Math in Living C O L O R !!

4.03 Quadratic Formula

Intermediate Algebra: One Step at a Time.
Page 323 -331 #10, 25, 26, 4 extra problems

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See Section 4.03, with explanations, examples, and exercises, coming soon!

P. 326: #10 $x^2 - 25 = 0$

Solution: Of course you probably recognize that this problem can be solved by factoring, and indeed this is the easiest way to solve it. However, you are asked to solve this page of problems by the Quadratic Formula as a means of confirming the method of Quadratic Formula.

Remember that the solution to the quadratic equation $ax^2 + bx + c = 0$ is given by the formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

In the equation: $x^2 - 25 = 0$

$a = 1, \quad b=0, \quad c = -25$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(0) \pm \sqrt{(0)^2 - 4 \bullet 1 \bullet (-25)}}{2 \bullet 1}$$

$$x = \frac{0 \pm \sqrt{0 + 100}}{2}$$

P. 326: #10 continued.

$$x = \frac{0 \pm 10}{2}$$

$$x = \frac{\pm 10}{2}$$

Final answer: $x = \pm 5$

Check by factoring: $x^2 - 25 = 0$

$$(x-5)(x+5) = 0$$

Same final answer: $x = 5;\ x = -5$

P. 330: #25. $2x(2-x) = 3$

Solution: $4x - 2x^2 = 3$

This is a quadratic equation, so the first step is to set it equal to zero, preferably with a positive x^2 term. This can be done by moving everything to the right side, by adding $+2x^2$ and $-4x$ to each side.

$$4x - 2x^2 + 2x^2 - 4x = 3 + 2x^2 - 4x$$

$$0 = 2x^2 - 4x + 3$$

This quadratic equation cannot be solved by factoring, so in this case, the quadratic formula is the best choice of methods.

Remember that the solution to the quadratic equation: $ax^2 + bx + c = 0$

is given by the formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

P. 330: #25 continued.

In the equation: $2x^2 - 4x + 3 = 0$

$$a = 2, \quad b = -4, \quad c = 3$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \bullet 2 \bullet 3}}{2 \bullet 2}$$

$$x = \frac{4 \pm \sqrt{16 - 24}}{4}$$

$$x = \frac{4 \pm \sqrt{-8}}{4}$$

$$x = \frac{4 \pm \sqrt{-4} \bullet \sqrt{2}}{4}$$

$$x = \frac{4 \pm 2i\sqrt{2}}{4}$$

Factor out the common factor of 2 in the numerator, and reduce the fraction.

$$x = \frac{\cancel{2}(2 \pm i\sqrt{2})}{\cancel{4}\ 2}$$

Final answer: $x = \dfrac{2 \pm i\sqrt{2}}{2}$

P. 330 #26. $2x(x+2)=-5$

Solution: The first step is to multiply out the parentheses, and notice that it is a quadratic equation. When you see this, you might as well go ahead and set it equal to zero by adding +5 to each side of the equation.

$$2x^2+4x+5=0$$

This quadratic equation cannot be solved by factoring, so the quadratic formula is the best method.

The solution to $ax^2+bx+c=0$ is given by the formula $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$.

In the equation: $2x^2+4x+5=0$, $a=2$, $b=4$, $c=5$

Write the formula $$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

and substitute $$x=\frac{-(4)\pm\sqrt{(4)^2-4\bullet 2\bullet 5}}{2\bullet 2}$$

$$x=\frac{-4\pm\sqrt{16-40}}{4}$$

$$x=\frac{-4\pm\sqrt{-24}}{4}$$

$$x=\frac{-4\pm\sqrt{-4}\bullet\sqrt{6}}{4}$$

$$x=\frac{-4\pm 2i\sqrt{6}}{4}$$

Factor out the common factor of 2 in the numerator, and reduce the fraction.

$$x=\frac{2(-2\pm i\sqrt{6})}{4}$$

Final answer: $$x=\frac{-2\pm i\sqrt{6}}{2}$$

Additional Problem #1 from Megan

In the equation: $-5x^2+4x-6=0$ because the coefficient of the x^2 term is negative, it's usually a good idea to multiply both sides by -1.

$$5x^2-4x+6=0 \qquad a=5,\quad b=-4,\quad c=-6$$

Write the formula $$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

and substitute $$x=\frac{-(-4)\pm\sqrt{(-4)^2-4\bullet 5\bullet 6}}{2\bullet 5}$$

$$x=\frac{4\pm\sqrt{16-120}}{10}$$

$$x=\frac{4\pm\sqrt{-104}}{10}$$

$$x=\frac{4\pm\sqrt{-4}\bullet\sqrt{26}}{10}$$

$$x=\frac{4\pm 2i\sqrt{26}}{10}$$

Factor out the common factor of 2 in the numerator, and reduce the fraction.

$$x=\frac{2(2\pm i\sqrt{26})}{10}$$

Final answer: $$x=\frac{2\pm i\sqrt{26}}{5}$$

Additional Problem #2 from Andre

Solve the equation: $4x(x-2)=5$

$$4x^2-8x-5=0 \qquad a=4,\quad b=-8,\quad c=-5$$

Write the formula $$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

and substitute $$x=\frac{-(-8)\pm\sqrt{(-8)^2-4\bullet 4\bullet(-5)}}{2\bullet 4}$$

$$x=\frac{8\pm\sqrt{64+80}}{8}$$

$$x=\frac{8\pm\sqrt{144}}{8}$$

NOTE: Because of the perfect square, this problem could have been solved by factoring!

$$x=\frac{8\pm 12}{8}$$

$$x=\frac{8+12}{8} \quad \text{or} \quad x=\frac{8-12}{8}$$

$$x=\frac{20}{8}=\frac{5}{2} \quad \text{or} \quad x=-\frac{4}{8}=-\frac{1}{2}$$

Factoring would have looked like this:

$$4x^2-8x-5=0$$

$$(2x-5)(2x+1)=0$$

$$2x=5 \ \ or \ \ 2x=-1$$

Final answer: $$x=\frac{5}{2} \ \ or \ \ x=-\frac{1}{2}$$

Additional Problem #3 from Andre

Solve the equation $6x^2 - 19x + 15 = 0$ $\quad a = 6, \quad b = -19, \quad c = 15$

Write the formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

and substitute $x = \dfrac{-(-19) \pm \sqrt{(-19)^2 - 4 \bullet 6 \bullet 15}}{2 \bullet 6}$

$$x = \frac{19 \pm \sqrt{361 - 360}}{12}$$

$$x = \frac{19 \pm \sqrt{1}}{12}$$

NOTE: This problem could have been solved by factoring!

$$x = \frac{19 \pm 1}{12}$$

$$x = \frac{19 + 1}{12} \quad \text{or} \quad x = \frac{19 - 1}{12}$$

$$x = \frac{20}{12} = \frac{5}{3} \quad \text{or} \quad x = \frac{18}{12} = \frac{3}{2}$$

Factoring would have been DIFFICULT, but it would have looked like this:

$$6x^2 - 19x + 15 = 0$$

$$(3x - 5)(2x - 3) = 0$$

$$3x = 5 \ \ or \ \ 2x = 3$$

Final answer: $x = \dfrac{5}{3} \ \ or \ \ x = \dfrac{3}{2}$

Additional Problem #4 from Andre

Solve the equation: $5x^2 + 30x + 20 = 0$

Because there is a COMMON FACTOR, it's usually a good idea to either factor the common factor or divide both sides by this common factor. In this case, divide both sides of the equation by 5!

Note: If you do NOT divide both sides by 5, the quadratic formula still works, but the numbers are larger. Dividing by 5 just makes things simpler.

WARNING: NEVER DIVIDE BOTH SIDES OF AN EQUATION BY A VARIABLE!!!

$$5x^2 + 30x + 20 = 0$$

$$x^2 + 6x + 4 = 0 \qquad a = 1, \quad b=6, \quad c = 4$$

Write the formula $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

and substitute $$x = \frac{-(6) \pm \sqrt{(6)^2 - 4 \bullet 1 \bullet 4}}{2 \bullet 1}$$

$$x = \frac{-6 \pm \sqrt{36 - 16}}{2}$$

$$x = \frac{-6 \pm \sqrt{20}}{2}$$

$$x = \frac{-6 \pm \sqrt{4} \bullet \sqrt{5}}{2}$$

$$x = \frac{-6 \pm 2\sqrt{5}}{2}$$

Factor out the common factor of 2 in the numerator, and reduce the fraction.

$$x = \frac{\cancel{2}(-3 \pm \sqrt{5})}{\cancel{2}}$$

Final answer: $x = -3 \pm \sqrt{5}$