Math in Living C O L O R !!

2.07 Fractional Equations

Intermediate Algebra: One Step at a Time. Pages 201 - 206: #16,18, 20,21,23.
Page 212: #21, 3 Extra Prob.

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See Section 2.07 with explanations, examples, and exercises, coming soon!
[NOTE: Equation boxes work in #16, 21, 23]

P. 204. # 16. $\frac{x}{3}-\frac{x+2}{2}=1$

Solution: Notice that this is a fractional equation. I call this the "fraction hater's delight" because in the very first step you get to eliminate ALL the fractions. What makes this possible? It is an EQUATION, and you are allowed to multiply both sides of an equation by some non-zero number. You multiply both sides of the equation by the LCD, which is 6 .

$$6\bullet\frac{x}{3}-6\bullet\frac{x+2}{2}=6\bullet 1$$

Notice that **ALL THE DENOMINATORS DIVIDE OUT!!** When you reduce all the fractions, this is what is left—**NO FRACTIONS!!**

$$2x-3(x+2)=6$$

$$2x-3x-6=6$$

$$-1x-6=6$$

$$-1x=12$$

$$x=-12$$

The check is optional, since there are no variables in the denominators. However, it's not a bad idea to check these:

$$\frac{x}{3}-\frac{x+2}{2}=1$$

$$\frac{-12}{3}-\frac{-12+2}{2}=1$$

$$-4-(-5)=1$$

$$-4+5=1$$ It checks!!

P. 204. # 18. $\frac{x(x+1)}{6} - \frac{x}{3} = 1$

Solution: This is a fractional equation. I call these problems the "fraction hater's delight" because in the very first step you get to eliminate ALL the fractions. What makes this possible? It is an EQUATION, and you are allowed to multiply both sides of an equation by some non-zero number. You multiply both sides of the equation by the LCD, which is 6.

$$6 \bullet \frac{x(x+1)}{6} - 6 \bullet \frac{x}{3} = 6 \bullet 1$$

Notice that **ALL THE DENOMINATORS DIVIDE OUT!!** When you reduce all the fractions, this is what is left—**NO FRACTIONS!!**

$$\cancel{6} \bullet \frac{x(x+1)}{\cancel{6}} - 2\cancel{6} \bullet \frac{x}{\cancel{3}} = 6 \bullet 1$$

$$x^2 + x - 2x = 6$$

$$x^2 - 1x - 6 = 0$$

$$(x-3)(x+2) = 0$$

Final Answer: $x = 3$, $x = -2$

The check is optional, since there are no variables in the denominators.

P. 205. # 20. $\frac{5}{x-2}-\frac{5}{x+2}=4$

Solution: That this is a fractional equation. The first step is to find the LCD, which is $(x-2)(x+2)$, and multiply both sides of the equation by the LCD, being careful NOT to allow any of the denominators to be zero. In other words, in this problem, you must be careful that $x \neq 2$ and $x \neq -2$.

$$\frac{5}{x-2}-\frac{5}{x+2}=4$$

$$(x-2)(x+2)\bullet\frac{5}{x-2}-(x-2)(x+2)\bullet\frac{5}{x+2}=4\bullet(x-2)(x+2)$$

This looks pretty ugly, but when you reduce all the fractions, it really is not bad. In fact, ALL THE DENOMINATORS DIVIDE OUT!! When you reduce all the fractions, this is what is left—NO FRACTIONS!!

$$5(x+2)-5(x-2)=4(x-2)(x+2)$$

$$5x+10-5x+10=4(x^2-4)$$

$$20=4x^2-16$$

There are at least two ways to solve this. Probably the easiest is to add $+16$ to each side:

$$20+16=4x^2-16+16$$

$$36=4x^2$$

$$4x^2=36$$

Divide both sides by 4: $x^2=9$

Therefore, $x=3$ or $x=-3$

Both answers are valid, since neither value of x makes the denominator zero.

P. 205. # 21. $\frac{3}{x-3}-\frac{7}{x+3}=2$

Solution: This is a fractional equation. The first step is to find the LCD, which is $(x-3)(x+3)$, and multiply both sides of the equation by the LCD, being careful NOT to allow any of the denominators to be zero. In other words, in this problem, you must be careful that $x \neq 3$ and $x \neq -3$.

$$\frac{3}{x-3}-\frac{7}{x+3}=2$$

$$(x-3)(x+3)\bullet\frac{3}{x-3}-(x-3)(x+3)\bullet\frac{7}{x+3}=2\bullet(x-3)(x+3)$$

This looks pretty ugly, but when you reduce all the fractions, it really is not bad. In fact, ALL THE DENOMINATORS DIVIDE OUT!! When you reduce all the fractions, this is what is left—NO FRACTIONS!!

$$3(x+3)-7(x-3)=2\bullet(x-3)(x+3)$$

Next, multiply out the parentheses, and on the right side, multiply the product of the binomials first, then distribute the 2.

$$3x+9-7x+21=2\bullet(x^2-9)$$

$$-4x+30=2x^2-18$$

This is a quadratic equation, so set it equal to zero. In order to keep the x^2 term positive, move everything to the right side of the equation:

$$-4x+30=2x^2-18$$
$$\underline{+4x-30 \quad +4x-30}$$
$$0=2x^2+4x-48$$

Factor completely beginning with the common factor of 2:

$$0=2(x^2+2x-24)$$

Next, factor the trinomial. It does factor, doesn't it??

$$0=2(x+6)(x-4)$$

$$x=-6, \qquad x=4$$

Checking: You do NOT have to check the answers, but you MUST check to make sure all the denominators are okay! Make sure no denominators are "accidentally" equal to zero! This is NOT allowed, and any values of x that cause one or more denominators to equal zero must be REJECTED!! These answers are acceptable, so the final answer is $x=-6, \quad x=4$.

P. 206. # 23. $\dfrac{1}{x^2-4x+3}-\dfrac{2}{x^2+4x-5}=\dfrac{4}{x^2+2x-15}$

Solution: This is a fractional equation. The first step is to factor each denominator in order to find the LCD.

$$\frac{1}{(x-3)(x-1)}-\frac{2}{(x+5)(x-1)}=\frac{4}{(x+5)(x-3)}$$

Now, you should see that the **LCD** $=(x-3)(x-1)(x+5)$. Multiply both sides of the equation by the LCD, being careful NOT to allow any of the denominators to be zero. In other words, in this problem, you must be careful that $x\neq 3$, $x\neq 1$ and $x\neq -5$.

$$\frac{1}{(x-3)(x-1)}-\frac{2}{(x+5)(x-1)}=\frac{4}{(x+5)(x-3)}$$

$$\cancel{(x-3)}\,\cancel{(x-1)}\,(x+5)\bullet\frac{1}{\cancel{(x-3)}\,\cancel{(x-1)}}-(x-3)\,\cancel{(x-1)}\,\cancel{(x+5)}\bullet\frac{2}{\cancel{(x+5)}\,\cancel{(x-1)}}=\cancel{(x-3)}\,(x-1)\,\cancel{(x+5)}\bullet\frac{4}{\cancel{(x+5)}\,\cancel{(x-3)}}$$

This looks incredibly BAD, but when you reduce all the fractions, it cleans up nicely. In fact, **ALL THE DENOMINATORS DIVIDE OUT!!** When you reduce all the fractions, this is what is left—**NO FRACTIONS!!**

$$\cancel{(x-3)}\,\cancel{(x-1)}\,(x+5)\bullet\frac{1}{\cancel{(x-3)}\,\cancel{(x-1)}}-(x-3)\,\cancel{(x-1)}\,\cancel{(x+5)}\bullet\frac{2}{\cancel{(x+5)}\,\cancel{(x-1)}}=\cancel{(x-3)}\,(x-1)\,\cancel{(x+5)}\bullet\frac{4}{\cancel{(x+5)}\,\cancel{(x-3)}}$$

When you divide out all the factors, this is all that is left:

$$1\bullet(x+5)-2\bullet(x-3)=4\bullet(x-1)$$

Next, multiply out the parentheses, and combine like terms:

$$x+5-2x+6=4x-4$$
$$-x+11=4x-4$$
$$+x\;+4\qquad +x+4$$
$$15=5x$$
$$x=3$$

Check: Since $x\neq 3$, this value must be rejected since $x=3$ would make a denominator (actually it would make TWO denominators!) zero! Since there are no other solutions, there is NO SOLUTION to this equation.

Final Answer: No Solution!!

P. 212. # 21. $\dfrac{x}{x-5}+\dfrac{12}{x-2}=\dfrac{15}{(x-5)(x-2)}$

Solution: This is a fractional equation. The first step is to find the LCD, which is $(x-5)(x-2)$, and multiply both sides of the equation by the LCD, being careful NOT to allow any of the denominators to be zero. In other words, in this problem, you must be careful that $x \neq 5$ and $x \neq 2$.

$$\frac{x}{x-5}+\frac{12}{x-2}=\frac{15}{(x-5)(x-2)}$$

$$(x-5)(x-2)\bullet\frac{x}{x-5}+(x-5)(x-2)\bullet\frac{12}{x-2}=(x-5)(x-2)\bullet\frac{15}{(x-5)(x-2)}$$

This looks pretty ugly, but when you reduce all the fractions, it really is not bad. In fact, ALL THE DENOMINATORS DIVIDE OUT!! When you reduce all the fractions, this is what is left—NO FRACTIONS!!

$$x(x-2)+12(x-5)=15$$

$$x^2-2x+12x-60=15$$

$$x^2+10x-60-15=0$$

$$x^2+10x-75=0$$

$$(x+15)(x-5)=0$$

$$x=-15, \quad x=5$$

The answer $x=-15$ is valid, since this value of x does NOT make the denominator zero. However, the value of $x=5$ must be rejected, since it DOES make the denominator zero. This is NOT allowed.

The final answer is $x=-15$.

Additional Problems:

Extra Problem #1. $\frac{1}{3x}-\frac{5}{9x}+\frac{1}{9}=0$

Solution: This is a fractional equation. I call this the "fraction hater's delight" because in the very first step you get to eliminate ALL the fractions. What makes this possible? It is an EQUATION, and you are allowed to multiply both sides of an equation by some non-zero number. You multiply both sides of the equation by the LCD, which is $9x$.

$$9x\bullet\frac{1}{3x}-9x\bullet\frac{5}{9x}+9x\bullet\frac{1}{9}=9x\bullet 0$$

Notice that ALL THE DENOMINATORS DIVIDE OUT!! When you reduce all the fractions, this is what is left—NO FRACTIONS!!

$$3-5+x=0$$

$$-2+x=0$$

$$x=2$$

The denominators are NOT zero when x=2. However, it's not a bad idea to check these, if you have time to do so:

$$\frac{1}{3x}-\frac{5}{9x}+\frac{1}{9}=0$$

$$\frac{1}{3(2)}-\frac{5}{9(2)}+\frac{1}{9}=0$$

$$\frac{1}{6}-\frac{5}{18}+\frac{1}{9}=0$$

$$\frac{3}{18}-\frac{5}{18}+\frac{2}{18}=0$$ It checks!!

Extra Problem #2. $\frac{6}{y+2}-\frac{1}{y-5}=\frac{4}{y^2-3y-10}$

Solution: Notice that this is a fractional equation. The first step is to factor the second denominator in order to find the LCD.

$$\frac{6}{y+2}-\frac{1}{y-5}=\frac{4}{(y-5)(y+2)}$$

In this case, the LCD = $(y+2)(y-5)$. Multiply both sides of the equation by the LCD, which is $(y+2)(y-5)$., being careful NOT to allow any of the denominators to be zero. In other words, in this problem, you must be careful that $y \neq -2$ and $y \neq 5$.

$$\frac{6}{y+2}-\frac{1}{y-5}=\frac{4}{(y-5)(y+2)}$$

$$\cancel{(y+2)}(y-5)\bullet\frac{6}{\cancel{y+2}}-(y+2)\cancel{(y-5)}\bullet\frac{1}{\cancel{y-5}}=\cancel{(y+2)}\,\cancel{(y-5)}\bullet\frac{4}{\cancel{(y-5)}\,\cancel{(y+2)}}$$

This looks pretty ugly, but when you reduce all the fractions, it really is not bad. In fact, **ALL THE DENOMINATORS DIVIDE OUT!!** When you reduce all the fractions, this is all that is left—**NO FRACTIONS!!**

$$6(y-5)-1(y+2)=4$$

$$6y-30-y-2=4$$

$$5y-32=4$$

$$5y=36$$

$$y=\frac{36}{5}$$

The answer $y=\frac{36}{5}$ is valid, since this value of x does NOT make the denominator zero.

Extra Problem #3. $\frac{2}{3x}+\frac{1}{4}=\frac{11}{6x}-\frac{1}{3}$

Solution: Notice that this is a fractional equation. This is the "fraction hater's delight" because in the very first step you get to eliminate ALL the fractions. What makes this possible? It is an EQUATION, and you are allowed to multiply both sides of an equation by some non-zero number. You multiply both sides of the equation by the LCD, which is $12x$.

$$12x\bullet\frac{2}{3x}+12x\bullet\frac{1}{4}=12x\bullet\frac{11}{6x}-12x\bullet\frac{1}{3}$$

Notice that **ALL THE DENOMINATORS DIVIDE OUT!!** When you reduce all the fractions, this is what is left—**NO FRACTIONS!!**

$$4\bullet 2+3x\bullet 1=2\bullet 11-4x\bullet 1$$

$$8+3x=22-4x$$

$$8+3x+4x=22-4x+4x$$

$$8+7x=22$$

$$8-8+7x=22-8$$

$$7x=14$$

$$x=2$$

The denominators are NOT zero when x=2. However, it's not a bad idea to check these, if you have time to do so:

$$\frac{2}{3x}+\frac{1}{4}=\frac{11}{6x}-\frac{1}{3}$$

$$\frac{2}{3(2)}+\frac{1}{4}=\frac{11}{6(2)}-\frac{1}{3}$$

$$\frac{2}{(6)}+\frac{1}{4}=\frac{11}{(12)}-\frac{1}{3}$$

$$\frac{4}{(12)}+\frac{3}{12}=\frac{11}{(12)}-\frac{4}{12}$$

$$\frac{7}{12}=\frac{7}{12}$$ It checks!!