Math in Living C O L O R !!

Complex Fractions – Part I

Intermediate Algebra: One Step at a Time, **Section 2.06, Pages 196-200:**
College Algebra: One Step at a Time, **Section 1.05, Pages 57-62:**
#6, 7, 9, 17, 18, 19, 21, 26, 28, 29, 30, 31, 33, 34, 36, 39, 40, 41, 43, 47, 49, 51, 53, extras.

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Intermediate Algebra:

To see Section 2.06 with explanations, examples, and exercises, click here!

College Algebra:

To see Section 1.05 with explanations, examples, and exercises, click here!

Problems #1-10 are all similar, in that they consist of a single fraction divided by a single fraction. This is a great opportunity to try out Method I, which I call the "Unstacking Method." All you do is "unstack" the fraction, and write it as a division problem!

Remember, in general, to "unstack" the problem, use these formulas:

$$\frac{\frac{a}{b}}{\frac{c}{d}} \text{ means } \frac{a}{b} \div \frac{c}{d}$$

and

$$\frac{a}{b} \div \frac{c}{d} \text{ means } \frac{a}{b} \bullet \frac{d}{c}$$

P. 191: #6. $$\frac{\frac{3x^3}{14y^2}}{\frac{21x^5}{8y}}$$

Solution: The first step is to "unstack" the problem:

$$\frac{\frac{3x^3}{14y^2}}{\frac{21x^5}{8y}} = \frac{3x^3}{14y^2} \div \frac{21x^5}{8y}$$

Now, just invert the second fraction, and multiply:

$$= \frac{3x^3}{14y^2} \bullet \frac{8y}{21x^5}$$

Divide out factors of any numerator with corresponding factors from the denominators. In particular, divide out the factors of 3, 2, x^3, and y:

$$= \frac{1x^3}{7y^2} \bullet \frac{4y}{7x^5}$$

$$= \frac{1}{7y} \bullet \frac{4}{7x^2}$$

$$= \frac{4}{49x^4y}$$

P. 191: #7. $$\frac{\frac{4x}{x-4}}{\frac{8y}{x^2-16}}$$

Solution: The first step is to "unstack" the problem:

$$\frac{\frac{4x}{x-4}}{\frac{8y}{x^2-16}} = \frac{4x}{x-4} \div \frac{8y}{x^2-16}$$

Just invert the second fraction, and multiply. While you are at it, factor what can be factored:

$$= \frac{\cancel{4}x}{\cancel{x-4}} \bullet \frac{\cancel{(x-4)}(x+4)}{2\cancel{8}y}$$

Divide out factors of any numerator with corresponding factors from the denominators. In particular, divide out the factors of 4 and (x − 4) :

$$= \frac{x(x+4)}{2y}$$

P. 191: #9. $$\frac{\dfrac{x^3-8}{x+2}}{\dfrac{x^2+2x+4}{x^2-4}}$$

Solution: Of course, as before, the first step is to "un-stack" the problem:

$$\frac{\dfrac{x^3-8}{x+2}}{\dfrac{x^2+2x+4}{x^2-4}} = \frac{x^3-8}{x+2} \div \frac{x^2+2x+4}{x^2-4}$$

Just invert the second fraction, and multiply. While you are at it, factor what can be factored. Notice that the first numerator is the difference of cubes. In the second fraction, the denominator is a difference of squares. Be careful! The numerator does NOT factor!!

š $$= \frac{x^3-8}{x+2} \bullet \frac{x^2-4}{x^2+2x+4}$$

$$= \frac{(x-2)(x^2+2x+4)}{x+2} \bullet \frac{(x-2)(x+2)}{x^2+2x+4}$$

Divide out factors of any numerator with corresponding factors from the denominators. In particular, divide out the factors of x^2+2x+4 and (x + 2) .

$$= \frac{(x-2)\cancel{(x^2+2x+4)}}{\cancel{x+2}} \bullet \frac{(x-2)\cancel{(x+2)}}{\cancel{x^2+2x+4}}$$

Only the two factors of $(x-2)$ remain.

$$= (x-2)^2$$

P. 195 #17. $$\frac{\frac{1}{x}+\frac{1}{2}}{\frac{4}{x^2}-1}$$

Solution: **Method I.** **Both methods work equally well here, but a lot of students just prefer the "unstacking" method, until there is time to see some of the advantages of Method II. This explanation will use Method I.**

$$\frac{\frac{1}{x}+\frac{1}{2}}{\frac{4}{x^2}-1} = \left(\frac{1}{x}+\frac{1}{2}\right) \div \left(\frac{4}{x^2}-1\right)$$

You must find the LCD for the fractions on the left side (RED), which is $2x$, and an LCD for the fractions on the right side (BLUE), which is x^2. You will have to play "what's missing" in these fractions.

$$\left(\frac{1}{x}+\frac{1}{2}\right) \div \left(\frac{4}{x^2}-1\right)$$

$$\left(\frac{1}{x}\bullet\frac{2}{2}+\frac{1}{2}\bullet\frac{x}{x}\right) \div \left(\frac{4}{x^2}-\frac{1}{1}\bullet\frac{x^2}{x^2}\right)$$

$$\left(\frac{2+x}{2x}\right) \div \left(\frac{4-x^2}{x^2}\right)$$

Invert the second fraction, factoring the difference of squares in the second fraction.

$$\frac{2+x}{2x} \bullet \frac{x^2}{(2-x)(2+x)}$$

Divide out an x from the first denominator with the x in the second numerator, and divide out the (2+x) factors. What is left is

$$\frac{\cancel{2+x}}{2\cancel{x}} \bullet \frac{x^{\cancel{2}}}{(2-x)\cancel{(2+x)}}$$

The final answer is: $$\frac{x}{2(2-x)}$$

P. 195 #18. $$\frac{\frac{1}{x^2}-1}{\frac{1}{x}-1}$$

Solution: **Method I.** **Both methods work equally well here, but a lot of students just prefer the "unstacking" method, until there is time to see some of the advantages of Method II. This explanation will use Method I.**

$$\frac{\frac{1}{x^2}-1}{\frac{1}{x}-1} = \left(\frac{1}{x^2}-1\right) \div \left(\frac{1}{x}-1\right)$$

You must find the LCD for the fractions on the left side (RED), which is x^2, and an LCD for the fractions on the right side (BLUE), which is x. You will have to play "what's missing in these fractions.

$$\left(\frac{1}{x^2}-1\right) \div \left(\frac{1}{x}-1\right)$$

$$\left(\frac{1}{x^2}-\frac{1}{1}\bullet\frac{x^2}{x^2}\right) \div \left(\frac{1}{x}-\frac{1}{1}\bullet\frac{x}{x}\right)$$

$$\left(\frac{1-x^2}{x^2}\right) \div \left(\frac{1-x}{x}\right)$$

Factor the first numerator, and invert the second fraction.

$$\frac{(1-x)(1+x)}{x^2} \bullet \frac{x}{1-x}$$

Divide out an x from the first denominator with the x in the second numerator, and divide out the (1-x) factors.

$$\frac{\cancel{(1-x)}(1+x)}{x\,\cancel{x^2}} \bullet \frac{\cancel{x}}{\cancel{1-x}}$$

What is left is

$$\frac{(1+x)}{x} \bullet \frac{1}{1} \text{ or } \frac{x+1}{x}$$

P. 195 #19

$$\frac{\left(3 - \dfrac{6}{x-2}\right)}{\left(8 - \dfrac{16}{x-2}\right)}$$

Solution: **Method II.** **The LCD for the whole problem is $x-2$, so multiply numerator and denominator by $x-2$.**

$$= \frac{\frac{x-2}{1}}{\frac{x-2}{1}} \frac{\left(3 - \dfrac{6}{x-2}\right)}{\left(8 - \dfrac{16}{x-2}\right)}$$

$$= \frac{\left(3(x-2) - (x-2)\bullet\dfrac{6}{x-2}\right)}{\left(8(x-2) - (x-2)\bullet\dfrac{16}{x-2}\right)}$$

Although this looks terrible, it is really quite simple, and you can do it in your head. Normally, you don't even write down the previous step. It all simplifies to this:

$$= \frac{(3x-6 - 6)}{(8x-16 - 16)}$$

Combine like terms in numerator and denominator:

$$= \frac{(3x-12)}{(8x-32)}$$

Factor the common factor within the numerator and the denominator:

$$= \frac{3(x-4)}{8(x-4)}$$

Reduce the fraction by dividing out the (x − 4) factor:

$$= \frac{3}{8}.$$

$$\frac{\left(4 + \frac{4}{x} - \frac{3}{x^2}\right)}{\left(4 - \frac{8}{x} + \frac{3}{x^2}\right)}$$

P. 195: #21.

Solution: **Method I.** **The "Unstacking Method."**

$$\frac{\left(4 + \frac{4}{x} - \frac{3}{x^2}\right)}{\left(4 - \frac{8}{x} + \frac{3}{x^2}\right)} = \left(4 + \frac{4}{x} - \frac{3}{x^2}\right) \div \left(4 - \frac{8}{x} + \frac{3}{x^2}\right)$$

Find the LCD for the RED part (which is x^2), and the LCD for the BLUE part (which is also x^2). Multiply each fraction by the "missing" factors:

$$= \left(\frac{4}{1} \bullet \frac{x^2}{x^2} + \frac{4}{x} \bullet \frac{x}{x} - \frac{3}{x^2}\right) \div \left(\frac{4}{1} \bullet \frac{x^2}{x^2} - \frac{8}{x} \bullet \frac{x}{x} + \frac{3}{x^2}\right)$$

$$= \left(\frac{4x^2 + 4x - 3}{x^2}\right) \div \left(\frac{4x^2 - 8x + 3}{x^2}\right)$$

$$= \left(\frac{4x^2 + 4x - 3}{x^2}\right) \bullet \left(\frac{x^2}{4x^2 - 8x + 3}\right)$$

$$= \frac{(2x-1)(2x+3)}{x^2} \bullet \frac{x^2}{(2x-3)(2x-1)}$$

Divide out the factors of (2x-1) and x^2, and the final answer is, as it was by Method II,

$$= \frac{(2x + 3)}{(2x - 3)}$$

P. 195 #21.

$$\frac{\left(4 + \frac{4}{x} - \frac{3}{x^2}\right)}{\left(4 - \frac{8}{x} + \frac{3}{x^2}\right)}$$

Solution: Method II.

The LCD for the whole problem is x^2, so multiply numerator and denominator by $\frac{x^2}{1}$.

$$= \frac{\left(\frac{x^2}{1}\right)\left(4 + \frac{4}{x} - \frac{3}{x^2}\right)}{\left(\frac{x^2}{1}\right)\left(4 - \frac{8}{x} + \frac{3}{x^2}\right)}$$

$$= \frac{\left(4 \bullet \left(\frac{x^2}{1}\right) + \frac{4}{x} \bullet \left(\frac{x^2}{1}\right) - \frac{3}{x^2} \bullet \left(\frac{x^2}{1}\right)\right)}{\left(4 \bullet \left(\frac{x^2}{1}\right) - \frac{8}{x} \bullet \left(\frac{x^2}{1}\right) + \frac{3}{x^2} \bullet \left(\frac{x^2}{1}\right)\right)}$$

As in the previous problem, although this looks terrible, it is really quite simple, and you can do it in your head. Normally, you don't even write down the previous step. It all simplifies to this:

$$= \frac{\left(4x^2 + 4x - 3\right)}{\left(4x^2 - 8x + 3\right)}$$

Factor the numerator and denominator:

$$= \frac{(2x - 1)(2x + 3)}{(2x - 1)(2x - 3)}$$

Reduce the fraction by dividing out the (2x − 1) factor:

$$= \frac{(2x + 3)}{(2x - 3)}.$$

P. 196 # 26.

$$\frac{\frac{x}{x+1}+1}{\frac{2x+1}{x-1}}$$

Solution: **Method I.** **The "Unstacking Method."**

$$\frac{\frac{x}{x+1}+1}{\frac{2x+1}{x-1}} = \left(\frac{x}{x+1}+\frac{1}{1}\right) \div \left(\frac{2x+1}{x-1}\right)$$

The LCD for the first (red) part is x+1. The second (blue) part is a single fraction, so you don't even need a common denominator!! Express the red part as a single fraction:

$$= \left(\frac{x}{x+1}+\frac{1}{1}\right) \div \left(\frac{2x+1}{x-1}\right)$$

$$= \left(\frac{x}{x+1}+\frac{1}{1} \bullet \frac{(x+1)}{(x+1)}\right) \div \left(\frac{2x+1}{x-1}\right)$$

$$= \left(\frac{x+x+1}{x+1}\right) \div \left(\frac{2x+1}{x-1}\right)$$

Combine like terms in the first fraction, and invert the second fraction:

$$= \left(\frac{2x+1}{x+1}\right) \bullet \left(\frac{x-1}{2x+1}\right)$$

Divide out factors of any numerator with corresponding factors from the denominators. In particular, divide out the factors of (2x+1):

$$= \frac{x-1}{x+1}$$

P. 197 # 28. $\dfrac{\dfrac{6x}{x-1}-3}{4-\dfrac{12}{x+4}}$

Solution: Method I. The "Unstacking Method."

$$\frac{\dfrac{6x}{x-1}-3}{4-\dfrac{12}{x+4}} = \left(\frac{6x}{x-1}-\frac{3}{1}\right)\div\left(\frac{4}{1}-\frac{12}{x+4}\right)$$

The LCD for the first (red) part is $x-1$, for the second (blue) part is $x+4$, so multiply numerator and denominator of each fraction by the appropriate missing factors:

$$= \left(\frac{6x}{x-1}-\frac{3}{1}\right)\div\left(\frac{4}{1}-\frac{12}{x+4}\right)$$

$$= \left(\frac{6x}{x-1}-\frac{3}{1}\bullet\frac{(x-1)}{(x-1)}\right)\div\left(\frac{4}{1}\bullet\frac{(x+4)}{(x+4)}-\frac{12}{x+4}\right)$$

Put each of these LCDs in place:

$$= \left(\frac{\qquad\qquad}{x-1}\right)\div\left(\frac{\qquad\qquad}{x+4}\right)$$

Multiply out the numerators:

$$= \left(\frac{6x\ -\ 3x+3}{x-1}\right)\div\left(\frac{4x+16\ -12}{x+4}\right)$$

Combine like terms for each of the numerators:

$$= \left(\frac{3x+3}{x-1}\right)\div\left(\frac{4x+4}{x+4}\right)$$

Factor the numerators (if possible!), invert the second fraction, and multiply:

$$= \left(\frac{3(x+1)}{x-1}\right)\bullet\left(\frac{x+4}{4(x+1)}\right)$$

Divide out factors of any numerator with corresponding factors from the denominators. In particular, divide out the factors of $(x+1)$:

$$= \frac{3(x+4)}{4(x-1)}$$