

# Math in Living C O L O R !!

## 1.06 Radicals and Fractional Exponents

### Rationalizing Binomial Denominators

*College Algebra: One Step at a Time*, Page 80-82: #3, 5, 6, 8, 9, 15, 18, 3 Extra.

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See Section 1.06, with explanations, examples, and exercises, coming soon!

**P. 80. # 3.** 
$$\frac{15}{\sqrt{5} + 5\sqrt{2}}$$

**Solution:** Rationalize the denominator by multiplying numerator and denominator of the fraction by the conjugate of the denominator. That is the same as the denominator by with opposite sign:  $(\sqrt{5} - 5\sqrt{2})$ .

$$\frac{15}{\sqrt{5} + 5\sqrt{2}} \frac{(\sqrt{5} - 5\sqrt{2})}{(\sqrt{5} - 5\sqrt{2})}$$

It is best to multiply out (FOIL) the denominator, but it's a good idea to leave the numerator in factored form.

$$\frac{15(\sqrt{5} - 5\sqrt{2})}{5 - 5\sqrt{10} + 5\sqrt{10} - 25 \cdot 2}$$

The middle term subtracts out, and the  $5 - 50$  equals  $-45$ :

$$\frac{15(\sqrt{5} - 5\sqrt{2})}{-45}$$

This reduces by dividing out the  $15$  with the  $-45$ :

$$\frac{(\sqrt{5} - 5\sqrt{2})}{-3}$$

The tradition in math is to avoid negative denominators, so let's multiply numerator and denominator times  $-1$ .

$$\frac{-1 \cdot (\sqrt{5} - 5\sqrt{2})}{-1 \cdot -3}$$

**Final answer:** 
$$\frac{-\sqrt{5} + 5\sqrt{2}}{3} \text{ or } \frac{5\sqrt{2} - \sqrt{5}}{3}$$

**P. 81. # 5.**  $\frac{12}{4+2\sqrt{3}}$

**Solution:** Before beginning this problem, notice that you could factor the denominator and reduce the fraction. It turns out that this step is well worthwhile.

$$\frac{12}{2(2+\sqrt{3})}$$

Reduce the fraction by dividing out the 12 and the 2.

$$\frac{6}{(2+\sqrt{3})}$$

Rationalize the denominator by multiplying numerator and denominator of the fraction by the conjugate of the denominator. That is the same as the denominator by with opposite sign:  $(2-\sqrt{3})$ .

$$\frac{6}{(2+\sqrt{3})} \frac{(2-\sqrt{3})}{(2-\sqrt{3})}$$

It is best to multiply out (FOIL) the denominator, but it's a good idea to leave the numerator in factored form.

$$\frac{6(2-\sqrt{3})}{4-2\sqrt{3}+2\sqrt{3}-3}$$

The middle term subtracts out, and the  $4-3$  equals 1

**Final answer:**  $\frac{6(2-\sqrt{3})}{1}$  or  $6(2-\sqrt{3})$  or  $12-6\sqrt{3}$ .

**P. 81. # 6.**  $\frac{12}{6-3\sqrt{3}}$

**Solution:** Before beginning this problem, notice that you could factor the denominator and reduce the fraction. It turns out that this step is well worthwhile.

$$\frac{12}{3(2-\sqrt{3})}$$

Reduce the fraction by dividing out the 12 and the 3.

$$\frac{4}{(2-\sqrt{3})}$$

Rationalize the denominator by multiplying numerator and denominator of the fraction by the conjugate of the denominator. That is the same as the denominator by with opposite sign:  $(2+\sqrt{3})$ .

$$\frac{4}{(2-\sqrt{3})} \frac{(2+\sqrt{3})}{(2+\sqrt{3})}$$

It is best to multiply out (FOIL) the denominator, but it's a good idea to leave the numerator in factored form.

$$\frac{4(2+\sqrt{3})}{4+2\sqrt{3}-2\sqrt{3}-3}$$

The middle term subtracts out, and the  $4-3$  equals 1

**Final answer:**  $\frac{4(2+\sqrt{3})}{1}$  or  $4(2+\sqrt{3})$  or  $8+4\sqrt{3}$ .

**P. 81. # 8.**  $\frac{6}{3\sqrt{2} + 4\sqrt{3}}$

**Solution:** Rationalize the denominator by multiplying numerator and denominator of the fraction by the conjugate of the denominator. That is the same as the denominator by with opposite sign:  $(3\sqrt{2} - 4\sqrt{3})$ .

$$\frac{6}{(3\sqrt{2} + 4\sqrt{3})} \cdot \frac{(3\sqrt{2} - 4\sqrt{3})}{(3\sqrt{2} - 4\sqrt{3})}$$

It is best to multiply out (FOIL) the denominator, but it's a good idea to leave the numerator in factored form.

$$\frac{6(3\sqrt{2} - 4\sqrt{3})}{9 \cdot 2 - 12\sqrt{6} + 12\sqrt{6} - 16 \cdot 3}$$

The middle term always subtracts out!

$$\frac{6(3\sqrt{2} - 4\sqrt{3})}{18 - 48}$$

Simplify the denominator

$$\frac{6(3\sqrt{2} - 4\sqrt{3})}{-30}$$

Divide out the 6 and -30.

$$\frac{(3\sqrt{2} - 4\sqrt{3})}{-5}$$

Avoid negative denominators, so multiply numerator and denominator by -1.

$$\frac{(-1)}{(-1)} \cdot \frac{(3\sqrt{2} - 4\sqrt{3})}{-5}$$

**Final answer:**  $\frac{-3\sqrt{2} + 4\sqrt{3}}{5}$  or  $\frac{4\sqrt{3} - 3\sqrt{2}}{5}$

**Extra Problem #1.**  $\frac{\sqrt{20}}{6-\sqrt{6}}$

**Solution:** Rationalize the denominator by multiplying numerator and denominator of the fraction by the conjugate of the denominator. That is the same as the denominator by with opposite sign:  $(6+\sqrt{6})$ .

$$\frac{\sqrt{20}}{6-\sqrt{6}} \frac{(6+\sqrt{6})}{(6+\sqrt{6})}$$

It is best to multiply out (FOIL) the denominator, and since there are also radicals in the numerator, it will be necessary to multiply out the numerator as well. It might be helpful to go ahead and simplify  $\sqrt{20} = \sqrt{4}\sqrt{5}$  or  $2\sqrt{5}$

$$\frac{2\sqrt{5}(6+\sqrt{6})}{36+6\sqrt{6}-6\sqrt{6}-6}$$

The middle term always subtracts out!

$$\frac{12\sqrt{5}+2\sqrt{30}}{36-6}$$

Simplify the numerator and denominator.

$$\frac{12\sqrt{5}+2\sqrt{30}}{30}$$

$$\frac{2(6\sqrt{5}+\sqrt{30})}{30}$$

$$\frac{\cancel{2}(6\sqrt{5}+\sqrt{30})}{\cancel{30} 15}$$

**Final answer:**  $\frac{6\sqrt{5}+\sqrt{30}}{15}$

Since this is a numerical problem, you can check with your calculator by calculating the decimal approximation of the problem and of the answer to see if they agree.

$$\frac{\sqrt{20}}{6-\sqrt{6}} = 1.259575563$$

$$\frac{6\sqrt{5}+\sqrt{30}}{15} = 1.259575563 \quad \text{It checks!!}$$

**Extra Problem #2.**  $\frac{\sqrt{20}}{6 + \sqrt{6}}$

**Solution:** Rationalize the denominator by multiplying numerator and denominator of the fraction by the conjugate of the denominator. That is the same as the denominator by with opposite sign:  $(6 - \sqrt{6})$ .

$$\frac{\sqrt{20}}{6 + \sqrt{6}} \frac{(6 - \sqrt{6})}{(6 - \sqrt{6})}$$

It is best to multiply out (FOIL) the denominator, and since there are also radicals in the numerator, it will be necessary to multiply out the numerator as well. It might be helpful to go ahead and simplify  $\sqrt{20} = \sqrt{4}\sqrt{5}$  or  $2\sqrt{5}$ .

$$\frac{2\sqrt{5}(6 - \sqrt{6})}{36 - 6\sqrt{6} + 6\sqrt{6} - 6}$$

The middle term always subtracts out!

$$\frac{12\sqrt{5} - 2 \cdot \sqrt{30}}{36 - 6}$$

Simplify the numerator and denominator.

$$\frac{12\sqrt{5} - 2 \cdot \sqrt{30}}{30}$$

$$\frac{2(6\sqrt{5} - \sqrt{30})}{30}$$

$$\frac{\cancel{2}(6\sqrt{5} - \sqrt{30})}{\cancel{30} 15}$$

$$\frac{6\sqrt{5} - \sqrt{30}}{15}$$

Since this is a numerical problem, you can check with your calculator by calculating the decimal approximation of the problem and of the answer to see if they agree.

$$\frac{\sqrt{20}}{6 + \sqrt{6}} = 0.5292788193$$

$$\frac{6\sqrt{5} - \sqrt{30}}{15} = 0.5292788193 \quad \text{It checks!!}$$

P. 81. #9.

$$\frac{\sqrt{27}}{2\sqrt{6} - 3\sqrt{3}}$$

**Solution:** Rationalize the denominator by multiplying numerator and denominator of the fraction by the conjugate of the denominator. That is the same as the denominator by with opposite sign:  $(2\sqrt{6} + 3\sqrt{3})$ .

$$\frac{\sqrt{27}}{(2\sqrt{6} - 3\sqrt{3})} \frac{(2\sqrt{6} + 3\sqrt{3})}{(2\sqrt{6} + 3\sqrt{3})}$$

It is best to multiply out (FOIL) the denominator, and since there are also radicals in the numerator, it will be necessary to multiply out the numerator as well. It might be helpful to go ahead and simplify  $\sqrt{27} = \sqrt{9}\sqrt{3}$  or  $3\sqrt{3}$ .

$$\frac{3\sqrt{3}(2\sqrt{6} + 3\sqrt{3})}{4 \cdot 6 + 6\sqrt{18} - 6\sqrt{18} - 9 \cdot 3}$$

The middle term always subtracts out!

$$\frac{6\sqrt{18} + 9 \cdot 3}{24 - 27}$$

Simplify the numerator and denominator.

$$\frac{6\sqrt{9}\sqrt{2} + 27}{-3}$$
$$\frac{6 \cdot 3\sqrt{2} + 27}{-3}$$
$$\frac{18\sqrt{2} + 27}{-3}$$
$$\frac{9(2\sqrt{2} + 3)}{-3}$$

**Final answer:**  $-3(2\sqrt{2} + 3)$  or  $-6\sqrt{2} - 9$

**Extra Problem #3.**  $\frac{\sqrt{6} - \sqrt{3}}{\sqrt{6} + \sqrt{3}}$

**Solution:** Rationalize the denominator by multiplying numerator and denominator of the fraction by the conjugate of the denominator. That is the same as the denominator by with opposite sign:  $(\sqrt{6} - \sqrt{3})$ .

$$\frac{\sqrt{6} - \sqrt{3}}{\sqrt{6} + \sqrt{3}} \cdot \frac{(\sqrt{6} - \sqrt{3})}{(\sqrt{6} - \sqrt{3})}$$

In this case, you must multiply out (**F O I L**) both the numerator and the denominator.

$$\frac{6 - \sqrt{18} - \sqrt{18} + 3}{6 - \sqrt{18} + \sqrt{18} - 3}$$

Simplify numerator and denominator.

$$\frac{9 - 2\sqrt{18}}{3}$$

$$\frac{9 - 2\sqrt{9} \cdot \sqrt{2}}{3}$$

$$\frac{9 - 6 \cdot \sqrt{2}}{3}$$

Factor the numerator in order to reduce the fraction.

$$\frac{\cancel{3}(3 - 2 \cdot \sqrt{2})}{\cancel{3}}$$

**Final answer:**  $3 - 2 \cdot \sqrt{2}$

This is a great place to check with the calculator! Use your calculator to calculate the problem. Be sure to put parentheses around the numerator and denominator, and if your calculator opens parentheses when you enter square root, be sure to close the parentheses after you enter the number!

Problem =  $\frac{(\sqrt{6} - \sqrt{3})}{(\sqrt{6} + \sqrt{3})}$  should give you **.1715728753 ...**

Answer =  $3 - 2 \cdot \sqrt{2}$  also should give you **.1715728753 ...**

This checks!!



**P. 82. #15.** 
$$\frac{4\sqrt{5} + 5\sqrt{2}}{3\sqrt{2} - 2\sqrt{5}}$$

**Solution:** Rationalize the denominator by multiplying numerator and denominator of the fraction by the conjugate of the denominator. That is the same as the denominator by with opposite sign:  $(3\sqrt{2} + 2\sqrt{5})$ .

$$\frac{4\sqrt{5} + 5\sqrt{2}}{3\sqrt{2} - 2\sqrt{5}} \frac{(3\sqrt{2} + 2\sqrt{5})}{(3\sqrt{2} + 2\sqrt{5})}$$

In this case, you must multiply out (F O I L) both the numerator and the denominator.

$$\frac{12 \bullet \sqrt{10} + 8\sqrt{25} + 15\sqrt{4} + 10 \bullet \sqrt{10}}{9 \bullet 2 + 6\sqrt{10} - 6\sqrt{10} - 4 \bullet 5}$$

Simplify numerator and denominator.

$$\frac{22\sqrt{10} + 8 \bullet 5 + 15 \bullet 2}{18 - 20}$$

$$\frac{22\sqrt{10} + 40 + 30}{-2}$$

$$\frac{22\sqrt{10} + 70}{-2}$$

Factor the numerator in order to reduce the fraction.

$$\frac{2(11\sqrt{10} + 35)}{-2}$$

**Final answer:**  $-1(11\sqrt{10} + 35)$  or  $-11\sqrt{10} - 35$

Finally, as a check, if you calculate the decimal approximation of the problem and of the answer, you should get approximately -69.78505.

**P. 82. #18.**

$$\frac{3\sqrt{10} - 2\sqrt{6}}{4\sqrt{10} + 5\sqrt{6}}$$

**Solution:** Rationalize the denominator by multiplying numerator and denominator of the fraction by the conjugate of the denominator. That is the same as the denominator by with opposite sign:  $(4\sqrt{10} - 5\sqrt{6})$ .

$$\frac{(3\sqrt{10} - 2\sqrt{6})(4\sqrt{10} - 5\sqrt{6})}{(4\sqrt{10} + 5\sqrt{6})(4\sqrt{10} - 5\sqrt{6})}$$

In this case, you must multiply out (**F O I L**) both the numerator and the denominator.

$$\frac{12 \cdot 10 - 15\sqrt{60} - 8\sqrt{60} + 10 \cdot 6}{16 \cdot 10 - 20\sqrt{60} + 20\sqrt{60} - 25 \cdot 6}$$

Simplify numerator and denominator.

$$\frac{120 - 23\sqrt{60} + 60}{160 - 150}$$
$$\frac{180 - 23\sqrt{4}\sqrt{15}}{10}$$
$$\frac{180 - 23 \cdot 2\sqrt{15}}{10}$$
$$\frac{180 - 46\sqrt{15}}{10}$$

Factor the numerator in order to reduce the fraction.

$$\frac{\cancel{2}(90 - 23\sqrt{15})}{\cancel{10} 5}$$
$$\frac{90 - 23\sqrt{15}}{5}$$

**Final answer:**