Math in Living C O L O R !!

1.06 Radicals and Fractional Exponents

Rationalizing Monomial Denominators

College Algebra: One Step at a Time, Page 77-79: #13, 17, 21, 23, 25, 27.

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See Section 1.06, with explanations, examples, and exercises, coming soon!

P.78. # 13. $$\frac{6}{\sqrt[3]{3}}$$

Solution: Notice that the denominator has a cube root that is not a perfect cube! The goal, in rationalizing the denominator, is to get a perfect cube for the denominator. In this case, it would be nice to get a denominator like $\sqrt[3]{3^3}$. To do this, you need to multiply numerator and denominator by $\sqrt[3]{3^2}$. It looks like this:

$$\frac{6}{\sqrt[3]{3}} \cdot \frac{\sqrt[3]{3^2}}{\sqrt[3]{3^2}}$$

$$\frac{6\sqrt[3]{9}}{\sqrt[3]{3^3}}$$

$$\frac{6\sqrt[3]{9}}{3}$$

Divide out the factor of 3:

Final answer: $2\sqrt[3]{9}$

As a check, calculate the value of the problem: $\frac{6}{\sqrt[3]{3}}$ = 4.160167646

then calculate the value of your answer : $2\sqrt[3]{9}$ = 4.160167646

P. 78. # 17. $\frac{35}{\sqrt[3]{49}}$

Solution: Notice that the denominator has a cube root that is not a perfect cube! The denominator does factor into $\sqrt[3]{7^2}$. The goal, in rationalizing the denominator, is to get a perfect cube for the denominator. In this case, it would be nice to get a denominator like $\sqrt[3]{7^3}$. To do this, you need to multiply numerator and denominator by $\sqrt[3]{7}$. It looks like this:

$$\frac{35}{\sqrt[3]{7^2}} \cdot \frac{\sqrt[3]{7}}{\sqrt[3]{7}}$$

$$\frac{35\sqrt[3]{7}}{\sqrt[3]{7^3}}$$

$$\frac{\overset{5}{\cancel{35}}\sqrt[3]{7}}{\cancel{7}}$$

Divide out the factor of 7:

Final answer: $5\sqrt[3]{7}$

As a check, calculate the value of the problem: $\frac{35}{\sqrt[3]{49}}$ = 9.564655914 . . .

then calculate the value of your answer : $5\sqrt[3]{7}$ = 9.564655914 . . .

P. 79: #21. $\dfrac{12x^4y^2}{\sqrt[3]{4x^5y}}$

Solution: Notice that the denominator has a cube root that is not a perfect cube! The denominator does contains 4 which is 2^2. The goal, in rationalizing the denominator, is to get a perfect cube for the denominator. In this case, it would be nice to get a denominator like $\sqrt[3]{2^3}$. Since you already have $\sqrt[3]{2^2}$, to get the $\sqrt[3]{2^3}$, you need to multiply numerator and denominator by $\sqrt[3]{2}$. As to the variables x^5y, for perfect cubes, it is necessary to get exponents that are divisible by 3, such as x^6y^3. Since you already have the $\sqrt[3]{x^5y}$, to get the $\sqrt[3]{x^6y^3}$, you will need to multiply numerator and denominator by $\sqrt[3]{xy^2}$. Altogether, it looks like this:

$$\frac{12x^4y^2}{\sqrt[3]{4x^5y}} \bullet \frac{\sqrt[3]{2xy^2}}{\sqrt[3]{2xy^2}}$$

$$\frac{12x^4y^2\sqrt[3]{2xy^2}}{\sqrt[3]{8x^6y^3}}$$

The denominator is (of course!) a perfect cube, so you can simplify this:

$$\frac{12x^4y^2\sqrt[3]{2xy^2}}{2x^2y}$$

The fraction reduces, so divide out the $2x^2y$.

Final answer: $6x^2y\sqrt[3]{2xy^2}$

P. 79. #23. $\dfrac{40xy^2}{\sqrt[3]{25x^2y^4}}$

Solution: Notice that the denominator has a cube root that is not a perfect cube! The denominator does contains 25 which is 5^2. The goal, in rationalizing the denominator, is to get a perfect cube for the denominator. In this case, it would be nice to get a denominator like $\sqrt[3]{5^3}$. Since you already have $\sqrt[3]{5^2}$, to get the $\sqrt[3]{5^3}$, you need to multiply numerator and denominator by $\sqrt[3]{5}$. As to the variables x^2y^4, for perfect cubes, it is necessary to get exponents that are divisible by 3, such as x^3y^6. Since you already have the $\sqrt[3]{x^2y^4}$, to get the $\sqrt[3]{x^3y^6}$, you will need to multiply numerator and denominator by $\sqrt[3]{xy^2}$. Altogether, it looks like this:

$$\frac{40xy^2}{\sqrt[3]{25x^2y^4}} \cdot \frac{\sqrt[3]{5xy^2}}{\sqrt[3]{5xy^2}}$$

$$\frac{40xy^2\sqrt[3]{5xy^2}}{\sqrt[3]{125x^3y^6}}$$

The denominator is (of course!) a perfect cube, so you can simplify this:

$$\frac{40xy^2\sqrt[3]{5xy^2}}{5xy^2}$$

The fraction reduces, so divide out the $5xy^2$.

Final answer: $8\sqrt[3]{5xy^2}$

P. 79: 25. $$\frac{12x^4y^2}{\sqrt[5]{4x^2y^3}} = \frac{12x^4y^2}{\sqrt[5]{2^2x^2y^3}}$$

Solution: Notice that the denominator has a fifth root that is not a perfect fifth power! The denominator does contains 4 which is 2^2. The goal, in rationalizing this denominator, is to get a perfect fifth power for the denominator. In this case, it would be nice to get a denominator like $\sqrt[5]{2^5}$. Since you already have $\sqrt[5]{2^2}$, to get the $\sqrt[5]{2^5}$, you need to multiply numerator and denominator by $\sqrt[5]{2^3}$. As to the variables x^2y^3, for perfect fifth powers, it is necessary to get exponents that are divisible by 5, such as x^5y^5. Since you already have the $\sqrt[5]{x^2y^3}$, to get the $\sqrt[5]{x^5y^5}$, you will need to multiply numerator and denominator by $\sqrt[5]{x^3y^2}$. Altogether, it looks like this:

$$\frac{12x^4y^2}{\sqrt[5]{2^2x^2y^3}} \bullet \frac{\sqrt[5]{2^3x^3y^2}}{\sqrt[5]{2^3x^3y^2}}$$

$$\frac{12x^4y^2\sqrt[5]{2^3x^3y^2}}{\sqrt[5]{2^5x^5y^5}}$$

The denominator is (of course!) a perfect cube, so you can simplify this:

$$\frac{12x^4y^2\sqrt[5]{2^3x^3y^2}}{2xy}$$

The fraction reduces, so divide out the $2xy$.

Final answer: $$6x^3y\sqrt[5]{8x^3y^2}$$

P. 79: #27. $$\frac{12x^4y^2}{\sqrt[5]{16x^4y^6}} = \frac{12x^4y^2}{\sqrt[5]{2^4x^4y^6}}$$

Solution: Notice that the denominator has a fifth root that is not a perfect fifth power! The denominator contains 16 which is 2^4. The goal, in rationalizing this denominator, is to get a perfect fifth power for the denominator. In this case, it would be nice to get a denominator like $\sqrt[5]{2^5}$. Since you already have $\sqrt[5]{2^4}$, to get the $\sqrt[5]{2^5}$, you need to multiply numerator and denominator by $\sqrt[5]{2}$. As to the variables x^4y^6, for perfect fifth powers, it is necessary to get exponents that are divisible by 5, such as x^5y^{10}. Since you already have the $\sqrt[5]{x^4y^6}$, to get the $\sqrt[5]{x^5y^{10}}$, you will need to multiply numerator and denominator by $\sqrt[5]{xy^4}$. Altogether, it looks like this:

$$\frac{12x^4y^2}{\sqrt[5]{2^4x^4y^6}} \cdot \frac{\sqrt[5]{2xy^4}}{\sqrt[5]{2xy^4}}$$

$$\frac{12x^4y^2\sqrt[5]{2xy^4}}{\sqrt[5]{2^5x^5y^{10}}}$$

The denominator is (of course!) a perfect cube, so you can simplify this:

$$\frac{12x^4y^2\sqrt[5]{2xy^4}}{2xy^2}$$

The fraction reduces, so divide out the $2xy^2$.

Final answer: $6x^3\sqrt[5]{2xy^4}$