Math in Living C O L O R !!

3.01 Introduction to Radicals

Intermediate Algebra: One Step at a Time. Page 243 - 248: #31, 32, 33, 44, 53, 59

See Section 3.01, with explanations, examples, and exercises, coming soon!
Explanations, examples, and exercises from Basic Algebra coming soon!

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Radicals are not as hard as you think they are! However, before you do anything with a cube root, you must have these special numbers in mind (or on a piece of paper!) in front of you:
2^3=8, 3^3=27, 4^3=64, 5^3=125.

Memorize them: 8, 27, 64, 125

Before you do anything with a 4th root, be thinking 2^4=16 or 3^4=81.
A 5th root problem will almost always involve 2^5=32.

P. 246, # 31. $\sqrt{45}$

Solution: Find a perfect square that divides into 45. That would be 9 times 5. Place the 9 in the first radical and the 5 in the second radical.

$$\sqrt{} \cdot \sqrt{}$$

$$\sqrt{9} \cdot \sqrt{5}$$

$$3 \cdot \sqrt{5}$$

Since this is a numerical problem you can check the answer by calculating the decimal value of the problem and then calculating the value of the answer to see if these values are the same.

$\sqrt{45}$ **= 6.708203932**

$3 \cdot \sqrt{5}$ **= 6.708203932 -- It checks!!**

Before attempting the following cube root problems, remember the perfect cube numbers:

8, 27, 64, 125

P. 248, # 32. $\sqrt[3]{54}$

Solution: Using the perfect cubes above, find the perfect cube that divides evenly into 54. The only perfect cube that divides evenly into 54 is 27. Break down the 54 into $27 \bullet 2$.

$$\sqrt[3]{} \bullet \sqrt[3]{}$$
$$\sqrt[3]{27} \bullet \sqrt[3]{2}$$
$$3 \bullet \sqrt[3]{2}$$

Since this is a numerical problem you can check the answer by using a calculator to calculate the decimal value of the problem and then calculate the value of the answer to see if these values are the same.

$\sqrt[3]{54}$ = 3.77976315

$3 \bullet \sqrt[3]{2}$ = 3.77976315 -- It checks!!

P. 248, # 33. $\sqrt[3]{80}$

Solution: Before you begin cube root problem, you must remember the perfect cube numbers, i.e., $2^3 = 8,\ \ 3^3 = 27,\ 4^3 = 64,\ \ 5^3 = 125$, and find the perfect cube that divides evenly into 80. The only perfect cube that divides evenly into 80 is 8. Break down the 80 into $8 \bullet 10$.

$$\sqrt[3]{} \bullet \sqrt[3]{}$$
$$\sqrt[3]{8} \bullet \sqrt[3]{10}$$
$$2 \bullet \sqrt[3]{10}$$

Optional numerical check: Use a calculator to calculate the approximate value of the problem, and the approximate value of the answer. See if they agree.

$\sqrt[3]{80}$ = 4.30886938

$2 \bullet \sqrt[3]{10}$ = 4.30886938 -- It checks!!

Preliminary thoughts about variables

Do you remember the law of exponents about raising a power to a power? When you raise a power to a power, you must multiply the exponents! Taking a root of a power is the opposite of raising a power to a power, so you must divide the exponent by the index of the radical. By the way, the index of a square root is 2, the index of a cube root is 3, the index of a fourth root is 4, etc. It is a very good thing when the index of the radical divides evenly into the exponent. A few simple examples might help.

$\sqrt{x^6} = x^3$; $\sqrt[3]{x^6} = x^2$; $\sqrt[4]{x^{12}} = x^3$; $\sqrt[5]{x^{20}} = x^4$; $\sqrt[6]{x^{12}} = x^2$;

$\sqrt{x^{10}} = x^5$; $\sqrt[3]{x^{12}} = x^4$; $\sqrt[4]{x^{20}} = x^5$; $\sqrt[5]{x^{30}} = x^6$; $\sqrt[6]{x^{30}} = x^5$.

If the index does not divide evenly into the exponent, then find an exponent that IS divisible by the index, and break it down as illustrated by the following examples.

$$\sqrt{x^3} = \sqrt{x^2}\sqrt{x} = x\sqrt{x} \qquad \sqrt{x^5} = \sqrt{x^4}\sqrt{x} = x^2\sqrt{x} \qquad \sqrt{x^9} = \sqrt{x^8}\sqrt{x} = x^4\sqrt{x}$$

$$\sqrt[3]{x^4} = \sqrt[3]{x^3}\sqrt[3]{x} = x\sqrt[3]{x} \qquad \sqrt[3]{x^5} = \sqrt[3]{x^3}\sqrt[3]{x^2} = x\sqrt[3]{x^2} \qquad \sqrt[3]{x^{14}} = \sqrt[3]{x^{12}}\sqrt[3]{x^2} = x^4\sqrt[3]{x^2}$$

$$\sqrt[4]{x^{13}} = \sqrt[4]{x^{12}}\sqrt[4]{x} = x^3\sqrt[4]{x} \qquad \sqrt[4]{x^9} = \sqrt[4]{x^8}\sqrt[4]{x} = x^2\sqrt[4]{x} \qquad \sqrt[5]{x^{17}} = \sqrt[5]{x^{15}}\sqrt[5]{x^2} = x^3\sqrt[5]{x^2}$$

P. 246, # 44. $\sqrt[4]{x^{13}y^9}$

Solution: **When taking a fourth root of variables raised to powers, you must divide the exponents by 4. In this case, the exponents are NOT divisible by 4, so find a power that IS divisible by 4, and break it down into two radicals as follows:**

$$\sqrt[4]{} \bullet \sqrt[4]{}$$

Find a power that IS divisible by 4 and place it in the first radical.

$$\sqrt[4]{x^{12}y^8} \bullet \sqrt[4]{}$$

Place all the "leftover" factors in the second radical:

$$\sqrt[4]{x^{12}y^8} \bullet \sqrt[4]{xy}$$

Simplify the first radical by the dividing exponents by the index. The second radical can't be simplified, so leave it alone.

$$x^3y^2 \bullet \sqrt[4]{xy}$$

P. 247, # 53. $\sqrt{98x^7y^{13}}$

Solution: Find a perfect square that divides into 98. That would be 49 times 2. Place the 49 in the first radical, along with powers of x and y (x^6 and y^{12}) that are divisible by 2. Put the "left-over" factors (2, x, and y) in the second radical.

$$\sqrt{} \bullet \sqrt{}$$

$$\sqrt{49x^6y^{12}} \bullet \sqrt{2xy}$$

Final answer: $7x^3y^6 \bullet \sqrt{2xy}$

P. 248, # 59. $\sqrt[4]{32x^8y^6}$

Solution: When taking a fourth root, you must first find a perfect fourth power factor (i.e., $2^4 = 16,\ \ 3^4 = 81$) that divides into 32. For the powers of x and y, you need to find exponents that are divisible by 4. In this case, the power of x is divisible by 4 so x^8 goes in the first radical with the perfect powers. Break down the 32 into $16 \bullet 2$.

$$\sqrt[4]{} \bullet \sqrt[4]{}$$

$$\sqrt[4]{16x^8} \bullet \sqrt[4]{2}$$

The power of y must be broken into $y^4 \bullet y^2$ $\bullet$. Place the perfect fourth power y^4 in the first radical, and the leftover factor y^2 in the second radical:

$$\sqrt[4]{16x^8y^4} \bullet \sqrt[4]{2y^2}$$

Simplify the first radical by taking the fourth root of 16, which is 2, and divide the exponents by the index of the radical which is 4. The second radical can't be simplified, so leave it alone.

$$2x^2y \bullet \sqrt[4]{2y^2}$$