Math in Living C O L O R !!

1.06 Radicals and Fractional Exponents

College Algebra: One Step at a Time
Page 66-76: #38, 39, 41, 42, 43, 45, 53, 67, 70, 80, 88, 89, 90.

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See Section 1.06, with explanations, examples, and exercises, **coming soon!**

Radicals are not as hard as you think they are! However, before you do anything with a cube root, you must have these special numbers in mind (or on a piece of paper!) in front of you:
$2^3=8$, $3^3=27$, $4^3=64$, $5^3=125$.

Memorize them: ***8, 27, 64, 125***

Before you do anything with a 4^{th} root, be thinking $2^4=16$ or $3^4=81$.
A 5^{th} root problem will almost always involve $2^5=32$.

P. 70: #38. $5\sqrt[3]{108} - 4\sqrt[3]{32}$

Solution: Find a perfect cube that divides into 108 (that would be 27) and a perfect cube that divides into 32 (that would be 8). 108=27•4 and 32=8•4.

$$5\sqrt[3]{108} - 4\sqrt[3]{32}$$

$$5\sqrt[3]{27\bullet 4} - 4\sqrt[3]{8\bullet 4}$$

$$5\sqrt[3]{27}\bullet\sqrt[3]{4} - 4\sqrt[3]{8}\bullet\sqrt[3]{4}$$

$$5\bullet 3\bullet\sqrt[3]{4} - 4\bullet 2\bullet\sqrt[3]{4}$$

Now multiply the numbers 5 times 3 and the 4 times 2.

$$15\sqrt[3]{4} - 8\sqrt[3]{4}$$

Combine like radicals.

Final answer: $7\sqrt[3]{4}$

In # 39, you have 4th roots, so keep in mind that $2^4=16$ and $3^4=81$.

P. 70: #39. $7\sqrt[4]{32} - 3\sqrt[4]{162}$

Solution: Find a perfect 4th power that divides into 32 (that would be 16) and one that divides into 162 (that would be 81). $32=16\cdot 2$ and $162=81\cdot 2$.

$$7\sqrt[4]{32} - 3\sqrt[4]{162}$$

$$7\sqrt[4]{16\bullet 2} - 3\sqrt[4]{81\bullet 2}$$

$$7\sqrt[4]{16}\bullet\sqrt[4]{2} - 3\sqrt[4]{81}\bullet\sqrt[4]{2}$$

$$7\bullet 2\bullet\sqrt[4]{2} - 3\bullet 3\bullet\sqrt[4]{2}$$

Now multiply the numbers 7 times 2 and the 3 times 3.

$$14\sqrt[4]{2} - 9\sqrt[4]{2}$$

Now combine like radicals.

Final answer: $5\sqrt[4]{2}$

P. 71: #41. $7x^2\sqrt{24xy^6} + 8y^3\sqrt{54x^5}$

$$7x^2\sqrt{}\sqrt{} + 8y^3\sqrt{}\sqrt{}$$

Solution: First, separate each of the square roots into two square roots. Sort out the square roots into perfect squares that go in the first (red) square root, and the left-over factors that go in the second (blue) square root.

$$7x^2\sqrt{4y^6}\sqrt{6x} + 8y^3\sqrt{9x^4}\sqrt{6x}$$

Everyone can take the square root of the first (red) radicals since they are perfect squares. Nobody knows what to do about the second (blue) radical since they cannot be simplified. So do what you can do (the red radicals), and leave the rest (blue radicals!) alone:

$$7x^2\bullet 2y^3\sqrt{6x} + 8y^3\bullet 3x^2\sqrt{6x}$$

As in the last step, you do what you are able to do next--multiply outside the radicals:

$$14x^2y^3\sqrt{6x} + 24x^2y^3\sqrt{6x}$$

Notice that you have like radicals and like terms. They combine together!

Final answer: $38x^2y^3\sqrt{6x}$

P. 71: #42. $5xy\sqrt{20x^7y^5} - 4\sqrt{45x^9y^7}$

Solution: $5xy\sqrt{\quad}\sqrt{\quad} - 4\sqrt{\quad}\sqrt{\quad}$

First, separate each of the square roots into two square roots. Sort out the square roots into perfect squares that go in the first (red) square root, and the left-over factors that go in the second (blue) square root.

$$5xy\sqrt{4x^6y^4}\sqrt{5xy} - 4\sqrt{9x^8y^6}\sqrt{5xy}$$

Everyone can take the square root of the first (red) radicals since they are perfect squares. Nobody knows what to do about the second (blue) radical since they cannot be simplified. So do what you can do (the red radicals), and leave the rest (blue radicals!) alone:

$$5xy \bullet 2x^3y^2\sqrt{5xy} - 4 \bullet 3x^4y^3\sqrt{5xy}$$

Multiply outside the radicals:

$$10x^4y^3\sqrt{5xy} - 12x^4y^3\sqrt{5xy}$$

Notice that you have like radicals and like terms. They combine together:

Final answer: $-2x^4y^3\sqrt{5xy}$

P. 71: #43. $5x^2y\sqrt[3]{54x^7y^5} - 4xy^2\sqrt[3]{16x^{10}y^2}$

Solution: $5x^2y\sqrt[3]{\quad}\sqrt[3]{\quad} - 4xy^2\sqrt[3]{\quad}\sqrt[3]{\quad}$

First, separate each of the cube roots into two cube roots. Sort out the factors that are perfect cubes and place them in the first (red) cube root, and place any left-over factors in the second (blue) cube root.

$$5x^2y\sqrt[3]{27x^6y^3}\sqrt[3]{2xy^2} - 4xy^2\sqrt[3]{8x^9}\sqrt[3]{2xy^2}$$

Everyone can take the cube root of the first (red) radicals since they are perfect cubes. Nobody knows what to do about the second (blue) radical since they cannot be simplified. So do what you can do (the red radicals), and leave the rest (blue radicals!) alone:

$$5x^2y \bullet 3x^2y\sqrt[3]{2xy^2} - 4xy^2 \bullet 2x^3\sqrt[3]{2xy^2}$$

As in the last step, you do what you are able to do next--multiply outside the radicals:

$$15x^4y^2\sqrt[3]{2xy^2} - 8x^4y^2\sqrt[3]{2xy^2}$$

Notice that these are like radicals and like terms. They combine together:

Final answer: $7x^4y^2\sqrt[3]{2xy^2}$

P. 71: #45. $3x^2y\sqrt{20xy^4} - 2x\sqrt{45x^3y^6}$

Solution: $3x^2y\sqrt{}\sqrt{} - 2x\sqrt{}\sqrt{}$

First, separate each of the square roots into two square roots. Sort out the perfect squares, and place these in the first (red) square root, and put the left-over factors in the second (blue) square root.

$$3x^2y\sqrt{4y^4}\sqrt{5x} - 2x\sqrt{9x^2y^6}\sqrt{5x}$$

Everyone can take the square root of the first (red) radicals since they are perfect squares. Nobody knows what to do about the second (blue) radical since they cannot be simplified. So do what you can do (the red radicals), and leave the rest (blue radicals!) alone:

$$3x^2y \bullet 2y^2\sqrt{5x} - 2x \bullet 3xy^3\sqrt{5x}$$

As in the last step, you do what you are able to do next--multiply outside the radicals:

$$6x^2y^3\sqrt{5x} - 6x^2y^3\sqrt{5x}$$

Notice that you have like radicals. They subtract out.

Final answer: 0

P. 72: #53. $\sqrt[3]{12} \bullet \sqrt[3]{6}$

Solution: Since there are no obvious perfect cube factors in this problem, use the product property of radicals to multiply 12 times 6. In this case, the numbers are small enough to just perform the multiplication.

$$\sqrt[3]{72}$$

The perfect cube that divides into 72 is 8, so break it down into $8 \bullet 9$:

$$\sqrt[3]{8} \bullet \sqrt[3]{9}$$

Final answer: $2\sqrt[3]{9}$

Since this is a numerical problem, you can check the answer by calculating the value of the problem:

$$\sqrt[3]{12} \bullet \sqrt[3]{6} = 4.160167646\ldots$$

and compare it to the decimal value of the answer that you obtained:

$$2\sqrt[3]{9} = 4.160167646\ldots$$

P. 73: #67.

$$4\sqrt{3} \bullet 6\sqrt{15}$$

Solution: Remember, you multiply the numbers that are OUTSIDE the radical together, and you keep them OUTSIDE the radical. Then you multiply the numbers that are INSIDE the radical together and keep them INSIDE the radical.

$$24\sqrt{45}$$

Now, simplify the radical 45. Break it down into 9 times 5.

$$24\sqrt{9}\sqrt{5}$$

$$24 \bullet 3\sqrt{5}$$

Final answer:

$$72\sqrt{5}$$

Since this is a numerical problem, you can check the answer by calculating the value of the problem and comparing to the decimal value of the answer you obtained.

$$4\sqrt{3} \bullet 6\sqrt{15} = 160.9968944\ldots$$

$$72\sqrt{5} = 160.9968944\ldots$$

P. 73: #70.

$$8\sqrt[3]{65} \bullet 2\sqrt[3]{50}$$

Solution: Remember, you multiply the numbers that are OUTSIDE the radical together, and you keep them OUTSIDE the radical. Then you multiply the numbers that are INSIDE the radical together and keep them INSIDE the radical.

$$8 \bullet 2\sqrt[3]{65 \bullet 50}$$

However, if you use a calculator and multiply out the numbers that are INSIDE the radical, you end up with a large number that you won't know how to simplify. It's better, instead of multiplying the numbers out, to break them down into prime factors, and for square roots, look for pairs of numbers, for cube roots, look for three of a kind, etc.

$$8 \bullet 2\sqrt[3]{5 \bullet 13 \bullet 5 \bullet 10}$$

$$16\sqrt[3]{5 \bullet 13 \bullet 5 \bullet 5 \bullet 2}$$

Notice that you have three factors of 5! That makes a perfect cube:

$$16\sqrt[3]{5^3 \bullet 13 \bullet 2}$$

Now, separate into two radicals, with the perfect cube in the first, and the leftover factors in the second radical.

$$16 \bullet 5 \bullet \sqrt[3]{13 \bullet 2}$$

Final answer:

$$80\sqrt[3]{26}$$

You can compare the decimal values of problem and the answer you obtained.

$8\sqrt[3]{65} \bullet 2\sqrt[3]{50} = 236.9996855\ldots$ $80\sqrt[3]{26} = 236.9996855\ldots$

P. 74: #80. $\left(4\sqrt{5}-5\sqrt{15}\right)\left(3\sqrt{5}+2\sqrt{15}\right)$

Solution:

F O I L

$$12\bullet 5+8\sqrt{75}-15\sqrt{75}-10\bullet 15$$

$$60 \quad - \quad 7\sqrt{75} \quad - \quad 150$$

$$-90 \quad - \quad 7\sqrt{25}\sqrt{3}$$

$$-90 \quad - \quad 7\bullet 5\sqrt{3}$$

Final answer: $-90 \quad - \quad 35\sqrt{3}$

As a check, calculate the values of the problem and the answer obtained:

$\left(4\sqrt{5}-5\sqrt{15}\right)\left(3\sqrt{5}+2\sqrt{15}\right)$ = -150.6217783 . . .

$-90 \quad - \quad 35\sqrt{3}$ = -150.6217783 . . .

P. 75: #88. $\left(5+\sqrt[3]{5}\right)\left(25-5\sqrt[3]{5}+\sqrt[3]{25}\right)$

Solution: Multiply the first (5) times everything in the second parentheses:

$$5\left(25-5\sqrt[3]{5}+\sqrt[3]{25}\right)$$

$$125 \; - \; 25\sqrt[3]{5} \; + \; 5\sqrt[3]{25}$$

Next, multiply the second ($\sqrt[3]{5}$) times everything in the second parentheses:

$$\sqrt[3]{5}\left(25-5\sqrt[3]{5}+\sqrt[3]{25}\right)$$

$$25\sqrt[3]{5}-5\sqrt[3]{25}+\sqrt[3]{125}$$

Now, put it ALL together and combine like terms:

$$\left(5+\sqrt[3]{5}\right)\left(25-5\sqrt[3]{5}+\sqrt[3]{25}\right)$$

$$= \; 125 \; - \; 25\sqrt[3]{5} \; + \; 5\sqrt[3]{25}$$

$$+ \; 25\sqrt[3]{5}-5\sqrt[3]{25}+\sqrt[3]{125}$$

$$= \; 125 \qquad + \sqrt[3]{125}$$

$$= \; 125 \qquad + 5$$

Final answer: $= \; 130$

P. 75: #89. $(3\sqrt{2} - 2\sqrt{3})^3$

Solution: $(3\sqrt{2} - 2\sqrt{3})(3\sqrt{2} - 2\sqrt{3})(3\sqrt{2} - 2\sqrt{3})$

First, multiply the second binomial times the third binomial:

$$(3\sqrt{2} - 2\sqrt{3})(9 \bullet 2 - 12\sqrt{6} + 4 \bullet 3)$$
$$(3\sqrt{2} - 2\sqrt{3})(18 - 12\sqrt{6} + 12)$$
$$(3\sqrt{2} - 2\sqrt{3})(30 - 12\sqrt{6})$$

Now, F OI L this out and simplify the result:

$$(3\sqrt{2} - 2\sqrt{3})(30 - 12\sqrt{6})$$

F O I L

$$90\sqrt{2} - 36\sqrt{12} - 60\sqrt{3} + 24\sqrt{18}$$
$$90\sqrt{2} - 36\sqrt{4}\sqrt{3} - 60\sqrt{3} + 24\sqrt{9}\sqrt{2}$$
$$90\sqrt{2} - 36 \bullet 2\sqrt{3} - 60\sqrt{3} + 24 \bullet 3\sqrt{2}$$
$$90\sqrt{2} - 72\sqrt{3} - 60\sqrt{3} + 72\sqrt{2}$$

Final answer: $162\sqrt{2} - 132\sqrt{3}$

As a check, calculate the values of the problem and the answer obtained.

$(3\sqrt{2} - 2\sqrt{3})^3$ = **0.471890505 . . .** $162\sqrt{2} - 132\sqrt{3}$ = **0.471890505 . . .**

P. 75: #90. $(4\sqrt{6} + 5\sqrt{3})^3$

Solution: $(4\sqrt{6} + 5\sqrt{3})(4\sqrt{6} + 5\sqrt{3})(4\sqrt{6} + 5\sqrt{3})$

First, multiply the second binomial times the third binomial:

$$(4\sqrt{6} + 5\sqrt{3})(16 \bullet 6 + 40\sqrt{18} + 25 \bullet 3)$$
$$(4\sqrt{6} + 5\sqrt{3})(96 + 40\sqrt{9} \bullet \sqrt{2} + 75)$$
$$(4\sqrt{6} + 5\sqrt{3})(171 + 120\sqrt{2})$$

Now, F OI L this out and simplify the result:

$$(4\sqrt{6} + 5\sqrt{3})(171 + 120\sqrt{2})$$

F O I L

$$684\sqrt{6} + 480\sqrt{12} + 855\sqrt{3} + 600\sqrt{6}$$
$$684\sqrt{6} + 480\sqrt{4} \bullet \sqrt{3} + 855\sqrt{3} + 600\sqrt{6}$$
$$684\sqrt{6} + 480 \bullet 2\sqrt{3} + 855\sqrt{3} + 600\sqrt{6}$$
$$684\sqrt{6} + 960\sqrt{3} + 855\sqrt{3} + 600\sqrt{6}$$

Final answer: $1284\sqrt{6} + 1815\sqrt{3}$

As a check, calculate the values of the problem and the answer obtained.

$(4\sqrt{6} + 5\sqrt{3})^3$ = **6288.817045 . . .** $1284\sqrt{6} + 1815\sqrt{3}$ = **6288.817045 . . .**