Math in Living C O L O R !!

1.05 Fractions

College Algebra: One Step at a Time **Section 1.05 Page 45 - 51: #11, 13, 19, 26**

Dr. Robert J. Rapalje, Retired
Central Florida, USA

See Section 1.05, with explanations, examples, and exercises, coming soon!

P. 47 # 11. $\dfrac{x^4-81}{x^4+5x^2-36} \div \dfrac{x^4-3x^3-27x+81}{x^3-2x^2-9x+18}$

Solution: Since this is a multiplication/division problem, the first step is to factor everything that can be factored. The first numerator is a difference of squares, the first denominator is a trinomial, and the numerator and denominator of the second fraction can be factored by grouping. It will take about three steps to factor completely. Meanwhile, you must invert the second fraction and multiply.

$$\frac{(x^2-9)(x^2+9)}{(x^2+9)(x^2-4)} \bullet \frac{x^2(x-2)-9(x-2)}{x^3(x-3)-27(x-3)}$$

$$\frac{(x-3)(x+3)(x^2+9)}{(x^2+9)(x-2)(x+2)} \bullet \frac{(x-2)(x^2-9)}{(x-3)(x^3-27)}$$

$$\frac{(x-3)(x+3)(x^2+9)}{(x^2+9)(x-2)(x+2)} \bullet \frac{(x-2)(x-3)(x+3)}{(x-3)(x-3)(x^2+3x+9)}$$

Now, you can divide out any factor from any numerator with any corresponding factor from any denominator. Here you can divide out two factors of $(x-3)$, and also factors of (x^2+9) and $(x-2)$.

$$\frac{\cancel{(x-3)}(x+3)\cancel{(x^2+9)}}{\cancel{(x^2+9)}\cancel{(x-2)}(x+2)} \bullet \frac{\cancel{(x-2)}\cancel{(x-3)}(x+3)}{\cancel{(x-3)}\cancel{(x-3)}(x^2+3x+9)}$$

$$\frac{(x+3)}{(x+2)} \bullet \frac{(x+3)}{(x^2+3x+9)}$$

Final answer: $\dfrac{(x+3)^2}{(x+2)(x^2+3x+9)}$.

P. 47 # 13. $\dfrac{x^8-16}{x^8-8x^4+16} \div \dfrac{x^8-1}{x^8-5x^4+4}$

Solution: Since this is a multiplication/division problem, the first step is to factor everything that can be factored. The first numerator is a difference of squares, the first denominator is a trinomial. Likewise, the second numerator is a difference of squares, and the second denominator is a trinomial. You must begin by factoring each numerator and denominator. Meanwhile, you must invert the second fraction and multiply.

$$\frac{(x^4-4)(x^4+4)}{(x^4-4)(x^4-4)} \bullet \frac{(x^4-4)(x^4-1)}{(x^4-1)(x^4+1)}$$

Now, you can divide out any factor from any numerator with any corresponding factor from any denominator. Here you can divide out two factors of (x^4-4), and also factors of (x^4+4) and (x^4-1).

$$\frac{\cancel{(x^4-4)}(x^4+4)}{\cancel{(x^4-4)}\,\cancel{(x^4-4)}} \bullet \frac{\cancel{(x^4-4)}\,\cancel{(x^4-1)}}{\cancel{(x^4-1)}(x^4+1)}$$

Final answer: $\dfrac{(x^4+4)}{(x^4+1)}$ or $\dfrac{x^4+4}{x^4+1}$.

P. 50 # 19. $\dfrac{x-2}{x-4} - \dfrac{x}{x+4} + 4$

Solution: Since this is an addition/subtraction problem, the first step is to find the LCD, which is obviously $(x-4)(x+4)$.

The next step is to play "What's Missing?" by observing which factors are missing from each of the three fractions. Notice that the first denominator is missing the factor: $(x+4)$. The second fraction is missing the $(x-4)$ factor, and the third fraction is missing both factors: $(x-4)(x+4)$. So, you must multiply numerators and denominators of each fraction by the respective missing factor:

$$\frac{x-2}{x-4} - \frac{x}{x+4} + \frac{4}{1}$$

$$\frac{(x-2)}{(x-4)}\bullet\frac{(x+4)}{(x+4)} - \frac{x}{x+4}\bullet\frac{(x-4)}{(x-4)} + \frac{4}{1}\bullet\frac{(x+4)(x-4)}{(x+4)(x-4)}$$

It may help in this rather complicated problem, to multiply the factors $(x+4)(x-4)$ in the third numerator first, and then continue multiplying out the rest of the numerators.

$$\frac{(x-2)}{(x-4)}\bullet\frac{(x+4)}{(x+4)} - \frac{x}{(x+4)}\bullet\frac{(x-4)}{(x-4)} + \frac{4}{1}\bullet\frac{(x^2-16)}{(x+4)(x-4)}$$

The denominator of the resulting fraction is of course the LCD $(x-4)(x+4)$:

$$\frac{- \qquad +}{(x-4)(x+4)}$$

$$\frac{x^2+2x-8 - x^2+4x + 4x^2-64}{(x-4)(x+4)}$$

Combine the like terms in the numerator!

Final answer: $\dfrac{4x^2+6x-72}{(x-4)(x+4)}$

While you could factor a common factor from the numerator, the fraction does not reduce, so the factoring is not necessary.

P. 51: 26. $\dfrac{x}{x+4} + \dfrac{2x}{x^2-4} - \dfrac{2}{(x+2)(x+4)}$

Solution: Since this is an addition/subtraction problem, the first step is to find the LCD, by factoring the middle fraction.

$$\frac{x}{x+4} + \frac{2x}{(x-2)(x+2)} - \frac{2}{(x+2)(x+4)}.$$

The LCD consists of three binomial factors $(x+4)(x-2)(x+2)$. The next step is to play "What's Missing?" by observing which factors are missing from each of the three fractions. Notice that the first denominator is missing two factors: $(x-2)(x+2)$. The second fraction is missing the $(x+4)$ factor, and the third fraction is missing the $(x-2)$. So, you must multiply numerators and denominators of each fraction by the respective missing factor

$$\frac{x}{x+4} + \frac{2x}{(x-2)(x+2)} - \frac{2}{(x+2)(x+4)}$$

$$\frac{x}{x+4}\cdot\frac{(x-2)(x+2)}{(x-2)(x+2)} + \frac{2x}{(x-2)(x+2)}\cdot\frac{(x+4)}{(x+4)} - \frac{2}{(x+2)(x+4)}\cdot\frac{(x-2)}{(x-2)}$$

It may help in this rather complicated problem, to multiply the factors $(x-2)(x+2)$ in the first numerator first, and then continue multiplying out the rest of the numerators.

$$\frac{x}{x+4}\cdot\frac{(x^2-4)}{(x-2)(x+2)} + \frac{2x}{(x-2)(x+2)}\cdot\frac{(x+4)}{(x+4)} - \frac{2}{(x+2)(x+4)}\cdot\frac{(x-2)}{(x-2)}$$

$$\frac{x^3-4x+2x^2+8x-2x+4}{(x+4)(x-2)(x+2)}$$

Combine the x-terms in the numerator!

$$\frac{x^3+2x^2+2x+4}{(x+4)(x-2)(x+2)}$$

Factor the numerator by grouping:

$$\frac{x^2(x+2)+2(x+2)}{(x+4)(x-2)(x+2)}$$

$$\frac{(x+2)(x^2+2)}{(x+4)(x-2)(x+2)}$$

Finally, divide out the factor of (x+2) in the numerator and denominator.

Final answer: $\dfrac{x^2+2}{(x+4)(x-2)}$