Math in Living C O L O R !!

5.03 Equation of a Line

Intermediate Algebra: One Step at a Time
Page 403: 24, Extra Problem, #33a) and b), 34a) and b)

Dr. Robert J. Rapalje, Retired
Central Florida, USA

See Section 5.03, with explanations, examples, and exercises, coming soon!

P. 401. # 24. Find the equation of the line through $(3, -2)$ and $(-4, 4)$.

Solution:

In order to find the equation of any line, you must have a **point (from which to start!)** and a **slope (a direction in which to go!)**. In this case, you are given two points. The first step is to find the slope between the two points, using the well-known formula for the slope between two points:

$$m = \frac{RISE}{RUN} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{RISE}{RUN} = \frac{4 - (-2)}{(-4) - 3}$$

$$m = \frac{RISE}{RUN} = \frac{6}{-7} \, or \, -\frac{6}{7}$$

Now, write down the formula : $y = mx + b$, where $m = -\frac{6}{7}$.

You can use either point. Let's say $x = 3, \quad y = -2$. Substitute the values.

$$-2 = -\frac{6}{7} \bullet 3 + b$$

1

P. 401. # 24 continued.

To clear the fraction, multiply by the denominator which is 7.

$$-2 = -\frac{6}{7} \cdot 3 + b$$

$$7 \cdot (-2) = 7 \cdot -\frac{6}{7} \cdot 3 + 7 \cdot b$$

$$7 \cdot (-2) = \cancel{7} \cdot -\frac{6}{\cancel{7}} \cdot 3 + 7 \cdot b$$

Divide out the 7:

Solve for b:

$$-14 = -18 + 7 \cdot b$$

$$4 = 7b$$

$$b = \frac{4}{7}$$

Be sure to answer the question! Find the equation of the line $y = mx + b$

$$y = -\frac{6}{7}x + \frac{4}{7}$$

Check your answer, be substituting $x = 3$ to see if $y = -2$.

$$y = -\frac{6}{7}x + \frac{4}{7}$$

$$y = -\frac{6}{7}3 + \frac{4}{7}$$

$$y = -\frac{18}{7} + \frac{4}{7}$$

$$y = -\frac{14}{7} = -2$$

It checks!!

Final answer: $y = -\frac{6}{7}x + \frac{4}{7}$

2

Extra Problem:

Find the equation of the line through $(-3, -4)$ and $(3, -6)$.

Solution:

In order to find the equation of any line, you must have a point (from which to start!) and a slope (a direction in which to go!). In this case, you are given TWO points. Your first step, obviously, is to find the slope of the line. You must use the formula for the slope between two points:

Find the slope of the given line by solving for y in terms of x.

$$m = \frac{RISE}{RUN} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-6 - (-4)}{3 - (-3)}$$

$$m = \frac{-6 + 4}{3 + 3}$$

$$m = \frac{-2}{6} = \frac{-1}{3}$$

Now that you have found the slope, there are two methods of finding the equation of the line. While I used to solve these problems by the method of the "point-slope formula," I believe another method is easier. Since this is MY website, I'll show you MY favorite method first, then I'll use the point-slope formula as well. Let me know what YOU think!!

The $y = mx + b$ Method:

The equation $y = mx + b$ has four unknowns: **x** and **y**, which are variables, and **m** and **b** which are constants. You know that $m = \dfrac{-1}{3}$, and you need to find **b**.

You can use either given point, either $(-3, -4)$ or $(3, -6)$ for the values of x and y, and $m = \dfrac{-1}{3}$ to find the value of **b**.

$$y = mx + b$$, where $x = -3$, $y = -4$, and $m = \dfrac{-1}{3}$

Solve for b:
$$-4 = \dfrac{-1}{3}(-3) + b$$

$$-4 = \dfrac{-1}{\cancel{3}}(-\cancel{3}^{-1}) + b$$

$$-4 = +1 + b$$

So, $b = -5$ and the equation is

$$y = mx + b$$

Final answer:
$$y = \dfrac{-1}{3}x - 5$$

You can check this answer by substituting the OTHER values of **x** and **y**, that is $(3, -6)$, into the equation to see if it actually works:

$$y = \dfrac{-1}{3}x - 5$$

$$-6 = \dfrac{-1}{3}(3) - 5$$

$$-6 = -1 - 5$$

$$-6 = -6 \quad \text{IT CHECKS!!}$$

Point-Slope Formula Method

This method uses the famous point-slope formula: $y - y_1 = m(x - x_1)$,

where (x_1, y_1) represents any point on the line. We already know that $m = \dfrac{-1}{3}$.

Let $x_1 = -3$, $y_1 = -4$.

$$y - y_1 = m(x - x_1)$$

$$y - (-4) = \frac{-1}{3}(x - (-3))$$

$$y + 4 = \frac{-1}{3}(x + 3)$$

$$y + 4 = \frac{-1}{3}x - 1$$

$$\underline{\quad -4 \qquad\quad -4 \quad}$$

$$y = \frac{-1}{3}x - 5$$

Final answer:

P. 403. # 33a) Find the equation of the line through $(4,2)$ and parallel to $4x + 3y = 12$.

Solution:

In order to find the equation of any line, you must have a point (from which to start!) and a slope (a direction in which to go!). In this case, you are given a point, but instead of being given the slope of the line, you are given the equation of a given line. Your line must be parallel to this given line, which means that the given line has the same slope as the line you need to find.

Find the slope of the given line by solving for y in terms of x.

$$4x + 3y = 12$$

Add $-4x$ to each side of the equation:

$$3y = -4x + 12$$

Divide both sides by 3:

$$\frac{3y}{3} = \frac{-4x}{3} + \frac{12}{3}$$

$$y = \frac{-4}{3}x + 4$$

P. 403. # 33a) continued.

The slope of the given line is $m = -\dfrac{4}{3}$, so the slope of a line parallel is $m = -\dfrac{4}{3}$.

Now find the equation of a line with $m = -\dfrac{4}{3}$ passing through $(4, 2)$,

Start with the formula: $y = mx + b$, where $m = -\dfrac{4}{3}$, $x = 4$, $y = 2$.

$$2 = \dfrac{-4}{3} \bullet 4 + b$$

To clear the fraction, multiply by the denominator which is 3.

$$3 \bullet 2 = 3 \bullet \dfrac{-4}{3} \bullet 4 + 3 \bullet b$$

Divide out the 3: $6 = -16 + 3b$

Solve for b: $22 = 3b$

$$b = \dfrac{22}{3}$$

Be sure to answer the question! Find the equation of the line $y = mx + b$

$$y = -\dfrac{4}{3}x + \dfrac{22}{3}$$

Check your answer, be substituting $x = 4$ to see if $y = 2$

$$y = -\dfrac{4}{3}x + \dfrac{22}{3}$$

$$y = -\dfrac{4}{3} \bullet 4 + \dfrac{22}{3}$$

$$y = -\dfrac{16}{3} + \dfrac{22}{3}$$

$$y = \dfrac{6}{3} = 2 \text{ It checks!!}$$

Final answer: $y = -\dfrac{4}{3}x + \dfrac{22}{3}$

6

P. 403. # 33b) Find the equation of the line through $(4,2)$ and
perpendicular to $4x + 3y = 12$.

Solution:

In the previous part of this problem, you found the slope of the given line to be
$m = -\dfrac{4}{3}$, so the slope of a line perpendicular to this line is the **negative**

reciprocal of this slope, which is $m = \dfrac{3}{4}$. Now find the equation of a line with

$m = \dfrac{3}{4}$ passing through $(4,2)$,

Start with the formula: $y = mx + b$, where $m = \dfrac{3}{4}$, $x = 4$, $y = 2$.

Substitute: $\qquad\qquad 2 = \dfrac{3}{4} \bullet 4 + b$

In this case, the fraction clears itself, so you can just divide out the 4. However, if
you prefer to work it like most of the other problems like this, you can clear the
fraction as before. The result will be the same. HOWEVER, let's do it the easy
way!!

$$2 = \dfrac{3}{4} \bullet 4 + b$$

Divide out the 4 : $\qquad 2 = 3 + b$

Solve for b: $\qquad\qquad b = -1$

Be sure to answer the question! Find the equation of the line $y = mx + b$

$$y = \dfrac{3}{4}x - 1$$

Check your answer, be substituting $x = 4$ to see if $y = 2$

$$y = \dfrac{3}{4}x - 1$$

Divide out the 4: $\qquad\qquad y = \dfrac{3}{4} \bullet 4 - 1$

$$y = 3 - 1$$

$$y = 2 \text{ It checks!!}$$

Final answer: $y = \dfrac{3}{4}x - 1$

P. 404. # 34a) Find the equation of the line through $(4, 2)$ and parallel to $4x - 3y = 12$.

Solution:

In order to find the equation of any line, you must have a point (from which to start!) and a slope (a direction in which to go!). In this case, you are given a point, but instead of being given the slope of the line, you are given the equation of a given line. Your line must be parallel to this given line, which means that the given line has the same slope as the line you need to find.

Find the slope of the given line by solving for *y* in terms of *x*.
$$4x - 3y = 12$$

Add $-4x$ to each side of the equation:
$$-3y = -4x + 12$$

Divide both sides by -3:
$$\frac{-3y}{-3} = \frac{-4x}{-3} + \frac{12}{-3}$$

$$y = \frac{4}{3}x - 4$$

The slope of the given line is $m = \dfrac{4}{3}$, so the slope of a line parallel is also $m = \dfrac{4}{3}$.

Now find the equation of a line with $m = \dfrac{4}{3}$ passing through $(4, 2)$,

Start with the formula: $y = mx + b$, where $m = \dfrac{4}{3}$, $x = 4$, $y = 2$.

Substitute $2 = \dfrac{4}{3} \bullet 4 + b$

To clear the fraction, multiply by the denominator which is 3.

$$3 \bullet 2 = \cancel{3} \bullet \frac{4}{\cancel{3}} \bullet 4 + 3 \bullet b$$

Divide out the 3: $6 = 16 + 3b$

Solve for b: $-10 = 3b$

$$b = -\frac{10}{3}$$

8

P. 404. # 34a) continued.

Be sure to answer the question! Find the equation of the line $y = mx + b$

$$y = \frac{4}{3}x - \frac{10}{3}$$

Check your answer, be substituting $x = 4$ **to see if** $y = 2$

$$y = \frac{4}{3}x - \frac{10}{3}$$

$$y = \frac{4}{3} \bullet 4 - \frac{10}{3}$$

$$y = \frac{16}{3} - \frac{10}{3}$$

$$y = \frac{6}{3} = 2 \quad \text{It checks!!}$$

Final answer: $y = \frac{4}{3}x - \frac{10}{3}$

P. 403. # 33b) Find the equation of the line through $(4, 2)$ and perpendicular to $4x - 3y = 12$.

Solution:

In the previous part of this problem, you found the slope of the given line to be $m = \dfrac{4}{3}$, so the slope of a line perpendicular to this line is the negative reciprocal of this slope, which is $m = -\dfrac{3}{4}$.

Now find the equation of a line with $m = -\dfrac{3}{4}$ passing through $(4, 2)$,

Start with the formula: $y = mx + b$, where $m = -\dfrac{3}{4}$, $x = 4$, $y = 2$.

$$2 = -\frac{3}{\cancel{4}} \bullet \cancel{4} + b$$

In this case, the fraction clears itself, so you can just divide out the 4. However, if you prefer to work it like most of the other problems like this, you can clear the fraction as before. The result will be the same. HOWEVER, let's do it the easy way!!

$$2 = -\frac{3}{\cancel{4}} \bullet \cancel{4} + b$$

Divide out the 4 : $2 = -3 + b$

Solve for b: $b = 5$

Be sure to answer the question! Find the equation of the line $y = mx + b$

$$y = -\frac{3}{4}x + 5$$

Check your answer, be substituting $x = 4$ to see if $y = 2$

$$y = -\frac{3}{4}x + 5$$

Divide out the 4: $\qquad\qquad y = -\dfrac{3}{\cancel{4}} \bullet \cancel{4} + 5$

$$y = -3 + 5$$

$$y = 2 \ \text{It checks!!}$$

Final answer: $y = -\dfrac{3}{4}x + 5$.