 liters of water
100 – x = 20 liters of 50% alcohol

STEP 5: Check.

Check: 20 liters @ 50% solution = 10 liters
100 liters @ 10% solution = 10 liters . It checks!!

P. 72:

45. A merchant mixes some candy worth \$3.50 per pound with cheap stuff worth \$1.00 per pound. There are 10 more pounds of the cheap stuff than the more expensive candy. If the total value of the mixture is \$28, how many pounds of each are there?

SOLUTION:

STEP 1: Let x =_____.

Let x = number of pounds of expensive candy

$x+10$ = number of cheap stuff

Type	No Pounds	Ea	Values in \$\$
Expensive	x	3.50	3.50(x)
Cheap	x+10	1	1(x+10)
			28

STEP 2: Write the equation from the last column of the chart above.

$3.50(x) + 1(x+10) = 28$

STEP 3: Solve the equation.

$$3.5x + 1.0x + 10 = 28$$
$$4.5x + 10 = 28$$
$$4.5x = 18$$
$$\frac{4.5x}{4.5} = \frac{18}{4.5}$$
$$x = \frac{180}{45}$$
$$x = 4$$

STEP 4: Answer the question.

x = 4 pounds Expensive Candy

x+10 = 4 +10 = 14 pounds Cheap Stuff

STEP 5: Check.

Check: 4 Expensive @ \$3.50 = \$ 14.00

14 Cheap Stuff @ \$1.00 = 14.00

TOTAL: \$ 28.00 It checks!!

P. 73:

47. Fifty tickets were sold to a chicken barbeque for a total of $219. Children's tickets sold for $2.50, youth tickets sold for $3.50, and adult tickets sold for $5.00. There were 10 more youth tickets than children's tickets. How many of each ticket were sold?

SOLUTION:

STEP 1: Let x =_____.

Let x = number of children's tickets (at the end of the sentence!!)

x+10 = number of youth tickets

2x+10 = number of children and youth tickets combined

50-(2x+10) = number of adult tickets

50 - 2x - 10= number of adult tickets

40 - 2x = number of adult tickets

Type	No Pounds	Each	Values in $$
Children	x	2.50	2.50(x)
Youth	x+10	3.50	3.50(x+10)
Adult	40 - 2x	5.00	5.00(40-2x)
			219

STEP 2: Write the equation from the last column of the chart above.

$2.50(x) + 3.50(x+10) + 5.00(40 - 2x) = 219$

STEP 3: Solve the equation.

$2.5x + 3.5x + 35 + 200 - 10x = 219$

$6.0x + 235 - 10x = 219$

$-4x + 235 = 219$

$-4x + 235 - 235 = 219 - 235$

$-4x = -16$

$x = 4$

STEP 4: Answer the question.

x = 4 Children's Tickets

x+10 = 4 +10 = 14 Youth Tickets

40 - 2x = 40 - 8 = 32 Adult Tickets

STEP 5: Check.

Check: 4 Children's @ $2.50 = $ 10.00

14 Youth @ $3.50 = 49.00

32 Adult @ $5.00 = 160.00

TOTAL = 50 Tickets TOTAL: $219.00 It checks!!

P. 735

50. Two cars are driving in opposite directions, one at 55 mph and the other at 65 mph (on the interstate!). How long will it take before the two cars are 300 miles apart?

SOLUTION:

The basic formula is D=RT, where D=Distance, R=Rate, and T=Time. Since the cars are driving in opposite directions, the basic equation is that the SUM of the distances of the two cars is 300 miles.

STEP 1: Let x =_____.

Let x = time of each car

	Rate	Time	Distance
Slower Car	55	x	55 (x)
Faster Car	65	x	65 (x)
			300

STEP 2: Write the equation from the last column of the chart above.

$$55(x) + 65(x) = 300$$

STEP 3: Solve the equation.

$$55(x) + 65(x) = 300$$
$$120x = 300$$
$$x = \frac{300}{120}$$
$$x = 2.5$$

STEP 4: Answer the question.

x = 2.5 hours or 2 hours 30 minutes

STEP 5: Check.

Check: Distance of slower car = 55 mph * 2.5 hr = 137.5 miles
Distance of faster car = 65 mph * 2.5 hr = 162.5 miles
TOTAL: 300.0 It checks!!

Extra Problem, from Bernard in Chicago.

If there is a stock 95% solution of ethanol, how much of this stock solution is needed to produce 700 ml. of a 5% solution? A. How much pure water is needed, and B. how much 95% ethanol is needed? We are dealing with a slightly diluted solute, literally making a weaker solution from a stronger solution.

SOLUTION: It doesn't matter whether the mixture is liquid or solid, it is still a mixture problem, and the methods of this section still appy!!

STEP 1: Let x =_____.

Let x = number of liters of pure water (which is 0% ethanol!)

Type	No Liters	Ea	Pure Stuff
Water	x	0	0(x)
95%	700-x	0.95	.95(700-x)
5%	700	0.05	.05 (700)

STEP 2: Write the equation from the last column of the chart above.

$$0 + .95(700-x) = .05\,(700)$$

STEP 3: Solve the equation.

$$665 - .95x = 35$$
$$-.95x = -630$$
$$\frac{-.95x}{-.95} = \frac{-630}{-.95}$$
$$x = \frac{12600}{19} \approx 663.16 \text{ ml. of water}$$

STEP 4: Answer the question.

$$x \approx 663.16 \text{ ml of water}$$
$$700 - x \approx 36.84 \text{ ml of 95\% ethanol}$$

STEP 5: Check.

Check: 36.84 ml @ 95% solution = 34.998 ml. (note round off error!)
700 ml @ 5% solution = 35 ml . Close enough! It checks!!

NOTE: Life doesn't always come out even, does it!!