

# Math in Living C O L O R !!

## 4.01 Absolute Value Equations and Inequalities

*Intermediate Algebra: One Step at a Time.* Page 306 - 314: #25, Extra Problem

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See Section 4.01, with explanations, examples, and exercises, coming soon!

P. 308: #25.  $|x - 2| < 4$

**Solution:** You know before you ever start this problem that it is a **BETWEENNESS** problem because:

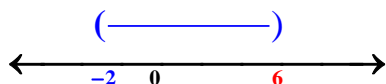
1. it has the **absolute value** sign,
2. it is a “<” or “≤” symbol, and
3. it is compared to a positive number **4**.

Now, you are probably asking, “Between what??” So you need to find the **ENDPOINTS**.

The endpoints will be at:

$$\begin{array}{r} x - 2 = 4 \\ \underline{+2 \quad +2} \\ x = 6 \end{array} \quad \text{and} \quad \begin{array}{r} x - 2 = -4 \\ \underline{+2 \quad +2} \\ x = -2 \end{array}$$

The solution for this inequality is the set of all values of  $x$  that are **BETWEEN** these endpoints, NOT including the endpoints. On a number line, it looks like this:



The final answer in interval notation (from LEFT to RIGHT!) is

$$(-2, 6).$$

**EXTRA PROBLEM**

$$4 - 7 \left| \frac{x}{2} - 3 \right| \geq -3$$

First, isolate the absolute value quantity by subtracting 4 from each side:

$$4 - 4 - 7 \left| \frac{x}{2} - 3 \right| \geq -3 - 4$$

$$-7 \left| \frac{x}{2} - 3 \right| \geq -7$$

Next, divide both sides by -7, remembering to reverse the inequality sign:

$$\frac{-7 \left| \frac{x}{2} - 3 \right|}{-7} \geq \frac{-7}{-7}$$

$$\left| \frac{x}{2} - 3 \right| \leq 1$$

You know that it is a **BETWEENNESS** problem because:

1. it has the **absolute value** sign,
2. it is a “<” or “≤” symbol, and
3. it is compared to a positive number **1**.

Now, you are wondering, “Between what??” So you need to find the **ENDPOINTS**. The endpoints will be at:

$$\frac{x}{2} - 3 = 1$$
$$\begin{array}{r} +3 \quad +3 \\ \hline \end{array}$$

$$\frac{x}{2} = 4$$

$$\frac{x}{2} = 4$$

and

$$\frac{x}{2} - 3 = -1$$
$$\begin{array}{r} +3 \quad +3 \\ \hline \end{array}$$

$$\frac{x}{2} = 2$$

$$\frac{x}{2} = 2$$

Multiply both sides by 2:

$$x = 8$$

$$x = 4$$

The solution for this inequality is the set of all values of x that are **BETWEEN** these endpoints, and **INCLUDING the endpoints**. On a number line, it looks like this:



The final answer in interval notation (from LEFT to RIGHT!) is **[4, 8]**.