Math in Living C O L O R !!

2.07 Literal Equations

Intermediate Algebra: One Step at a Time
Page 207 - 209: #8, Extra Problem, 11, 12, 15, 17, 19

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See Section 2.07 with explanations, examples, and exercises, coming soon!

Explanations, examples, and exercises from Basic Algebra, coming soon!

P. 135. # 8.

$V = \frac{1}{3}\pi r^2 h$, solve for r.

Solution: Since there is a denominator of 3, multiply both sides by 3 to clear the fraction!

$$3 \bullet V = 3 \bullet \frac{1}{3}\pi r^2 h$$

$$3V = \pi r^2 h$$

Next, remember that you are solving for r, and the r^2 has been multiplied by π and h. In order to "undo" the multiplication, you must divide both sides by π and h:

$$\frac{3V}{\pi h} = \frac{\cancel{\pi} r^2 \cancel{h}}{\cancel{\pi}\cancel{h}}$$

$$\frac{3V}{\pi h} = r^2$$

$$r^2 = \frac{3V}{\pi h}$$

In order to "undo" the square, you must take the square root of both sides:

$$r = \sqrt{\frac{3V}{\pi h}}$$

There is no need for a $\pm$ in this case, since r is a radius, and it cannot be negative.

Extra Problem (from Chris).

Solve for x: $a(x-b)=cx+ab$.

Solution: First, remove parentheses by the distributive property.

$$ax-ab=cx+ab$$

Next, get all the x terms on the left side by subtracting cx from each side. At the same time, add $+ab$ to each side to get all the non-x terms on the right side of the equation

$$\begin{array}{rcr} ax-ab & = & cx+ab \\ -cx+ab & & -cx+ab \\ \hline ax-cx & = & 2ab \end{array}$$

Now, factor the common factor of x:

$$x(a-c) = 2ab$$

Finally, since the x has been multiplied by $(a-c)$, you must divide both sides of the equation by $(a-c)$.

$$\frac{x \bullet \cancel{(a-c)}}{\cancel{(a-c)}} = \frac{2ab}{(a-c)}$$

$$x = \frac{2ab}{a-c}$$

P135: #11, $F = \frac{9}{5}C + 32$, solve for C.

There are at least two ways to solve for C in this problem. Both are equally correct, but one is much easier than the other. The easy way to solve this is to notice that the C has had two operations performed on it.

METHOD I:

First, C is multiplied by the fraction $\frac{9}{5}$, and then 32 was added. To solve for C, you must UNDO these two operations in reverse order. So, first undo the $+32$, by subtracting 32 from each side:

$$F = \frac{9}{5}C + 32$$

$$F - 32 = \frac{9}{5}C + 32 - 32$$

$$F - 32 = \frac{9}{5}C$$

Now, undo the multiplication by $\frac{9}{5}$ by multiplying both sides of the equation by the reciprocal of $\frac{9}{5}$ which is $\frac{5}{9}$:

$$F - 32 = \frac{9}{5}C$$

$$\frac{5}{9} \bullet (F - 32) = \frac{5}{9} \bullet \left(\frac{9}{5}C\right)$$

$$\frac{5}{9} \bullet (F - 32) = C$$

METHOD II: P135: #11, $F = \frac{9}{5}C + 32$, **solve for C.**

The other method involves multiplying both sides of the equation by the denominator which is 5**:**

$$F = \frac{9}{5}C + 32$$

$$5 \bullet F = 5 \bullet \frac{9}{5}C + 5 \bullet 32$$

$$5F = 9C + 160$$

Subtract 160 **from each side:**

$$5F - 160 = 9C + 160 - 160$$

$$5F - 160 = 9C$$

To solve for C **just divide both sides by** 9**:**

$$\frac{5F - 160}{9} = \frac{9C}{9}$$

$$C = \frac{5F - 160}{9}$$

This is a slightly different form of the answer obtained in the first method, if you factor out the 5, the answer will be the same as above:

$$C = \frac{5(F - 32)}{9}.$$

P. 135. # 12. $C = \frac{5}{9}(F - 32)$, solve for F.

Notice that this problem is the same as the answer to #11, so the problem will be solved in reverse!

$$C = \frac{5}{9}(F - 32)$$

Begin by "undoing" the fraction $\frac{5}{9}$ by multiplying both sides by $\frac{9}{5}$

$$\frac{9}{5}C = \frac{9}{5} \bullet \frac{5}{9}(F - 32)$$

$$\frac{9}{5}C = F - 32$$

Next, "undo" the -32 by adding a $+32$ to each side of the equation.

$$\frac{9}{5}C = F - 32$$

$$\frac{9}{5}C + 32 = F - 32 + 32$$

$$\frac{9}{5}C + 32 = F$$

$$F = \frac{9}{5}C + 32$$

In conclusion, notice that the problem for #11 is the answer for #12, and vice-versa.

P. 136. # 15.

$$P = \frac{xy}{a+bx}$$, solve for a.

There are two ways to begin this problem, both of which result in the same first step. You might want to "undo" the fraction by multiplying both sides of the equation by $a+bx$, which looks like this:

$$P \bullet (a+bx) = (a+bx) \bullet \frac{xy}{a+bx}$$

$$P \bullet (a+bx) = \cancel{(a+bx)} \bullet \frac{xy}{\cancel{a+bx}}$$

$$Pa + Pbx = xy$$

ALTERNATE METHOD OF ELIMINATING THE FRACTION
The other way to eliminate the fraction is to write it as a fraction equal to a fraction and "cross-multiply." $$\frac{P}{1} = \frac{xy}{a+bx}$$ $$P \bullet (a+bx) = xy \bullet 1$$

The result is EXACTLY the same:

$$Pa + Pbx = xy$$

Now, to solve for a, you must get all the a terms on one side, and the "non-a" terms on the other side. To do this subtract Pbx from each side.

$$Pa + \cancel{Pbx} - \cancel{Pbx} = xy - Pbx$$

$$Pa = xy - Pbx$$

P. 136. # 15 continued.

The last step is to divide both sides by P,

$$\frac{Pa}{P} = \frac{xy - Pbx}{P}$$

The P on the left side divides out, but the P on the right side does NOT divide out! Why??? (See answer below!)

$$\frac{\cancel{P}a}{\cancel{P}} = \frac{xy - Pbx}{P}$$

$$a = \frac{xy - Pbx}{P}$$

Answer to question above: (Because they are **FACTORS** on the left side, but **TERMS** on the right side!!)

P. 136. # 17.

[Notice the similarity between this exercise and #15. At first it looks like exactly the same problem. The difference is that in #15 you were solving for a, whereas in #17 you are solving for x. The problems begin the same, but they are NOT the same!]

$$P = \frac{xy}{a + bx} \quad , \text{ solve for } x.$$

There are two ways to begin this problem, both of which result in the same first step. You might want to "undo" the fraction by multiplying both sides of the equation by $a + bx$, which looks like this:

$$P \bullet (a + bx) = (a + bx) \bullet \frac{xy}{a + bx}$$

$$P \bullet (a + bx) = \cancel{(a + bx)} \bullet \frac{xy}{\cancel{a + bx}}$$

$$Pa + Pbx = xy$$

P. 136. # 17 continued

ALTERNATE METHOD OF ELIMINATING THE FRACTION
The other way to eliminate the fraction, as in #15, is to write it as a fraction equal to a fraction and "cross-multiply." $$\frac{P}{1} = \frac{xy}{a+bx}$$ $$P \bullet (a+bx) = xy \bullet 1$$

The result is EXACTLY the same:

$$Pa + Pbx = xy$$

Now, to solve for x, you must get all the x terms on one side, and the "non-x" terms on the other side. To do this subtract Pbx from each side, exactly as in #15.

$$Pa + \cancel{Pbx} - \cancel{Pbx} = xy - Pbx$$

$$Pa = xy - Pbx$$

Now, to solve for x, you must factor the common factor of x, so as to get the x in one place.

$$Pa = x(y - Pb)$$

$$\frac{Pa}{(y-Pb)} = \frac{x(y-Pb)}{(y-Pb)}$$

Divide out the factors of $(y - Pb)$.

$$\frac{Pa}{y-Pb} = \frac{x\cancel{(y-Pb)}}{\cancel{(y-Pb)}}$$

$$\frac{Pa}{y-Pb} = x$$

P. 137. # 19. $\frac{1}{F} = \frac{1}{S} + \frac{1}{U}$

The first step is to find the LCD, which is FSU (to all the Florida Gator and Miami Hurricane fans out there, GO FLORIDA STATE!!)

$$FSU \bullet \frac{1}{F} = FSU \bullet \frac{1}{S} + FSU \bullet \frac{1}{U}$$

In the first position, the F divides out, leaving SU.

In the second position the S divides out, leaving UF!

In the third position, the U divides out, leaving FS.

$$\cancel{F}\, SU \bullet \frac{1}{\cancel{F}} = F\,\cancel{S}U \bullet \frac{1}{\cancel{S}} + FS\,\cancel{U} \bullet \frac{1}{\cancel{U}}$$
$$SU = UF + FS$$

Now, in order to solve for S, you have to get all the S terms on one side of the equation. You can do that by subtracting FS from each side of the equation.

$$\begin{array}{l} SU = UF + FS \\ \underline{-FS \qquad\qquad\quad -FS} \\ SU - FS = UF \end{array}$$

Now, to solve for S, you have to factor out the S on the left side of the equation:

$$SU - FS = UF$$

$$S(U - F) = UF$$

and divide both sides by $(U-F)$: $\frac{S\,\cancel{(U-F)}}{\cancel{(U-F)}} = \frac{UF}{(U-F)}$

$$S = \frac{UF}{U-F}$$

IMPORTANT NOTE: This problem is very much like my own career, in that after I started (and graduated!) at FSU and I ended up (and graduated also!) at UF—except that I did NOT change colors!!