Math in Living C O L O R !!

2.01 Factoring the Common Factor

Intermediate Algebra: One Step at a Time. Pages 112 - 115: 42, 46, 47, 48

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See Section 2.01 with explanations, examples, and exercises, coming soon!

See also : "Basic Algebra: Factoring the Common Factor", coming soon!

Guidelines to Factoring
1. Common Factor 2. Trinomials 3. Difference of Squares; Difference and Sum of Cubes 4. Grouping

P. 115. # 42. $(x+y)^2 - y(x+y)$

The first step is to recognize that there is a common factor. You must take out the $(x+y)$ that is common to both terms. This leaves a factor of $(x+y)$ in the FIRST position, and a factor of y in the SECOND position!

$$(x+y)\,[\,(x+y)\;-\;y]$$

Next, you can drop the red parentheses terms within the brackets. Notice that the y terms subtract out, and all that is left is $x+y\;-\;y$ which is x.

$$(x+y)\,[\,x+y\;-\;y]$$
$$(x+y)\,[x]$$

It looks nicer if you write it like this:

Final Answer: $x(x+y)$

p. 115. # 46. $(x+y)^2+(x-y)(x+y)$

The first step is to recognize that there is a common factor. You must take out the $(x+y)$ that is common to both terms. This leaves a factor of $(x+y)$ in the **FIRST** position, and a factor of $(x-y)$ in the **SECOND** position!

$$(x+y)\,[\ (x+y)\ +\ (x-y)\ ]$$

Next, combine like terms within the brackets. Notice that the y terms subtract out, and all that is left is $(x+y)\ +\ (x-y)$ which is $x+x$ or $2x$

$$(x+y)\,[\ (x+y)\ +\ (x-y)\ ]$$
$$(x+y)\,[(2x)]$$

It looks nicer if you write it like this:

Final Answer: $2x(x+y)$

p. 115. # 47. $(3x-2y)^2-(3x-2y)(x-5y)$

The first step is to recognize that there is a common factor. You must take out the $(3x-2y)$ that is common to both terms. This leaves a factor of $(3x-2y)$ in the **FIRST** position, and a factor of $(x-5y)$ in the **SECOND** position!

$$(3x-2y)\,[\ (3x-2y)\ -\ 2(x-5y)\ ]$$

Next, remove the parentheses within the brackets by use of the distributive property, and combine like terms.

$$(3x-2y)\,[\ 3x-2y\ -\ 2x+10y\ ]$$
$$(3x-2y)\,[\ x+8y\ ]$$

Final Answer: $(3x-2y)\,(x+8y)$

p. 115. # 48. $(5x+3y)^2 - 4(5x+3y)(x+3y)$

The first step is to recognize that there is a common factor. You must take out the $(5x+3y)$ that is common to both terms. This leaves a factor of $(5x+3y)$ in the FIRST position, and factors of $4(x+3y)$ in the SECOND position!

$$(5x+3y)\ [\ (5x+3y)\ -\ 4(x+3y)\ ]$$

Next, remove the parentheses within the brackets by use of the distributive property, and combine like terms.

$$(5x+3y)\ [\ 5x+3y\ -\ 4x-12y\ ]$$

$$(5x+3y)\ [\ x-9y\ ]$$

Final Answer: $(5x+3y)(x-9y)$

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2.01 Difference of Squares
Perfect Square Trinomials

Intermediate Algebra: One Step at a Time. Pages 122-125: 43, 48, 50, 51, 52

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See Section 2.01 with explanations, examples, and exercises, coming soon!

Guidelines to Factoring
1. Common Factor 2. Trinomials 3. Difference of Squares; Difference and Sum of Cubes 4. Grouping

P. 124: 43. $x^4 - 16$ Notice that x^4 and 16 are both perfect squares.

The First times First must be x^4: x^2 times x^2

$$(x^2 \qquad)(x^2 \qquad)$$

The Last times Last must be 16: 4 times 4.

$$(x^2 \quad 4)(x^2 \quad 4)$$

Because the 16 is negative, use opposite signs.

$$(x^2 - 4)(x^2 + 4)$$

The factor $(x^2 - 4)$ is itself a difference of squares, and so it must be re-factored. However, the factor $(x^2 + 4)$ is the SUM of squares. It does not re-factor, and it must be left as it is in the final answer.

$$(x^2 - 4)(x^2 + 4)$$

$(x - 2)(x + 2)(x^2 + 4)$ Final Answer!!

P. 124: 48. $16x^4 - 81$ Notice that $16x^4$ and 81 are both perfect squares.

The First times First must be $16x^4$: $4x^2$ times $4x^2$

$(4x^2 \quad)(4x^2 \quad)$

The Last times Last must be 81: 9 times 9.

$(4x^2 \quad 9)(4x^2 \quad 9)$

Because the 16 is negative, use opposite signs.

$(4x^2 - 9)(4x^2 + 9)$

The factor $(4x^2 - 9)$ is itself a difference of squares, and so it must be re-factored. However, the factor $(4x^2 + 9)$ is the SUM of squares. It does not re-factor, and it must be left as it is in the final answer.

$(4x^2 - 9)(4x^2 + 9)$

$(2x - 3)(2x + 3)(4x^2 + 9)$ **Final Answer!!**

P. 125: 50. $x^4 + 13x^2 + 36$ Notice that First times First must be x^4: x^2 times x^2

Last times Last must be 36

OI term must add up to $13x^2$

$(x^2 \quad)(x^2 \quad)$

The Last times Last must be 36: 9 times 4.

$(x^2 \quad 9)(x^2 \quad 4)$

Because the 36 is positive, use SAME signs.

$(x^2 + 9)(x^2 + 4)$

The factors $(x^2 + 9)$ and $(x^2 + 4)$ are both the SUM of squares. They do not re-factor and must be left as it is in the final answer.

Final answer: $(x^2 + 9)$ $(x^2 + 4)$.

P. 125: 51.

$x^4 - 13x^2 + 36$ Notice that First times First must be x^4: x^2 times x^2

$(x^2 \quad)(x^2 \quad)$

The Last times Last must be 36 and the
OI term must add up to 13x^2
(Try 9 • 4, both negative)

$(x^2 \quad 9)(x^2 \quad 4)$

$(x^2 - 9)(x^2 - 4)$ O term is $-4x^2$, and the I term is $-9x^2$, for a total of 13x^2

The factors $(x^2 - 9)$ and $(x^2 - 4)$ are both difference of squares. Each must be re-factored so this is NOT the final answer.

$(x^2 - 9)(x^2 - 4)$

$(x-3)(x+3)(x-2)(x+2)$ Final Answer!!

P. 125: #52.

$x^4 - 29x^2 + 100$ The First times First must be x^4: x^2 times x^2 .

$(x^2 \quad)(x^2 \quad)$

The Last times Last must be 100
and the OI term must add up to $-29x^2$
(Try 25 • 4!!)

$(x^2 \quad 25)(x^2 \quad 4)$

O term is $-4x^2$, and the I term is $-25x^2$, for a total of $-29x^2$

$(x^2 - 25)(x^2 - 4)$

The factors $(x^2 - 25)$ and $(x^2 - 4)$ are each difference of squares. Each must be re-factored.

$(x^2 - 25)(x^2 - 4)$

$(x-5)(x+5)(x-2)(x+2)$ Final Answer!!

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2.01 Factoring by Sum/Difference of Cubes

Intermediate Algebra: One Step at a Time

Pages 126 - 130: #2, 10, 11, 18, 24, 28, 30, 32, 34

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Guidelines to Factoring
1. **Common Factor** 2. **Trinomials** 3. **Difference of Squares; Difference and Sum of Cubes** $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$ $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$ 4. **Grouping**

P. 127. # 2. $x^3 - 125$

Solution:

Notice that this is a difference of two cubes! The FIRST is x^3 and the SECOND is 125 which can be written 5^3. Remember also that the difference of two cubes factors into the product of a binomial times a trinomial in this form, according to the formula $\left(x^3 - y^3\right) = (x - y)\left(x^2 + xy + y^2\right)$.

What you have is:

$$x^3 - 125$$

$$(x)^3 - (5)^3$$

$$(\_\_ - \_\_)(\_\_ + \_\_ + \_\_)$$

$$(x - 5)(x^2 + 5x + 5^2)$$

$$(x - 5)(x^2 + 5x + 25)$$

The trinomial CANNOT be factored, so this is your final answer!!

P. 127. # 10. $27x^3 - 8y^3$

Solution:

First you must see that this is a difference of two cubes! The **FIRST** is $27x^3$ which can be written $(3x)^3$ and the **SECOND** is $8y^3$ which can be written $(2y)^3$. Remember also that the difference of two cubes factors into the product of a binomial times a trinomial in this form, according to the formula

$$\left(x^3 - y^3\right) = (x - y)(x^2 + xy + y^2).$$

What you have is:

$$27x^3 - 8y^3$$
$$(3x)^3 - (2y)^3$$
$$(\underline{\quad} - \underline{\quad})(\underline{\quad} + \underline{\quad} + \underline{\quad})$$
$$(\ 3x\ -\ 2y\ )(\ 9x^2 + 3x \bullet 2y + 4y^2\ )$$
$$(\ 3x\ -\ 2y\ )(\ 9x^2 + 6xy + 4y^2\ )$$

The trinomial CANNOT be factored, so this is your final answer!!

P. 127. # 11. $64x^3 + 125$

Solution:

Do you see that this is a sum of two cubes? The **FIRST** is $64x^3$ which can be written $(4x)^3$ and the **SECOND** is 125 which can be written 5^3. Remember also that the sum of two cubes factors into the product of a binomial times a trinomial in this form, according to the formula $\left(x^3 + y^3\right) = (x + y)(x^2 - xy + y^2)$.

$$64x^3 + 125$$
$$(4x)^3 + (5)^3$$
$$(\underline{\quad} - \underline{\quad})(\underline{\quad} + \underline{\quad} + \underline{\quad})$$
$$(\ 4x\ -\ 5\ )(\ 16x^2 + 4x \bullet 5 + 25\ )$$
$$(\ 4x\ -\ 5\ )(\ 16x^2 + 20x + 25\ )$$

The trinomial CANNOT be factored, so this is your final answer!!

P. 128. # 18. $3x^3 - 24y^3$

Solution:

The first step is to recognize that there is a common factor. Take out the 3 that is common to both terms.

$$3(x^3 - 8y^3)$$

Notice that what is left the parentheses is a difference of two cubes! The FIRST is x^3 and the SECOND is $8y^3$ which can be written $(2y)^3$. Remember also that the difference of two cubes factors into the product of a binomial times a trinomial in this form, according to the formula

$$\left(x^3 - y^3\right) = \left(x - y\right)\left(x^2 + xy + y^2\right).$$

What you have is:

$$3(x^3 - 8y^3)$$

$$3(x - 2y)(\_\_\_\_\_ + \_\_\_\_\_ + \_\_\_\_\_)$$

$$3(x - 2y)(x^2 + 2xy + 4y^2)$$

The trinomial CANNOT be factored, so again this is your final answer!!

P. 128. # 24. $x^6 + y^9$

Solution:

Next, notice that this is a SUM of two cubes! The FIRST is x^6, which is actually $\left(x^2\right)^3$ and the SECOND is y^9 which can be written $\left(y^3\right)^3$. Remember that the sum of two cubes factors into the product of a binomial times a trinomial in this form, according to the formula $\left(x^3 + y^3\right) = (x + y)(x^2 - xy + y^2)$.
What you have is:

$$x^6 + y^9$$

$$\left(x^2\right)^3 + \left(y^3\right)^3$$

$$(\_\_ + \_\_)(\_\_ - \_\_ + \_\_)$$

$$(x^2 + y^3)(x^4 - x^2y^3 + y^6)$$

P. 130. # 28. $5x^7 - 40xy^9$

Solution:

The first step is to recognize that there is a common factor. Take out the 5 and the x factor that are common to both terms.

$$5x(x^6 - 8y^9)$$

Next, notice that what is left the parentheses is a difference of two cubes!

The FIRST is x^6 which can be written as $\left(x^2\right)^3$ and

the SECOND is $8y^9$ which can be written $(2y^3)^3$.

Remember also that the difference of two cubes is a binomial times a trinomial according to the formula $\left(x^3 - y^3\right) = (x - y)(x^2 + xy + y^2)$.

What you have is:

$$5x(x^6 - 8y^9)$$

$$5x(x^2 - 2y^3)(\_\_\_\_\_ + \_\_\_\_\_ + \_\_\_\_\_)$$

$$5x(x^2 - 2y^3)\left(\left(x^2\right)^2 + x^2 \bullet 2y^3 + \left(2y^3\right)^2\right)$$

$$5x(x^2 - 2y^3)\left(x^4 + 2x^2y^3 + 4y^6\right)$$

P. 130. # 30. $25x^5 + 200x^8y^9$

Solution:

As always, the first step is to recognize that there is a common factor. Take out the 25 and the x^5 factor that are common to both terms.

$$25x^5(1+8x^3y^9)$$

Next, notice that what is left the parentheses is a sum of two cubes!

The FIRST is 1 which can be written as 1^3 and

the SECOND is $8x^3y^9$ which can be written $(2xy^3)^3$.

Remember also that the sum of two cubes is a binomial times a trinomial according to the formula $\left(x^3+y^3\right)=(x+y)(x^2-xy+y^2)$.

What you have is:

$$25x^5\left(1^3+(2xy^3)^3\right)$$

$$25x^5(1+2xy^3)(\_\_\_\_\_ - \_\_\_\_\_ + \_\_\_\_\_)$$

$$25x^5(1+2xy^3)\left((1)^2 - 1\bullet 2xy^3 + \left(2xy^3\right)^2\right)$$

$$25x^5(1+2xy^3)\left(1-2xy^3+4x^2y^6\right)$$

P. 130. # 32. $x^6 - 9x^3 + 8$

Solution:

In this case, there are NO common factors. So, try a trinomial. Notice that there are three terms, so it IS a trinomial, which can be factored into the product of two binomials.

$$x^6 - 9x^3 + 8$$
$$\left(x^3 - \_\_\right)\left(x^3 - \_\_\right)$$
$$\left(x^3 - 8\right)\left(x^3 - 1\right)$$

Notice that each of these binomials are actually differences of two cubes, and each factors:

$$\left(x-2\right)\left(x^2+2x+4\right)\left(x-1\right)\left(x^2+x+1\right)$$

P. 130. # 34. $x^6 - 64$

Solution:

This problem is easiest to factor as a difference of two squares:

$$x^6 - 64$$
$$\left(x^3 - \_\_\right)\left(x^3 + \_\_\right)$$
$$\left(x^3 - 8\right)\left(x^3 + 8\right)$$

Notice that each of these binomials are actually difference and also sum of two cubes:

You finish it:

$$\left(x - \_\_\right)\left(x^2 + \_\_ + \_\_\right)\left(x + \_\_\right)\left(x^2 - \_\_ + \_\_\right)$$

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2.01 Factoring by Grouping

Intermediate Algebra: One Step at a Time.
Pages 131 - 138: 5, 11, 16, 33, 35, 45, 48.

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See also, coming soon : Basic Algebra: Factoring by Grouping

Guidelines to Factoring
1. Common Factor 2. Trinomials 3. Difference of Squares; Difference and Sum of Cubes 4. Grouping

While there are many different types of factoring by grouping, a good place to start, especially if there are four terms in the problem, is to try grouping the first two terms and the last two terms. When you group the first two and the last two terms together, you MUST get a common factor! If you don't get a common factor—that is, if the second binomial factor does not match the first binomial factor EXACTLY, then you may have made an error.

INTRODUCTORY PROBLEM. $ax - bx \ + ay - by$

Solution: Notice that there are NO common factors, it is NOT a trinomial, and it is NOT the difference of squares or sum/difference of cubes! The only thing left to do is to try grouping the first two terms and the last two terms, and hope you get a common factor.

$$ax - bx \ + ay - by$$
$$x(a-b) \ + y(a-b)$$

Notice that you DO have a common factor $(a-b)$, so take out the $(a-b)$.

$$(a-b)(x+y)$$

P. 132. #5. $xy - 5x \; - \; 2y + 10$

Solution: Notice that there are NO common factors, it is NOT a trinomial, and it is NOT the difference of two squares! The only thing left to do is to try a grouping of the first two terms and the last two terms, and hope you get a common factor.

$$xy - 5x \; - \; 2y + 10$$
$$x(y-5) \; - \; 2(y-5)$$

Notice that now you have a common factor $(y-5)$, so take out the $(y-5)$.

Final answer: $(y-5)(x \; - \; 2)$

P. 132. #11. $x^3 + 5x^2 - 25x - 125$

Solution: Notice that there are NO common factors, it is NOT a trinomial, and it is NOT the difference of two squares! Group the first two terms and the last two terms, and make sure that you get a common factor!

$$x^3 + 5x^2 - 25x - 125$$
$$x^2(x+5) \; - \; 25(x+5)$$

Notice that now you have a common factor $(x+5)$, so take out the $(x+5)$.

$$(x+5)(x^2 \; - \; 25)$$

Now, you have a difference of two squares, which factors again!

$(x+5)(x-5)(x+5)$ **Final answer!!**

P. 133. #16. $x^3 - 9x^2 - 4x + 36$

Solution: Notice that there are NO common factors, it is NOT a trinomial, and it is NOT the difference of two squares! Group the first two terms and the last two terms, and make sure that you get a common factor!

$$x^3 - 9x^2 - 4x + 36$$
$$x^2(x-9) \; - \; 4(x-9)$$

Notice that you DO have a common factor $(x-9)$, so take out the $(x-9)$.

$$(x-9)(x^2 \; - \; 4)$$

Now, you have a difference of two squares, which factors again!

$(x-9)(x-2)(x+2)$ **Final answer!!**

P. 136. #33. $(x^2-7x)^2+16(x^2-7x)+60$

Solution: The first step is to recognize that this is a trinomial. Can you see that it is in three parts?

$$(x^2-7x)^2+16(x^2-7x)+60$$

The FIRST times FIRST must be $(x^2-7x)^2$, the LAST times LAST must be 60, and the OUTER times OUTER and INNER times INNER must add up to $16(x^2-7x)$. $(x^2-7x)^2+16(x^2-7x)+60$. You must find two numbers whose product is 60 and whose sum is 16.

The FIRST times FIRST must be $(x^2-7x)^2$

$$\left[(x^2-7x) \qquad\qquad \right]\left[(x^2-7x) \qquad\qquad \right]$$

Next, find two numbers whose product is 60 and whose sum is 16. That would be 10 and 6:

$$\left[(x^2-7x) \;+\; 10\right]\left[(x^2-7x) \;+\; 6\right]$$

This can be "cleaned up" to make it look like the product of two regular trinomials,

$$(x^2-7x \;+\; 10)(x^2-7x \;+\; 6)$$

and as "chance" would have it, these trinomials each can be factored!!

$$(x-5)\;(x-2)\;\;(x-6)\;(x-1)$$ **Final Answer!!**

P. 136. #35. $(x^2 - 5x)^2 - 36$

Solution: The first step is to recognize that this is a difference of two squares!

The FIRST times FIRST must be $(x^2 - 5x)^2$, the LAST times LAST is the perfect square 36 which is 6 times 6, and MIDDLE TERM must subtract out!

The FIRST times FIRST must be $(x^2 - 5x)^2$

$$\left[(x^2 - 5x) \qquad \right]\left[(x^2 - 5x) \qquad \right]$$

$$\left[(x^2 - 5x) - 6\right]\left[(x^2 - 5x) + 6\right]$$

This can be "cleaned up" to make it look like the product of two regular trinomials,

$$(x^2 - 5x - 6)(x^2 - 5x + 6)$$

and as "chance" would have it, these trinomials each can be factored!!

$(x - 6)\ (x + 1)\ (x - 2)\ (x - 3)$ Final Answer!!

P. 138. #45. $x^2 + 2xy + y^2 + 7x + 7y + 10$

Solution: Group the first three terms, the next two, and keep the last term separate. Using colors, this begins to look like a trinomial!

$$x^2 + 2xy + y^2 + 7x + 7y + 10$$

$$(x + y)^2 + 7(x + y) + 10$$

$$[(x + y) \qquad][(x + y) \qquad]$$

Find two numbers whose product is 10 and whose sum is 7.

$$[(x + y) + 5\ ][(x + y) + 2\ ]$$

This cleans up to give you this for the final answer:

$$(x + y + 5\ )\ (x + y + 2\ )$$

P. 138: #48. $x^2-4xy+4y^2+3x-6y+2$

Solution: First notice that, because of the number of terms involved here, this must be a grouping problem. Did you notice that the first three terms look good together? It turns out that these first three terms form a perfect square trinomial. Then try grouping the next two terms together from which you can factor out a common factor of 3. The last term stays by itself. Putting this into grouping by color may help you see it better:

$$x^2-4xy+4y^2+3x-6y+2$$

Rewrite it in this form

$$(x-2y)^2+3(x-2y)+2$$

and recognize that this is a trinomial. Can you see that it is in three parts?

$$(x-2y)^2+3(x-2y)+2$$

The FIRST times FIRST must be $(x-2y)^2$, the LAST times LAST must be 2, and the OUTER times OUTER and INNER times INNER must add up to $3(x-2y)$. To factor $(x-2y)^2+3(x-2y)+2$, you must find two numbers whose product is 2 and whose sum is 3.

The FIRST times FIRST must be $(x-2y)^2$

$$\left[(x-2y)\qquad\right]\left[(x-2y)\qquad\right]$$

Next, find two numbers whose product is 2 and whose sum is 3. That would be 2 and 1:

$$\left[(x-2y)+2\right]\left[(x-2y)+1\right]$$

This can be "cleaned up" to make it look like the product of two regular trinomials,

$$(x-2y+2)(x-2y+1)$$

P. 142. # 51. $x^5 + 9x^4 - x - 9$

Solution:

This is a grouping problem in which it works to group the first two terms together and the second two term together. From the first two terms you can take out a common factor of x^4. From the second two terms, there really isn't a common factor to take out, so just factor out a -1.

$$x^4(x+9) - 1(x+9)$$

Now there is a common factor of $(x+9)$, so take out the common factor:

$$(x+9)(x^4 - 1)$$

Next, factor the difference of two squares:

$$(x+9)(x^2-1)(x^2+1)$$

And again, the difference of two squares:

$$(x+9)(x-1)(x+1)(x^2+1)$$ **Final answer!!**

P. 142. # 52. $(x^2 + 2xy + y^2) + (4x + 4y) + 4$

Solution:

You may have guessed from the grouping by parentheses and by the colors used in this problem, that it is a trinomial. Factor the first grouping as a trinomial, then factor the middle two terms by taking out a common factor of 4, and leave the last 4 alone:

$$(x^2 + 2xy + y^2) + (4x + 4y) + 4$$
$$(x+y)(x+y) + 4(x+y) + 4$$
$$(x+y)^2 + 4(x+y) + 4$$

Now, the entire problem is a trinomial, which factors:

$$[(x+y)+2] \bullet [(x+y)+2]$$

It looks nicer if you clean it up:

$$[x+y+2] \bullet [x+y+2]$$

And even better, you can write it like this:

$$(x+y+2)^2$$ **Final answer!!**

P. 142. # 56. $x^5 - 9x^4 - x + 9$

Solution:

This is a grouping problem in which it works to group the first two terms together and the second two term together. From the first two terms you can take out a common factor of x^4. From the second two terms, there really isn't a common factor to take out, so just factor out a -1.

$$x^4(x-9) - 1(x-9)$$

Now there is a common factor of $(x-9)$, so take out the common factor:

$$(x-9)(x^4-1)$$

Next, factor the difference of two squares:

$$(x-9)(x^2-1)(x^2+1)$$

And again, the difference of two squares:

$$(x-9)(x-1)(x+1)(x^2+1)$$ **Final answer!!**

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2.01 Review of Factoring

Intermediate Algebra: One Step at a Time. **Pages 139 - 142: #33, 37, 51, 52, 56**

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See Section 2.01 with explanations, examples, and exercises, coming soon!

Guidelines to Factoring
1. **Common Factor** 2. **Trinomials** 3. **Difference of Squares; Difference and Sum of Cubes** 4. **Grouping**

P. 140. #33. $x^6 + 27y^9$

Solution: Notice that this is a SUM of two cubes! The FIRST is x^6, which is actually $\left(x^2\right)^3$ and the SECOND is $27y^9$ which can be written $\left(3y^3\right)^3$. Remember that the sum of two cubes factors into the product of a binomial times a trinomial in this form, according to the formula $\left(x^3 + y^3\right) = (x + y)(x^2 - xy + y^2)$.

What you have is:

$$x^6 + 27y^9$$

$$\left(x^2\right)^3 + \left(3y^3\right)^3$$

$$(\_\_ + \_\_)(\_\_ - \_\_ + \_\_)$$

$$(x^2 + 3y^3)(x^4 - x^2 3y^3 + 9y^6)$$

$$(x^2 + 3y^3)(x^4 - 3x^2y^3 + 9y^6) \quad \textbf{Final answer!!}$$

P. 141. #37. $3x^{10}+81xy^6$

Solution: The first step in any factoring problem should be to take out the common factor. In this case the common factor is $3x$

$$3x^{10}+81xy^6$$
$$3x\left(x^9+27y^6\right)$$

Next, notice that this is a SUM of two cubes! The FIRST is x^9 , which is actually $\left(x^3\right)^3$ and the SECOND is $27y^6$ which can be written $\left(3y^2\right)^3$. Remember that the sum of two cubes factors into the product of a binomial times a trinomial in this form, according to the formula

$$\left(x^3+y^3\right)=(x+y)(x^2-xy+y^2).$$

What you have is:

$$3x\left(x^9+27y^6\right)$$
$$3x\left(\left(x^3\right)^3+\left(3y^2\right)^3\right)$$
$$3x(\_\_+\_\_)(\_\_-\_\_+\_\_)$$
$$3x(x^3+3y^2)(\_\_\_-\_\_\_+\_\_\_)$$
$$3x(x^3+3y^2)(x^6-x^3 3y^2+9y^4)$$
$$3x(x^3+3y^2)(x^6-3x^3y^2+9y^4)$$ **Final answer!!**

P. 142. # 51. x^5+9x^4-x-9

Solution: This is a grouping problem in which it works to group the first two terms together and the second two term together. From the first two terms you can take out a common factor of x^4. From the second two terms, there really isn't a common factor to take out, so just factor out a -1.

$$x^4(x+9)-1(x+9)$$

Now there is a common factor of $(x+9)$, so take out the common factor:

$$(x+9)\left(x^4-1\right)$$

Next, factor the difference of two squares:

$$(x+9)\left(x^2-1\right)\left(x^2+1\right)$$

And again, the difference of two squares:

$$(x+9)(x-1)(x+1)\left(x^2+1\right)$$ **Final answer!!**

P. 142. # 52. $\left(x^2+2xy+y^2\right)+\left(4x+4y\right)+4$

Solution: You may have guessed from the grouping by parentheses and by the colors used in this problem, that it is a trinomial. Factor the first grouping as a trinomial, then factor the middle two terms by taking out a common factor of 4, and leave the last 4 alone:

$$\left(x^2+2xy+y^2\right)+\left(4x+4y\right)+4$$
$$\left(x+y\right)\left(x+y\right)+4\left(x+y\right)+4$$
$$\left(x+y\right)^2+4\left(x+y\right)+4$$

Now, the entire problem is a trinomial, which factors:

$$\left[\left(x+y\right)+2\right]\bullet\left[\left(x+y\right)+2\right]$$

It looks nicer if you clean it up:

$$\left[x+y+2\right]\bullet\left[x+y+2\right]$$

And even better, you can write it like this:

$$\left(x+y+2\right)^2$$ **Final answer!!**

P. 142. # 56. x^5-9x^4-x+9

Solution: This is a grouping problem in which it works to group the first two terms together and the second two term together. From the first two terms you can take out a common factor of x^4. From the second two terms, there really isn't a common factor to take out, so just factor out a -1.

$$x^4\left(x-9\right)-1\left(x-9\right)$$

Now there is a common factor of $\left(x-9\right)$, so take out the common factor:

$$\left(x-9\right)\left(x^4-1\right)$$

Next, factor the difference of two squares:

$$\left(x-9\right)\left(x^2-1\right)\left(x^2+1\right)$$

And again, the difference of two squares:

$$\left(x-9\right)\left(x-1\right)\left(x+1\right)\left(x^2+1\right)$$ **Final answer!!**

From College Algebra: $x^6 - 9x^3 + 8$

Solution: The first step is to recognize that this is a trinomial. Can you see that it is in three parts?

The FIRST times FIRST must be x^6, the LAST times LAST must be 8, and the OUTER times OUTER and INNER times INNER must add up to $-9x^3$. You must find two numbers whose product is 8 and whose sum is −9.

The FIRST times FIRST must be x^6. Try $x^3 \bullet x^3$

$$\left(x^3 \qquad\right)\left(x^3 \qquad\right)$$

Next, find two numbers whose product is 8 and whose sum is −9. That would be −8 and −1:

$$\left(x^3 - 8\right)\left(x^3 - 1\right)$$

Each of these factors represent the difference of cubes, which can be factored using the formula: $\left(x^3 - y^3\right) = \left(x - y\right)\left(x^2 + xy + y^2\right)$

$$\left(x^3 - 8\right)\left(x^3 - 1\right)$$

$$\left(x-2\right)\left(x^2+2x+2^2\right)\left(x-1\right)\left(x^2+1x+1^2\right)$$

$$\left(x-2\right)\left(x^2+2x+4\right)\left(x-1\right)\left(x^2+x+1\right)$$

These trinomials CANNOT be factored, so this is your final answer!!

From College Algebra: $x^6 - 64$

Solution: The first step is to recognize that this is a difference of two squares!

The FIRST times FIRST must be x^6 , the LAST times LAST is the perfect square 64 which is 8 times 8, and MIDDLE TERM must subtract out!

The FIRST times FIRST must be x^6, which would be $x^3 \bullet x^3$

$$\left(x^3 \qquad\right)\left(x^3 \qquad\right)$$

$$\left(x^3 - 8\right)\left(x^3 + 8\right)$$

Each of these factors represent the difference or sum of cubes, which can be factored using the formulas: $\left(x^3 - y^3\right) = (x - y)\left(x^2 + xy + y^2\right)$
and $\left(x^3 + y^3\right) = (x + y)\left(x^2 - xy + y^2\right)$

$$\left(x^3 - 8\right)\left(x^3 + 8\right)$$

$$(x-2)\left(x^2 + 2x + 2^2\right)(x+2)\left(x^2 - 2x + 2^2\right)$$

$$(x-2)\left(x^2 + 2x + 4\right)(x+2)\left(x^2 - 2x + 4\right)$$

These trinomials CANNOT be factored, so this is your final answer!!