# Math in Living C O L O R !!

# 2.05 Adding and Subtracting Fractions

*Intermediate Algebra: One Step at a Time*

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**See Section 2.05 with explanations, examples, and exercises, coming soon!**
**Explanations, examples, and exercises from Basic Algebra, coming soon!**

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## ADDITION AND SUBTRACTION OF FRACTIONS
## Summary

**I. FIND THE LEAST COMMON DENOMINATOR (LCD).**

A. Factor each denominator to determine what factors are needed for the common denominator.

B. For each of the denominator factors, determine the highest power of each factor. The LCD is the product of each factor raised to its highest power.

C. The LCD becomes <u>the</u> denominator of the fraction.

**II. PLAY "WHAT'S MISSING?"**

A. Compare each denominator to the LCD, and determine the missing factors for each denominator.

B. Multiply each numerator <u>and</u> denominator by "What's missing!"

**III. ADD OR SUBTRACT NUMERATORS.**

A. Add (or subtract) numerators, and place over the common denominator.

B. Combine like terms and reduce the resulting fraction, if possible.

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**P. 175 # 19.** $\dfrac{x^2-12x}{x-6} + \dfrac{x^2-5x+30}{x-6}$

**Solution:** Notice that you already have a common denominator. This means that the LCD (which is $x-6$) becomes the denominator of the fraction, and you ADD the numerators together:

$$\frac{x^2-12x + x^2-5x+30}{x-6}$$

Now, combine like terms in the numerator, remembering that $x^2 + x^2 = 2x^2$

$$\frac{2x^2-17x+30}{x-6}$$

The most difficult part of this problem is probably factoring the numerator. It might make it easier if you realize that the denominator has a factor of $x-6$, which means that the only reason you would need to factor the numerator would be if it too had a factor of $x-6$. If it does, it will have to start off like this, with both signs negative:

$$\frac{(2x-\underline{\quad})(x-\underline{\quad})}{x-6}$$

Next, notice that to get the last times last to be 30, with one of the numbers -6, the other number must be -5. It must be this way:

$$\frac{(2x-5)(x-6)}{x-6}$$

Notice that the middle term in this trinomial was $-12x-5x$, which is $-17x$ as it should be. Now, divide out the $x-6$ factors.

The final answer is $\dfrac{2x-5}{1}$ or $2x-5$.

**P. 175:** **20.** $$\frac{y^2+4y}{3y+5} + \frac{2y^2+4y+5}{3y+5}$$

**Solution:** First notice that this is the addition of two fractions. The first priority must be to have a common denominator, which in this case is $3y+5$! The LCD becomes THE denominator of the entire problem, and it looks like this:

$$\frac{\quad + \quad}{3y+5}$$

Next, ADD the numerators.

$$\frac{(y^2+4y)+(2y^2+4y+5)}{3y+5}$$

Combine like terms: $$\frac{3y^2+8y+5}{3y+5}$$

Factor the numerator: $$\frac{(3y+5)(y+1)}{3y+5}$$

Reduce the fraction: $$\frac{\cancel{(3y+5)}(y+1)}{\cancel{3y+5}}$$

Final answer: $\frac{(y+1)}{1}$ or $y+1$.

**P. 176: 22.** $\dfrac{5x^2}{x-2} - \dfrac{4x^2+3x-2}{x-2}$

**Solution:** First notice that this is a "Fraction Subtraction"!! The first priority must be to have a common denominator, which in this case is $x-2$! The LCD becomes THE denominator of the entire problem, and it looks like this:

$$\frac{\quad - \quad}{x-2}$$

Next, subtract the numerators. Be careful to distribute the negative through the second numerator!

$$\frac{5x^2-(4x^2+3x-2)}{x-2}$$

$$\frac{5x^2-4x^2-3x+2}{x-2}$$

Combine like terms: $$\frac{x^2-3x+2}{x-2}$$

Factor: $$\frac{(x-2)(x-1)}{x-2}$$

Reduce the fraction: $$\frac{\cancel{(x-2)}(x-1)}{\cancel{x-2}}$$

Final answer: $\dfrac{(x-1)}{1}$ or $x-1$.

**P. 176: 24.** $$\frac{2x^3-6x^2-3}{x^3+x^2-6x}-\frac{x^3-8x-3}{x^3+x^2-6x}$$

**Solution:** First notice that this is a "Fraction Subtraction", very similar to the other exercises on this page, but this one is probably "uglier"!! The first priority must be to have a common denominator, which in this case is $x^3+x^2-6x$! There is NO need to factor the numerators at all. The LCD is the priority here. As before, the LCD becomes THE denominator of the entire problem, and it looks like this:

$$\frac{\quad - \quad}{x^3+x^2-6x}$$

Next, subtract the numerators. Be careful to distribute the negative through the second numerator!

$$\frac{(2x^3-6x^2-3)-(x^3-8x-3)}{x^3+x^2-6x}$$

$$\frac{2x^3-6x^2-3-x^3+8x+3}{x^3+x^2-6x}$$

Combine like terms: $$\frac{x^3-6x^2+8x}{x^3+x^2-6x}$$

Factor the common factor of x from the numerator and the denominator:

$$\frac{x(x^2-6x+8)}{x(x^2+x-6)}$$

The x factors divide out: $$\frac{(x^2-6x+8)}{(x^2+x-6)}$$

Factor the trinomials in the numerator and denominator.

$$\frac{(x-4)(x-2)}{(x+3)(x-2)}$$

Reduce the fraction: $$\frac{(x-4)\cancel{(x-2)}}{(x+3)\cancel{(x-2)}}$$

Final answer: $$\frac{x-4}{x+3}.$$

## Extra Problem #1: $\dfrac{7}{8x} + \dfrac{3}{5x}$

**Step I: (Find the LCD):** The LCD is $40x$

**Step II: (What's Missing?):** 1st denominator has $8x$, missing $5$.
2nd denominator has $5x$, missing $8$.

**Multiply numerator and denominator of each fraction by "What's Missing":**

$$\frac{7}{8x} + \frac{3}{5x}$$

$$\frac{7}{8x} \cdot \frac{5}{5} + \frac{3}{5x} \cdot \frac{8}{8}$$

**Step III: (Add or Subtract):** Add the numerators and place over the LCD.

$$\frac{\quad + \quad}{40x}$$

$$\frac{35 + 24}{40x}$$

$$\frac{59}{40x}$$

## Extra Problem #2: $\dfrac{7}{8x} - \dfrac{5}{6y}$

**Step I: (Find the LCD):** The LCD is $24xy$

**Step II: (What's Missing?):** 1st denominator has $8x$, missing $3y$.
2nd denominator has $6y$, missing $4x$.

**Multiply numerator and denominator of each fraction by "What's Missing":**

$$\frac{7}{8x} - \frac{5}{6y}$$

$$\frac{7}{8x} \cdot \frac{3y}{3y} - \frac{5}{6y} \cdot \frac{4x}{4x}$$

**Step III: (Add or Subtract):** Subtract the numerators and place over the LCD.

$$\frac{\quad - \quad}{24xy}$$

$$\frac{21y - 20x}{24xy}$$

**This is the final answer. It does NOT simplify! Be careful! Do NOT combine unlike terms, and do NOT "cancel" terms!!**

**Extra Problem #3:** $\frac{3}{8xy^2} + \frac{5}{6x^2}$

**Step I: (Find the LCD):** The **LCD** is $24x^2y^2$

**Step II: (What's Missing?):** 1st denominator has $8xy^2$, missing $3x$.

2nd denominator has $6x^2$, missing $4y^2$.

**Multiply numerator and denominator of each fraction by "What's Missing":**

$$\frac{3}{8xy^2} + \frac{5}{6x^2}$$

$$\frac{3}{8xy^2} \bullet \frac{3x}{3x} + \frac{5}{6x^2} \bullet \frac{4y^2}{4y^2}$$

**Step III: (Add or Subtract):** Add the numerators and place over the **LCD**.

$$\frac{\quad + \quad}{24x^2y^2}$$

$$\frac{9x + 20y^2}{24x^2y^2}$$

**Since the numerator does not have like terms, and it does not factor, this is the final answer!**

**P. 181: 37.** $\dfrac{5}{x^2-10x+25} - \dfrac{3}{x^2-5x}$

**Step I: (Find the LCD):** Factor the denominators.

$$\frac{5}{(x-5)(x-5)} - \frac{3}{x(x-5)}$$

$$\frac{5}{(x-5)^2} - \frac{3}{x(x-5)}$$

The LCD = $x(x-5)^2$

**Step II: (What's Missing?):**

$$\frac{5}{(x-5)^2} \cdot \frac{x}{x} - \frac{3}{x(x-5)} \cdot \frac{(x-5)}{(x-5)}$$

**Step III: (Add or Subtract):** Add the numerators and place over the LCD.

$$\frac{5 \bullet (x) \quad - \quad 3 \bullet (x-5)}{x\,(x-5)^2}$$

$$\frac{5x \quad - \quad 3x+15}{x\,(x-5)^2}$$

$$\frac{2x + 15}{x\,(x-5)^2}$$

37. (Alternate Problem--No Extra Charge!!)

Suppose the problem had been written this way:

$$\frac{5}{x^2 - 10x + 25} + \frac{3}{x^2 - 5x}$$

Step I: (Find the LCD): Factor the denominators.

$$\frac{5}{(x-5)(x-5)} + \frac{3}{x(x-5)}$$

$$\frac{5}{(x-5)^2} + \frac{3}{x(x-5)}$$

The LCD = $x(x - 5)^2$

Step II: (What's Missing?):

$$\frac{5}{(x-5)^2} \cdot \frac{x}{x} + \frac{3}{x(x-5)} \cdot \frac{(x-5)}{(x-5)}$$

Step III: (Add or Subtract): Add the numerators and place over the LCD.

$$\frac{5 \bullet (x) + 3 \bullet (x-5)}{x\,(x-5)^2}$$

$$\frac{5x + 3x - 15}{x\,(x-5)^2}$$

$$\frac{8x - 15}{x\,(x-5)^2}$$

**P. 181: 38.** $$\frac{3x^2-4}{2x^2-4x} - \frac{x}{x-2} - \frac{3}{2x}$$

**Step I: (Find the LCD):** Factor the first denominator!

$$\frac{3x^2-4}{2x(x-2)} - \frac{x}{x-2} - \frac{3}{2x}$$

The LCD = $2x(x-2)$

**Step II: (What's Missing?):** 1st denominator has $2x(x-2)$, nothing missing.

2nd denominator has $x-2$, missing $2x$.

3rd denominator has $2x$, missing $(x-2)$.

Multiply numerator and denominator of each fraction by "What's Missing":

$$\frac{3x^2-4}{2x(x-2)} - \frac{x}{x-2}\frac{\bullet\ 2x}{\bullet\ 2x} - \frac{3}{2x}\frac{\bullet\ (x-2)}{\bullet\ (x-2)}$$

**Step III: (Add or Subtract):** Subtract the numerators and place over the LCD.

$$\frac{(3x^2-4) - x\bullet 2x - 3\bullet(x-2)}{2x(x-2)}$$

$$\frac{3x^2-4 - 2x^2 - 3x+6}{2x(x-2)}$$

$$\frac{x^2-3x+2}{2x(x-2)}$$

Factor the numerator: $$\frac{(x-2)(x-1)}{2x(x-2)}$$

Reduce the fraction: $$\frac{\cancel{(x-2)}(x-1)}{2x\cancel{(x-2)}}$$

$$\frac{x-1}{2x}$$

**P. 181 # 39.** $\frac{2a^2-7ab-12b^2}{2a(3a-4b)} + \frac{2a+4b}{3a-4b}$

**Solution:**

Since this is an addition problem, the first step is to find the LCD, which is $2a(3a-4b)$. The next step is to play "What's Missing?" by observing that the first denominator isn't missing any factors, but the second fraction is missing a $2a$. Therefore, you must multiply numerator and denominator of the second fraction by $2a$.

$$\frac{2a^2-7ab-12b^2}{2a(3a-4b)} + \frac{2a+4b}{3a-4b}$$

$$\frac{2a^2-7ab-12b^2}{2a(3a-4b)} + \frac{2a\bullet(2a+4b)}{2a\bullet(3a-4b)}$$

$$\frac{2a^2-7ab-12b^2+2a\bullet(2a+4b)}{2a(3a-4b)}$$

$$\frac{2a^2-7ab-12b^2+4a^2+8ab}{2a(3a-4b)}$$

$$\frac{6a^2+ab-12b^2}{2a(3a-4b)}$$

Now comes the hard part! This numerator might factor, then again, it may not! The only reason it would be necessary to factor this would be if there were a factor of $(3a-4b)$. This helps narrow down some of the choices for factoring the numerator, into

$$\frac{(3a-4b)(2a+3b)}{2a(3a-4b)}$$

Check it out! It really works. Now divide out the factors of $(3a-4b)$, and you have: $\frac{2a+3b}{2a}$, which is the final answer!

By the way, DO NOT divide out terms, like $2a$!

**P. 183 # 43.** $$\frac{x}{x^3+8} + \frac{4}{x+2}$$

**Solution:**

**Step I: (Find the LCD):** **Factor the first denominator!!**

$$\frac{x}{(x+2)(x^2-2x+4)} + \frac{4}{x+2}$$

**Step II: (What's Missing?):**

$$\frac{x}{(x+2)(x^2-2x+4)} + \frac{4}{x+2} \bullet \frac{(x^2-2x+4)}{(x^2-2x+4)}$$

**Step III: (Add or Subtract):** **Add the numerators and place over the LCD.**

$$\frac{x + 4(x^2-2x+4)}{(x+2)(x^2-2x+4)}$$

$$\frac{x + 4x^2-8x+16}{(x+2)(x^2-2x+4)}$$

$$\frac{4x^2-7x+16}{(x+2)(x^2-2x+4)}$$

**P. 183 # 47.** $\dfrac{x}{x^3-125} + \dfrac{10}{x^2-25}$

**Solution:**

**Step I: (Find the LCD):** Factor each denominator in order to find the **LCD**!!

$$\frac{x}{(x-5)(x^2+5x+25)} + \frac{10}{(x-5)(x+5)}$$

The **LCD** is $(x-5)(x+5)(x^2+5x+25)$, which becomes the denominator of the fraction.

**Step II: (What's Missing?):**

The first fraction is missing the factor $(x+5)$, while the second fraction is missing the factor $(x^2+5x+25)$. You must multiply numerator and denominator of the first fraction by $(x+5)$, and numerator and denominator of the second fraction by $(x^2+5x+25)$.

$$\frac{x}{(x-5)(x^2+5x+25)}\,\frac{\bullet\,(x+5)}{\bullet\,(x+5)} + \frac{10}{(x-5)(x+5)}\,\frac{\bullet\,(x^2+5x+25)}{\bullet\,(x^2+5x+25)}$$

**Step III: (Add or Subtract):** The denominator of the answer is the **LCD**.

$$\frac{\qquad\qquad}{(x-5)(x+5)(x^2+5x+25)}$$

**ADD** the numerators together:

$$\frac{x\bullet\,(x+5) \quad + \quad 10\bullet\,(x^2+5x+25)}{(x-5)(x+5)(x^2+5x+25)}$$

Multiply out the parentheses in the numerator, and combine like terms

$$\frac{x^2+5x \quad + \quad 10x^2+50x+250}{(x-5)(x+5)(x^2+5x+25)}$$

**Final Answer:** $\dfrac{11x^2+55x+250}{(x-5)(x+5)(x^2+5x+25)}$

**P. 184 # 49.** $\dfrac{x+4}{4x^2-16x+15} - \dfrac{x-4}{4x^2-4x-15}$

**Solution:** The first step is to factor each denominator in order to find the LCD for the problem.

$$\frac{x+4}{(2x-5)(2x-3)} - \frac{x-4}{(2x-5)(2x+3)}$$

The LCD is $(2x-5)(2x-3)(2x+3)$, which becomes the denominator of the fraction. The first fraction is missing the factor $(2x+3)$, while the second fraction is missing the factor $(2x-3)$. You must multiply numerator and denominator of the first fraction by $(2x+3)$, and numerator and denominator of the second fraction by $(2x-3)$.

$$\frac{x+4}{(2x-5)(2x-3)}\,\frac{\bullet\,(2x+3)}{\bullet\,(2x+3)} - \frac{x-4}{(2x-5)(2x+3)}\,\frac{\bullet\,(2x-3)}{\bullet\,(2x-3)}$$

The LCD is the denominator of the fraction!

$$\frac{\qquad\qquad\qquad\qquad}{(2x-3)(2x+3)(2x-5)}$$

and you SUBTRACT the numerators:

$$\frac{(x+4)\bullet\,(2x+3)-(x-4)\bullet\,(2x-3)}{(2x-3)(2x+3)(2x-5)}$$

Now, multiply out the two products of binomials in the numerator, but keep the parentheses in place in the first step. Be careful!! This will take TWO steps!!

$$\frac{(2x^2+11x+12)-(2x^2-11x+12)}{(2x-3)(2x+3)(2x-5)}$$

Now remove the parentheses:

$$\frac{2x^2+11x+12-2x^2+11x-12}{(2x-3)(2x+3)(2x-5)}$$

$$\frac{\cancel{2x^2}+11x\cancel{+12}\cancel{-2x^2}+11x\cancel{-12}}{(2x-3)(2x+3)(2x-5)}$$

**Final Answer:** $\dfrac{22x}{(2x-3)(2x+3)(2x-5)}$

**P. 184 #51.** $$\frac{3}{10a^2+a-3} - \frac{4}{2a^2+7a-4}$$

## Solution:

**This is a "fraction subtraction", so the first step is to find the Least Common Denominator for the two denominators. The first step of the first step is to factor the two denominators. This step is probably the most difficult part of the entire problem!**

**If you have trouble with this factoring, please click on [this link] for a more detailed explanation of "Advanced Trinomial Factoring":**

$$\frac{3}{(5a+3)(2a-1)} - \frac{4}{(2a-1)(a+4)}$$

**From this you can see that the LCD is $(5a+3)(2a-1)(a+4)$, and the first fraction is "missing" a factor of $(a+4)$, while the second fraction is "missing" a factor of $(5a+3)$. Now, multiply the numerator and denominator of each fraction by the respective missing factor:**

$$\frac{3}{(5a+3)(2a-1)}\frac{\bullet\,(a+4)}{\bullet\,(a+4)} - \frac{4}{(2a-1)(a+4)}\frac{\bullet\,(5a+3)}{\bullet\,(5a+3)}$$

**The LCD will be THE denominator of the entire fraction, and then multiply out the numerators being careful of the signs in the second part.**

$$\frac{3a+12-20a-12}{(5a+3)(2a-1)(a+4)}$$

**Combine like terms:** $$\frac{-17a}{(5a+3)(2a-1)(a+4)}$$ **Final Answer!**

**P. 185 # 52.** $\dfrac{1}{x-2} + \dfrac{1}{x^2-3x+2} - \dfrac{2}{x^2-4x+3}$

**Solution:** The first step is to factor each denominator in order to find the LCD for the problem.

$$\frac{1}{x-2} + \frac{1}{(x-2)(x-1)} - \frac{2}{(x-3)(x-1)}$$

The LCD is $(x-2)(x-1)(x-3)$, which becomes the denominator of the fraction. The first fraction is missing the factorS $(x-1)(x-3)$, the second fraction is missing the factor $(x-3)$, and the third fraction is missing the factor $(x-2)$. You must multiply numerator and denominator of each fraction by the "missing factors." It will look like this:

$$\frac{1}{x-2}\frac{\bullet(x-1)(x-3)}{\bullet(x-1)(x-3)} + \frac{1}{(x-2)(x-1)}\frac{\bullet(x-3)}{\bullet(x-3)} - \frac{2}{(x-3)(x-1)}\frac{\bullet(x-2)}{\bullet(x-2)}$$

Put down the LCD:

$$\frac{\qquad\qquad\qquad}{(x-1)(x-2)(x-3)}$$

and you ADD or SUBTRACT the numerators:

$$\frac{1\bullet(x-1)(x-3) + 1\bullet(x-3) - 2\bullet(x-2)}{(x-1)(x-2)(x-3)}$$

Now, multiply out the parentheses in the numerator, and combine like terms

$$\frac{x^2-4x+3+x-3-2x+4}{(x-1)(x-2)(x-3)}$$

$$\frac{x^2-5x+4}{(x-1)(x-2)(x-3)}$$

The numerator factors, so this MIGHT reduce. You have to factor it and try to reduce the fraction.

$$\frac{(x-4)(x-1)}{(x-1)(x-2)(x-3)}$$

Divide out the factor of $(x-1)$

$$\frac{(x-4)\cancel{(x-1)}}{\cancel{(x-1)}(x-2)(x-3)}$$

**Final Answer:** $\dfrac{x-4}{(x-2)(x-3)}$

## P. 185 # 53.

Notice that this problem came from a book called *New School Algebra* by G.A. Wentworth. It was published in 1898 by Ginn and Company.

$$\frac{1}{a^2-7a+12}+\frac{2}{a^2-4a+3}-\frac{3}{a^2-5a+4}$$

Since this is an addition/subtraction problem, the first step is to factor each denominator in order to find the LCD.

$$\frac{1}{(a-3)(a-4)}+\frac{2}{(a-3)(a-1)}-\frac{3}{(a-4)(a-1)}.$$

The LCD consists of three binomial factors $(a-3)(a-4)(a-1)$. The next step is to play "What's Missing?" by observing which factors are missing from each of the three fractions. Notice that the first denominator is missing the $(a-1)$ factor, the second fraction is missing the $(a-4)$ factor, and the third fraction is missing the $(a-3)$. So, you must multiply numerators and denominators of each fraction by the respective missing factor:

$$\frac{1}{(a-3)(a-4)}\quad+\quad\frac{2}{(a-3)(a-1)}\quad-\quad\frac{3}{(a-4)(a-1)}$$

$$\frac{1}{(a-3)(a-4)}\frac{\bullet\,(a-1)}{\bullet\,(a-1)}+\frac{2}{(a-3)(a-1)}\frac{\bullet\,(a-4)}{\bullet\,(a-4)}-\frac{3}{(a-4)(a-1)}\frac{\bullet\,(a-3)}{\bullet\,(a-3)}$$

$$\frac{a-1\quad+\quad 2a-8\quad-\quad 3a+9}{(a-3)(a-4)(a-1)}$$

When you combine like terms in the numerator, everything subtracts out, leaving

$$\frac{0}{(a-3)(a-4)(a-1)}$$

which is just $0$.

**P. 186 # 55.** $$\frac{x}{x+4} + \frac{2x}{x^2-4} - \frac{2}{(x+2)(x+4)}$$

Since this is an addition/subtraction problem, the first step is to find the LCD, by factoring the middle fraction.

$$\frac{x}{x+4} + \frac{2x}{(x-2)(x+2)} - \frac{2}{(x+2)(x+4)}.$$

The LCD consists of three binomial factors $(x+4)(x-2)(x+2)$. The next step is to play "What's Missing?" by observing which factors are missing from each of the three fractions. Notice that the first denominator is missing two factors: $(x-2)(x+2)$. The second fraction is missing the $(x+4)$ factor, and the third fraction is missing the $(x-2)$. So, you must multiply numerators and denominators of each fraction by the respective missing factor:

$$\frac{x}{x+4} + \frac{2x}{(x-2)(x+2)} - \frac{2}{(x+2)(x+4)}$$

$$\frac{x}{x+4}\,\frac{\bullet\,(x-2)(x+2)}{\bullet\,(x-2)(x+2)} + \frac{2x}{(x-2)(x+2)}\,\frac{\bullet\,(x+4)}{\bullet\,(x+4)} - \frac{2}{(x+2)(x+4)}\,\frac{\bullet\,(x-2)}{\bullet\,(x-2)}$$

It may help in this rather complicated problem, to multiply the factors $(x-2)(x+2)$ in the first numerator first, and then continue multiplying out the rest of the numerators.

$$\frac{x}{x+4}\,\frac{\bullet\,(x^2-4)}{\bullet\,(x-2)(x+2)} + \frac{2x}{(x-2)(x+2)}\,\frac{\bullet\,(x+4)}{\bullet\,(x+4)} - \frac{2}{(x+2)(x+4)}\,\frac{\bullet\,(x-2)}{\bullet\,(x-2)}$$

$$\frac{x^3-4x+2x^2+8x-2x+4}{(x+4)(x-2)(x+2)}$$

Combine the x-terms in the numerator!

$$\frac{x^3+2x^2+2x+4}{(x+4)(x-2)(x+2)}$$

Factor the numerator by grouping:

$$\frac{x^2(x+2)+2(x+2)}{(x+4)(x-2)(x+2)}$$

$$\frac{(x+2)(x^2+2)}{(x+4)(x-2)(x+2)}$$

Finally, divide out the factor of (x+2) in the numerator and denominator.

$$\frac{x^2+2}{(x+4)(x-2)}$$

**P. 187 # 9.** $\dfrac{x^3-64}{16-x^2}$

**Solution:**

The first step in reducing any fraction is to factor the numerator and denominator. Sometimes there will be common factors that divide out. In this case, the numerator is the difference of two cubes and the denominator will be the difference of two squares, which you should be really good at factoring. If NOT, then go back and re-study the section on factoring.

$$\frac{x^3-64}{16-x^2}$$

$$\frac{(x-4)(x^2+4x+16)}{(4-x)(4+x)}$$

Notice that the factors of $(x-4)$ and $(4-x)$ are negatives of one another, so they divide out and leave a factor of -1, left preferably in the numerator.

$$\frac{-1(x^2+4x+16)}{(4+x)} \text{ or } -\frac{x^2+4x+16}{x+4}$$

**P. 188 #11.** $\dfrac{5}{x-5}+\dfrac{5}{5-x}$

**Solution:**

In this case, notice that the second denominator is NOT the same as the first denominator, but it IS similar. In fact, the $x-5$ and the $5-x$ are negatives of one another. It might be helpful to multiply the numerator and denominator of the second fraction by $-1$ .

$$\frac{5}{x-5}+\frac{-1\bullet}{-1\bullet}\frac{5}{(5-x)}$$

$$\frac{5}{x-5}+\frac{-5}{-5+x} \text{ or } \frac{5}{x-5}+\frac{-5}{x-5}$$

Now, you have a common denominator of $x-5$ , so you can add the numerators together:

$$\frac{5-5}{x-5} \text{ which is } \frac{0}{x-5} \text{ or } 0$$

**P. 188 # 15.** $\dfrac{x^2}{x-2} + \dfrac{4}{2-x}$

**Solution:**

Notice that the second denominator $2-x$ is very similar to the $x-2$ factor in the first denominator. It will be very helpful to multiply the numerator and denominator of the second fraction by $-1$ .

$$\frac{x^2}{x-2} + \frac{-1\bullet}{-1\bullet}\frac{4}{2-x}$$

$$\frac{x^2}{x-2} + \frac{-4}{x-2}$$

The LCD is the denominator of the problem. Then, add numerators:

$$\frac{x^2-4}{x-2}$$

Next, factor the difference of squares:

$$\frac{(x-2)(x+2)}{x-2}$$

Reduce the fraction by dividing out the $(x-2)$.

The final answer is: $\dfrac{(x+2)}{1}$ or $x+2$.

**P. 189 # 17.** $\dfrac{x^3}{x-2} + \dfrac{4x}{2-x}$

**Solution:**

Notice that the second denominator $2-x$ is very similar to the $x-2$ factor in the first denominator. It will be very helpful to multiply the numerator and denominator of the second fraction by $-1$.

$$\frac{x^3}{x-2} + \frac{-1\bullet}{-1\bullet}\frac{4x}{2-x} \quad \text{or} \quad \frac{x^3}{x-2} + \frac{-4x}{x-2}$$

Put down the LCD as the denominator of the problem, and add (or subtract) numerators:

$$\frac{x^3-4x}{x-2}$$

Factor the common factor of x from the numerator:

$$\frac{x(x^2-4)}{x-2}$$

Next, factor the difference of squares, and reduce by dividing out the $(x-2)$:

$$\frac{x\,\cancel{(x-2)}\,(x+2)}{\cancel{x-2}}$$

The final answer is: $\dfrac{x(x+2)}{1}$ or $x(x+2)$.

**P. 189 # 23.** $$\frac{x}{x^2-25} - \frac{5}{5-x}$$

Find the LCD by factoring the first denominator:

$$\frac{x}{(x-5)(x+5)} - \frac{5}{5-x}$$

Notice that the second denominator $5-x$ is very similar to the $x-5$ factor in the first denominator. It will be very helpful to multiply the numerator and denominator by −1.

$$\frac{x}{(x-5)(x+5)} - \frac{-1\bullet}{-1\bullet}\frac{5}{(5-x)}$$

$$\frac{x}{(x-5)(x+5)} - \frac{-5}{x-5}$$

$$\frac{x}{(x-5)(x+5)} + \frac{5}{x-5}$$

Now, it should be clear that the LCD = (x-5)(x+5). The first fraction has the common denominator, but the second fraction needs a factor of $(x+5)$, so multiply numerator and denominator of the second fraction by $(x+5)$.

$$\frac{x}{(x-5)(x+5)} + \frac{5}{(x-5)}\frac{\bullet(x+5)}{\bullet(x+5)}$$

Put down the LCD as the denominator of the problem, and add numerators:

$$\frac{x+5x+25}{(x-5)(x+5)}$$

Combine like terms in the numerator:

**Final Answer:** $$\frac{6x+25}{(x-5)(x+5)}$$