

Math in Living C O L O R !!

1.09 Radical Equations

College Algebra: One Step at a Time.

Pages 138 - 147: #11, 13, 15, 16, 19, 20, 21, 22, 23, 27, 29, 31, 33, 35, 37.

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See Section 1.09, with explanations, examples, and exercises, coming soon!

P. 141. # 11. $\sqrt{x^2 - 3x + 7} = 2 - x$

Solution: The first step in solving radical equations is to isolate the radical. In this case, the radical is alone on the left side of the equation, so it is already isolated. You may proceed by squaring both sides, in order to “undo” the radical.

$$\left(\sqrt{x^2 - 3x + 7}\right)^2 = (2 - x)^2$$

On the left side, when you square the square root, you simply remove the radical sign. On the right side, remember that you are squaring a binomial, which is the first squared, twice the product, and the last squared:

$$x^2 - 3x + 7 = 4 - 4x + x^2$$

At first, it looks like a quadratic equation, but subtract x^2 from each side, and the x^2 term subtracts out leaving

$$-3x + 7 = 4 - 4x$$

To solve for x , add $+4x$ to each side, and subtract 7 from each side, to get the x terms on the left side, and the number terms on the right side:

$$\begin{array}{r} -3x + 7 = 4 - 4x \\ +4x - 7 \quad -7 + 4x \\ \hline x = -3 \end{array}$$

Since you squared both sides, the answer is NOT guaranteed. You must check the answers:

Check:

$$\begin{array}{l} \sqrt{x^2 - 3x + 7} = 2 - x \\ x = -3 \quad \sqrt{(-3)^2 - 3(-3) + 7} = 2 - (-3) \\ \quad \quad \quad \sqrt{9 + 9 + 7} = 2 + 3 \\ \quad \quad \quad \sqrt{25} = 5 \quad \text{It checks!} \end{array}$$

P. 142. # 13. $\sqrt{2x+15} = 2x+3$

Solution: Since the radical term is isolated on the right side of the equation, you may proceed by squaring both sides, in order to “undo” the radical.

$$(\sqrt{2x+15})^2 = (2x+3)^2$$

On the left side, when you square the square root, you simply remove the radical sign. On the right side, remember that you are squaring a binomial, which is the first squared, twice the product, and the last squared:

$$2x+15 = 4x^2+12x+9$$

This one is a quadratic equation, so you must set it equal to zero by subtracting $2x$ and 15 from each side:

$$\begin{array}{r} 2x+15 = 4x^2+12x+9 \\ -2x-15 \qquad \qquad -2x-15 \\ \hline 0 = 4x^2+10x-6 \end{array}$$

Factor the right side of the equation, beginning by factoring out the common factor of 2:

$$\begin{aligned} 0 &= 2(2x-1)(x+3) \\ 2x &= 1 \quad x = -3 \\ x &= \frac{1}{2} \end{aligned}$$

Since you squared both sides, the answers are NOT guaranteed. You must check the answers:

Check:

$$\begin{aligned} \sqrt{2x+15} &= 2x+3 \\ x = \frac{1}{2} \quad \sqrt{2 \cdot \frac{1}{2} + 15} &= 2 \cdot \frac{1}{2} + 3 \\ \sqrt{1+15} &= 1+3 \\ \sqrt{16} &= 4 \quad \text{It checks!} \end{aligned}$$

Check:

$$\begin{aligned} \sqrt{2x+15} &= 2x+3 \\ x = -3 \quad \sqrt{2 \cdot (-3) + 15} &= 2 \cdot (-3) + 3 \\ \sqrt{(-6) + 15} &= -6+3 \\ \sqrt{9} &= -3 \quad \text{Reject this answer!!} \end{aligned}$$

The **final answer** is $x = \frac{1}{2}$

P. 142. # 15. $\sqrt{3x+1} = \frac{1}{2}x + 1$

Solution: It's a good idea to begin by multiplying both sides of the equation by the denominator which is 2 in order to clear the fractions.

$$2 \cdot \sqrt{3x+1} = 2 \cdot \left(\frac{1}{2}x + 1 \right)$$

$$2 \cdot \sqrt{3x+1} = x + 2$$

Since the radical term is isolated on the right side of the equation, you may proceed by squaring both sides, in order to "undo" the radical.

$$\left(2 \cdot \sqrt{3x+1} \right)^2 = (x+2)^2$$

On the left side, when you square the square root, you simply remove the radical sign. On the right side, remember that you are squaring a binomial, which is the first squared, twice the product, and the last squared:

$$4 \cdot (3x+1) = x^2 + 4x + 4$$

$$12x + 4 = x^2 + 4x + 4$$

This is a quadratic equation, so you must set it equal to zero by subtracting $12x$ and 4 from each side:

$$\begin{array}{r} 12x + 4 = x^2 + 4x + 4 \\ -12x - 4 \quad -12x - 4 \\ \hline 0 = x^2 - 8x \end{array}$$

Factor the right side of the equation by taking out the common factor of x :

$$0 = x(x - 8)$$

$$x = 0 \quad x = 8$$

Since you squared both sides, the answers are NOT guaranteed. You must check the answers:

Check: $\sqrt{3x+1} = \frac{1}{2}x + 1$

$$x = 0 \quad \sqrt{3 \cdot 0 + 1} = \frac{1}{2} \cdot 0 + 1$$

$$\sqrt{1} = 0 + 1 \quad \text{It checks!}$$

Check: $\sqrt{3x+1} = \frac{1}{2}x + 1$

$$x = 8 \quad \sqrt{3 \cdot 8 + 1} = \frac{1}{2} \cdot 8 + 1$$

$$\sqrt{25} = 4 + 1 \quad \text{It also checks!!}$$

Final Answer: $x = 0 \quad x = 8$

P. 142. # 16. $\sqrt{x+3} = \frac{1}{3}x + 1$

Solution: It's a good idea to begin by multiplying both sides of the equation by the denominator which is 3 in order to clear the fractions.

$$3 \cdot \sqrt{x+3} = 3 \cdot \left(\frac{1}{3}x + 1 \right)$$

$$3 \cdot \sqrt{x+3} = x + 3$$

Since the radical term is isolated on the right side of the equation, you may proceed by squaring both sides, in order to "undo" the radical.

$$\left(3 \cdot \sqrt{x+3} \right)^2 = (x+3)^2$$

On the left side, when you square the square root, you simply remove the radical sign. On the right side, remember that you are squaring a binomial, which is the first squared, twice the product, and the last squared:

$$9 \cdot (x+3) = x^2 + 6x + 9$$

$$9x + 27 = x^2 + 6x + 9$$

This is a quadratic equation, so you must set it equal to zero by subtracting $9x$ and 27 from each side:

$$\begin{array}{r} 9x + 27 = x^2 + 6x + 9 \\ -9x - 27 \quad -9x - 27 \\ \hline 0 = x^2 - 3x - 18 \end{array}$$

Factor the right side of the equation:

$$\begin{aligned} 0 &= (x-6)(x+3) \\ x &= 6 \quad x = -3 \end{aligned}$$

Since you squared both sides, the answers are NOT guaranteed. You must check the answers:

Check: $\sqrt{x+3} = \frac{1}{3}x + 1$

$$x = 6 \quad \sqrt{6+3} = \frac{1}{3} \cdot 6 + 1$$

$$\sqrt{9} = 2 + 1 \quad \text{It checks!}$$

Check: $\sqrt{x+3} = \frac{1}{3}x + 1$

$$x = -3 \quad \sqrt{(-3)+3} = \frac{1}{3} \cdot (-3) + 1$$

$$\sqrt{0} = (-1) + 1 \quad \text{It also checks!!}$$

Final Answer: $x = 6 \quad x = -3$

P. 143: #19. $\sqrt{x+40} - \sqrt{x} = 4$

Solution: You must first isolate one of the radicals on the left side. Add $+\sqrt{x}$ to each side. Now, you can write this:

$$\sqrt{x+40} = \sqrt{x} + 4$$

Next, square both sides of the equation in order to eliminate the radical on the left side:

$$(\sqrt{x+40})^2 = (\sqrt{x} + 4)^2$$

On the left side, you just remove the square root sign. On the right side, square the first, take twice the product, and square the second.

$$x + 40 = x + 8\sqrt{x} + 16$$

Isolate the radical on the right side by adding $-x$ and -16 to each side:

$$\begin{array}{r} x + 40 = x + 8\sqrt{x} + 16 \\ -x \quad -16 \quad -x \qquad \qquad -16 \\ \hline \end{array}$$

$$24 = 8\sqrt{x}$$

Since there is a common factor, divide both sides by 8.

$$3 = \sqrt{x}$$

Now, square both sides again to eliminate the second radical.

$$9 = x$$

This answer still must be checked in the original equation:

Check: $\sqrt{x+40} = \sqrt{x} + 4$

$$\sqrt{9+40} = \sqrt{9} + 4$$

$$\sqrt{49} = \sqrt{9} + 4$$

$$7 = 3 + 4 \quad \text{This checks!}$$

The **final answer** is $x = 9$.

P. 143. # 20. $\sqrt{5x+1} = \sqrt{3x} + 1$

Solution: You can square both sides of the equation in order to eliminate the radical on the left side:

$$(\sqrt{5x+1})^2 = (\sqrt{3x} + 1)^2$$

On the left side, you just remove the square root sign. On the right side, square the first, take twice the product, and square the second.

$$5x+1 = 3x + 2\sqrt{3x} + 1$$

Isolate the radical on the right side by adding $-3x$ and -1 to each side:

$$\begin{aligned} 5x+1-3x-1 &= 3x-3x + 2\sqrt{3x} + 1-1 \\ 2x &= 2\sqrt{3x} \end{aligned}$$

Since there is a common factor, divide both sides by 2.

$$x = \sqrt{3x}$$

Now, square both sides again to eliminate the second radical.

$$x^2 = 3x$$

Set equal to zero:

$$x^2 - 3x = 0$$

Solve the quadratic equation, by factoring the common factor of x :

$$\begin{aligned} x(x-3) &= 0 \\ x = 0, x &= 3 \end{aligned}$$

These answers still must be checked in the original equation:

Check:

$$\begin{aligned} \sqrt{5x+1} &= \sqrt{3x} + 1 \\ x = 0 \quad \sqrt{5(0)+1} &= \sqrt{3(0)} + 1 \\ \sqrt{(1)} &= \sqrt{(0)} + 1 \\ 1 &= 0 + 1 \quad \text{This checks!!} \end{aligned}$$

Check:

$$\begin{aligned} \sqrt{5x+1} &= \sqrt{3x} + 1 \\ x = 3 \quad \sqrt{5(3)+1} &= \sqrt{3(3)} + 1 \\ \sqrt{(16)} &= \sqrt{(9)} + 1 \\ 4 &= 3 + 1 \quad \text{This one does NOT check.} \end{aligned}$$

The **final answer** is $x = 0, x = 3$

P. 143: 21. $\sqrt{5x-4} - \sqrt{x} = 2$

Solution: The first step is to isolate one of the radicals. Add \sqrt{x} to each side:

$$\sqrt{5x-4} = \sqrt{x} + 2$$

Now, square both sides of the equation to eliminate the radical on the left side:

$$(\sqrt{5x-4})^2 = (\sqrt{x} + 2)^2$$

On the left side, you just remove the square root sign. On the right side, square the first, take twice the product, and square the second.

$$5x - 4 = x + 4\sqrt{x} + 4$$

Isolate the radical on the right side by adding $-x$ and -4 to each side:

$$5x - 4 - x - 4 = x - x + 4\sqrt{x} + 4 - 4$$

$$4x - 8 = 4\sqrt{x}$$

Since there is a common factor, divide both sides by 4.

$$x - 2 = \sqrt{x}$$

Now, square both sides again to eliminate the second radical.

$$x^2 - 4x + 4 = x$$

Set equal to zero:

$$x^2 - 5x + 4 = 0$$

Solve the quadratic equation, by factoring:

$$(x - 4)(x - 1) = 0$$

$$x = 4, x = 1$$

These answers still must be checked in the original equation:

Check:

$$\begin{aligned} \sqrt{5x-4} - \sqrt{x} &= 2 \\ \sqrt{5(4)-4} - \sqrt{(4)} &= 2 \\ \sqrt{(16)} - \sqrt{(4)} &= 2 \\ 4 - 2 &= 2 \quad \text{This checks!!} \end{aligned}$$

Check:

$$\begin{aligned} \sqrt{5x-4} - \sqrt{x} &= 2 \\ \sqrt{5(1)-4} - \sqrt{(1)} &= 2 \\ \sqrt{(1)} - \sqrt{(1)} &= 2 \\ 1 - 1 &= 2 \quad \text{This one does NOT check.} \end{aligned}$$

The final answer is $x = 4$

P. 143. # 22. $\sqrt{2x} - \sqrt{3x+1} = 1$

Solution: This is an interesting problem! In my first attempts to solve this problem, I tried to isolate the $\sqrt{2x}$ by adding $\sqrt{3x+1}$ to each side. This solution can be seen on page 305 of my Intermediate Algebra book. However, I don't know why, but it turns out to be an easier solution if you isolate the $\sqrt{3x+1}$ by subtracting $\sqrt{2x}$ from each side.

$$-\sqrt{3x+1} = 1 - \sqrt{2x}$$

You may proceed by squaring both sides, in order to "undo" the radical on the left side.

$$\left(-\sqrt{3x+1}\right)^2 = \left(1 - \sqrt{2x}\right)^2$$

Of course when you square the left side, you must square the negative, which is a positive, and remove the radical sign. On the right side, remember that you are squaring a binomial, which is the quantity times itself.

$$\left(-\sqrt{3x+1}\right)^2 = \left(1 - \sqrt{2x}\right)\left(1 - \sqrt{2x}\right)$$

$$3x + 1 = 1 - 2\sqrt{2x} + 2x$$

Subtract $2x$ and 1 from each side to isolate the other radical, and you have

$$\begin{array}{r} 3x + 1 = 1 - 2\sqrt{2x} + 2x \\ -2x - 1 - 1 \qquad \qquad - 2x \\ \hline x = -2\sqrt{2x} \end{array}$$

Now, square both sides of the radical.

$$(x)^2 = \left(-2\sqrt{2x}\right)^2$$

$$x^2 = 4(2x)$$

$$x^2 = 8x$$

$$x^2 - 8x = 0$$

$$x(x - 8) = 0$$

$$x = 0 \quad x = 8$$

Since you squared both sides, the answers are NOT guaranteed. You must check the answers:

Check: $\sqrt{2x} - \sqrt{3x+1} = 1$

$x = 0$ $\sqrt{2 \cdot 0} - \sqrt{3 \cdot 0 + 1} = 1$

$0 - 1 \neq 1$ It does NOT check!

$x = 8$ $\sqrt{2 \cdot 8} - \sqrt{3 \cdot 8 + 1} = 1$

$4 - 5 \neq 1$ It does NOT check!

Final Answer: NO SOLUTION!!

P. 144. # 23. $\sqrt{3x+1} + \sqrt{2x} = 1$

Solution: The first step is to isolate one of the radicals by subtracting $\sqrt{2x}$ from each side of the equation.

$$\sqrt{3x+1} = 1 - \sqrt{2x}$$

Next, square both sides of the equation to eliminate the radical on the left side:

$$(\sqrt{3x+1})^2 = (1 - \sqrt{2x})^2$$

On the left side, you just remove the square root sign. On the right side, square the first, take twice the product, and square the second.

$$3x + 1 = 1 - 2\sqrt{2x} + 2x$$

Now, isolate the radical on the right side by adding -1 and $-2x$ to each side

$$3x - 2x + 1 - 1 = -2\sqrt{2x}$$
$$x = -2\sqrt{2x}$$

Now, square both sides again to eliminate the second radical.

$$x^2 = 4(2x)$$

$$x^2 = 8x$$

Set equal to zero by subtracting $8x$ from each side:

$$x^2 - 8x = 0$$

Of course it factors! In this case, take out the common factor of x :

$$x(x - 8) = 0$$

$$x = 0, x = 8$$

These answers still must be checked.

$$\sqrt{3x+1} + \sqrt{2x} = 1$$

Check: $x = 0$ $\sqrt{3 \cdot (0) + 1} + \sqrt{2 \cdot (0)} = 1$

$$\sqrt{1} + \sqrt{0} = 1$$

$$1 - 0 = 1 \text{ It checks!!}$$

Check: $x = 8$ $\sqrt{3 \cdot (8) + 1} + \sqrt{2 \cdot (8)} = 1$

$$\sqrt{25} + \sqrt{16} = 1$$

$$5 + 4 = 1 \text{ It does NOT check!!}$$

The **final answer** is $x = 0$

P. 145. # 27.

$$\sqrt{2x + 20} = \sqrt{1 - 6x} - 5$$

Solution: The first step is to square both sides of the equation in order to eliminate the radical on the left side:

$$(\sqrt{2x + 20})^2 = (\sqrt{1 - 6x} - 5)^2$$

On the left side, you just remove the square root sign. On the right side, square the first, take twice the product, and square the second.

$$2x + 20 = 1 - 6x - 10\sqrt{1 - 6x} + 25$$

$$2x + 20 = 26 - 6x - 10\sqrt{1 - 6x}$$

Isolate the radical on the right side by adding $+6x$ and -26 to each side:

$$2x + 20 - 26 + 6x = -10\sqrt{1 - 6x}$$

$$8x - 6 = -10\sqrt{1 - 6x}$$

Since there is a common factor, divide both sides by 2.

$$4x - 3 = -5\sqrt{1 - 6x}$$

Now, square both sides again to eliminate the second radical.

$$16x^2 - 24x + 9 = 25(1 - 6x)$$

$$16x^2 - 24x + 9 = 25 - 150x$$

Set equal to zero by adding $+150x - 25$ to each side:

$$16x^2 - 24x + 9 + 150x - 25 = 25 - 150x + 150x - 25$$

$$16x^2 + 126x - 16 = 0$$

Solve the quadratic equation! You can use factoring, the calculator program "POLYSMLT", or the quadratic formula! By factoring, it looks like this. It's a trinomial, so start by taking out the common factor of 2:

$$2(8x^2 + 63x - 8) = 0$$

$$2(8x - 1)(x + 8) = 0$$

$$x = \frac{1}{8}, x = -8$$

These answers still must be checked, and it turns out that the $x = \frac{1}{8}$ does NOT check.

However, the $x = -8$ DOES check, since

$$\sqrt{2(-8) + 20} = \sqrt{1 - 6(-8)} - 5.$$

$$\sqrt{(-16) + 20} = \sqrt{1 + 48} - 5$$

$$\sqrt{4} = \sqrt{49} - 5$$

$$2 = 7 - 5$$

P. 145. # 29. $\sqrt{4x+7} - \sqrt{2x+3} = 1$

Solution: Isolate one radical by adding $+\sqrt{2x+3}$ to each side of the equation.

$$\sqrt{4x+7} = 1 + \sqrt{2x+3}$$

Next, square both sides of the equation in order to eliminate the radical on the left side:

$$(\sqrt{4x+7})^2 = (1 + \sqrt{2x+3})^2$$

On the left side, you just remove the square root sign. On the right side, square the first, take twice the product, and square the second.

$$4x+7 = 1 + 2\sqrt{2x+3} + 2x+3$$

The next step is a small, but important one. Combine the 1 and the 3 on the right side.

$$4x+7 = 4 + 2\sqrt{2x+3} + 2x$$

Now, you must isolate the radical on the right side by adding -4 and $-2x$ to each side

$$4x - 2x + 7 - 4 = 2\sqrt{2x+3}$$

$$2x + 3 = 2\sqrt{2x+3}$$

Now, square both sides again to eliminate the second radical.

$$4x^2 + 12x + 9 = 4(2x+3)$$

$$4x^2 + 12x + 9 = 8x + 12$$

Set equal to zero by adding $-8x - 12$ to each side:

$$4x^2 + 12x + 9 - 8x - 12 = 8x + 12 - 8x - 12$$

$$4x^2 + 4x - 3 = 0$$

Hopefully(??!!) it factors, but it can also be solved using the calculator program "POLYSMLT" or the quadratic formula! The method of factoring is given here.

$$(2x+3)(2x-1) = 0$$

$$x = -\frac{3}{2}, \quad x = \frac{1}{2}$$

These answers still must be checked.

To be continued on next page!!

P. 145. # 29 continued.

$$\begin{aligned} \sqrt{4x+7} - \sqrt{2x+3} &= 1 \\ \text{Check: } x = -\frac{3}{2} \quad &\sqrt{4 \cdot \left(-\frac{3}{2}\right) + 7} - \sqrt{2 \cdot \left(-\frac{3}{2}\right) + 3} = 1 \\ &\sqrt{-6+7} - \sqrt{-3+3} = 1 \\ &\sqrt{1} - \sqrt{0} = 1 \\ &1 - 0 = 1 \quad \text{It checks!!} \end{aligned}$$

$$\begin{aligned} \text{Check: } x = \frac{1}{2} \quad &\sqrt{4 \cdot \left(\frac{1}{2}\right) + 7} - \sqrt{2 \cdot \left(\frac{1}{2}\right) + 3} = 1 \\ &\sqrt{2+7} - \sqrt{1+3} = 1 \\ &\sqrt{9} - \sqrt{4} = 1 \\ &3 - 2 = 1 \quad \text{This also checks!} \end{aligned}$$

The final answer is $x = -\frac{3}{2}$, $x = \frac{1}{2}$

P. 146. # 31. $\sqrt{5x+1} + \sqrt{x+2} = 3$

Solution: The first step is to isolate one of the radicals by subtracting $\sqrt{x+2}$ from each side of the equation.

$$\sqrt{5x+1} = 3 - \sqrt{x+2}$$

Next, square both sides of the equation in order to eliminate the radical on the left side:

$$(\sqrt{5x+1})^2 = (3 - \sqrt{x+2})^2$$

On the left side, you just remove the square root sign. On the right side, square the first, take twice the product, and square the second.

$$5x+1 = 9 - 6\sqrt{x+2} + x+2$$

The next step is a small, but important one. Combine the 9 and the 2 on the right side.

$$5x+1 = 11 - 6\sqrt{x+2} + x$$

Now, you must isolate the radical on the right side by subtracting 11 and x from each side

$$\begin{aligned} 5x - x + 1 - 11 &= -6\sqrt{x+2} \\ 4x - 10 &= -6\sqrt{x+2} \end{aligned}$$

To be continued next page!!

P. 146. # 31 continued.

Divide both sides of the equation by 2:

$$2x - 5 = -3\sqrt{x + 2}$$

Now, square both sides again to eliminate the second radical.

$$4x^2 - 20x + 25 = 9(x + 2)$$

$$4x^2 - 20x + 25 = 9x + 18$$

Set equal to zero by adding $-9x - 18$ to each side:

$$\begin{array}{r} 4x^2 - 20x + 25 = 9x + 18 \\ -9x - 18 \quad -9x - 18 \\ \hline \end{array}$$

$$4x^2 - 29x + 7 = 0$$

Hopefully(??!!) it factors, but it can also be solved using the calculator program "POLYSMLT" or the quadratic formula! The method of factoring is given here.

$$(4x - 1)(x - 7) = 0$$

$$x = \frac{1}{4}, x = 7$$

These answers still must be checked.

$$\sqrt{5x + 1} + \sqrt{x + 2} = 3$$

Check $x = \frac{1}{4}$:

$$\sqrt{5 \cdot \frac{1}{4} + 1} + \sqrt{\frac{1}{4} + 2} = 3$$

$$\sqrt{\frac{5}{4} + \frac{4}{4}} + \sqrt{\frac{1}{4} + 2} = 3$$

$$\sqrt{\frac{9}{4}} + \sqrt{\frac{9}{4}} = 3$$

$$\frac{3}{2} + \frac{3}{2} = 3 \quad \text{It checks!!}$$

Check $x = 7$:

$$\sqrt{5 \cdot 7 + 1} + \sqrt{7 + 2} = 3$$

$$\sqrt{36} + \sqrt{9} = 3$$

$$6 + 3 = 3 \quad \text{This does NOT check!!}$$

The final answer is $x = \frac{1}{4}$.

P. 146: 33. $2\sqrt{3x+4} - 3\sqrt{x} = 2$

Solution: The first step is to isolate one of the radicals by adding $+3\sqrt{x}$ to each side of the equation.

$$2\sqrt{3x+4} = 2 + 3\sqrt{x}$$

Next, square both sides of the equation to eliminate the radical on the left side:

$$(2\sqrt{3x+4})^2 = (2 + 3\sqrt{x})^2$$

On the left side, you just remove the square root sign. On the right side, square the first, take twice the product, and square the second.

$$4(3x+4) = 4 + 12\sqrt{x} + 9x$$

$$12x + 16 = 4 + 12\sqrt{x} + 9x$$

Now, isolate the radical on the right side by adding -4 and $-9x$ to each side

$$\begin{array}{r} 12x + 16 = 4 + 12\sqrt{x} + 9x \\ -9x \quad -4 \quad -4 \qquad \qquad -9x \\ \hline \end{array}$$

$$3x + 12 = 12\sqrt{x}$$

You will save yourself some work if you notice that you can divide each side by 3 before you square both sides to eliminate the second radical.

$$x + 4 = 4\sqrt{x}$$

Now, square both sides again to eliminate the second radical.

$$x^2 + 8x + 16 = 16x$$

Set equal to zero by adding $-16x$ to each side:

$$\begin{array}{r} x^2 + 8x + 16 = 16x \\ -16x \qquad \qquad -16x \\ \hline \end{array}$$

$$x^2 - 8x + 16 = 0 \quad \text{Of course it factors!!}$$

$$(x - 4)^2 = 0$$

$$x = 4$$

To be continued next page!!

P. 146: 33 continued.

This answer still must be checked.

Check:

$$2\sqrt{3x+4} - 3\sqrt{x} = 2$$

$x = 4$

$$2\sqrt{3(4)+4} - 3\sqrt{4} = 2$$
$$2\sqrt{16} - 3\sqrt{4} = 2$$
$$2 \cdot 4 - 3 \cdot 2 = 2$$
$$8 - 6 = 2 \quad \text{It checks!!}$$

The final answer is $x = 4$.

P. 147: #35.

$$\sqrt{7x+4} = \sqrt{2x+3} + 2$$

Solution: The radical on the left side is already isolated, so the first step is to square both sides: equation.

$$\left(\sqrt{7x+4}\right)^2 = \left(\sqrt{2x+3} + 2\right)^2$$

On the left side, you just remove the square root sign. On the right side, square the first, take twice the product, and square the second.

$$7x+4 = 2x+3 + 4\sqrt{2x+3} + 4$$

The next step is a small, but important one. Combine the 3 and the 4 on the right side.

$$7x+4 = 2x + 7 + 4\sqrt{2x+3}$$

Now, you must isolate the radical on the right side by adding -7 and $-2x$ to each side

$$7x+4 = 2x+7 + 4\sqrt{2x+3}$$
$$\frac{-2x-7 \quad -2x-7}{5x-3} = 4\sqrt{2x+3}$$

Now, square both sides again to eliminate the second radical.

To be continued next page!!

P. 146: 35 continued.

$$5x - 3 = 4\sqrt{2x + 3}$$

$$25x^2 - 30x + 9 = 16(2x + 3)$$

$$25x^2 - 30x + 9 = 32x + 48$$

Set equal to zero by adding $-32x - 48$ to each side:

$$25x^2 - 30x + 9 - 32x - 48 = 32x + 48 - 32x - 48$$

$$25x^2 - 62x - 39 = 0$$

Hopefully(??!!) it factors, but it can also be solved using the calculator program "POLYSMLT" or the quadratic formula! The method of factoring is given here.

$$(25x + 13)(x - 3) = 0$$

$$x = -\frac{13}{25}, x = 3$$

These answers still must be checked.

Check:

$$x = -\frac{13}{25} \quad \sqrt{7x + 4} = \sqrt{2x + 3} + 2$$
$$\sqrt{7\left(-\frac{13}{25}\right) + 4} = \sqrt{2\left(-\frac{13}{25}\right) + 3} + 2$$

$$\sqrt{\left(-\frac{91}{25}\right) + \frac{100}{25}} = \sqrt{\left(-\frac{26}{25}\right) + \frac{75}{25}} + 2$$

$$\sqrt{\frac{9}{25}} = \sqrt{\frac{49}{25}} + 2$$

$$\frac{3}{5} = \frac{7}{5} + 2 \quad \text{It DOES NOT check, so it is rejected!!}$$

Check:

$$x = 3 \quad \sqrt{7x + 4} = \sqrt{2x + 3} + 2$$
$$\sqrt{7(3) + 4} = \sqrt{2(3) + 3} + 2$$

$$\sqrt{25} = \sqrt{9} + 2$$

$$5 = 3 + 2 \quad \text{This checks!}$$

The final answer is $x = 3$

P. 147: #37. $3\sqrt{2x+5} - 2\sqrt{7-x} = 3$

Solution: First, isolate one of the radicals on the left side of the equation by adding $+2\sqrt{7-x}$ to each side of the equation.

$$3\sqrt{2x+5} = 3 + 2\sqrt{7-x}$$

Then, square both sides of the equation in order to eliminate the radical on the left side.

$$(3\sqrt{2x+5})^2 = (3 + 2\sqrt{7-x})^2$$

On the left side, you just remove the square root sign. On the right side, square the first, take twice the product, and square the second.

$$9 \cdot (2x+5) = 9 + 12\sqrt{7-x} + 4 \cdot (7-x)$$

Eliminate the parentheses using the distributive property.

$$18x + 45 = 9 + 12\sqrt{7-x} + 28 - 4x$$

The next step is a small, but important one. Combine the 9 and the 28 on the right side.

$$18x + 45 = 37 - 4x + 12\sqrt{7-x}$$

Now, you must isolate the radical on the right side by adding -37 and $+6x$ to each side

$$\begin{array}{r} 18x + 45 = 37 - 4x + 12\sqrt{7-x} \\ +4x \quad -37 \quad -37 +4x \\ \hline 22x + 8 = 12\sqrt{7-x} \end{array}$$

Divide both sides by 2: $11x + 4 = 6\sqrt{7-x}$

Now, square both sides again to eliminate the second radical.

$$121x^2 + 88x + 16 = 36(7-x)$$

$$121x^2 + 88x + 16 = 252 - 36x$$

Set equal to zero by adding -252 and $+36x$ to each side:

$$\begin{array}{r} 121x^2 + 88x + 16 = 252 - 36x \\ +36x - 252 \quad -252 +36x \\ \hline 121x^2 + 124x - 236 = 0 \end{array}$$

Hopefully(?!?) it factors, but it can also be solved using the calculator program "POLYSMLT" or the quadratic formula! The method of factoring is given here.

$$(121x - 118)(x + 2) = 0$$

To be continued next page!!

P. 147: 37 continued.

$$(121x - 118)(x + 2) = 0$$

$$x = \frac{118}{121}, \quad x = -2$$

These answers still must be checked.

$$3\sqrt{2x+5} - 2\sqrt{7-x} = 3$$

Check: $x = -2$ $3\sqrt{2(-2)+5} - 2\sqrt{7-(-2)} = 3$

$$3\sqrt{(-4)+5} - 2\sqrt{7+2} = 3$$

$$3\sqrt{1} - 2\sqrt{9} = 3$$

$$3 - 6 = 3 \quad \text{It DOES NOT check, so it is rejected!!}$$

Check: $x = \frac{118}{121}$ (NOTE: This is the hard one!! A calculator might help!!)

$$3\sqrt{2x+5} - 2\sqrt{7-x} = 3$$

$$3\sqrt{2\left(\frac{118}{121}\right)+5} - 2\sqrt{7-\left(\frac{118}{121}\right)} = 3$$

$$3\sqrt{\left(\frac{236}{121}\right)+5\left(\frac{121}{121}\right)} - 2\sqrt{7\left(\frac{121}{121}\right)-\left(\frac{118}{121}\right)} = 3$$

$$3\sqrt{\left(\frac{236}{121}\right)+\left(\frac{605}{121}\right)} - 2\sqrt{\left(\frac{847}{121}\right)-\left(\frac{118}{121}\right)} = 3$$

$$3\sqrt{\left(\frac{841}{121}\right)} - 2\sqrt{\left(\frac{729}{121}\right)} = 3$$

$$3\frac{29}{11} - 2\frac{27}{11} = 3$$

$$\frac{87}{11} - \frac{54}{11} = 3$$

$$\frac{33}{11} = 3 \quad \text{This checks!!}$$

The final answer is $x = \frac{118}{121}$

A very interesting (and simple!!) solution can be obtained with a graphing calculator:

Extra Problem from Elizabeth in Texas.

$$\frac{3\sqrt{20x+4}}{4} = 6$$

Solution: This is a classic problem of solving for x by “undoing” all of the operations. If you start with x , you can see that it was:

1. Multiplied by 20
2. Four was added
3. A square root was taken
4. The result was multiplied by 3
5. This result was divided by 4

The final result is 6.

You need to “undo” these steps **in reverse order**!!

$$\frac{3\sqrt{20x+4}}{4} = 6$$

5. Clear the fraction (undo division!). Multiply by 4:

$$4 \cdot \frac{3\sqrt{20x+4}}{4} = 6 \cdot 4$$

$$3\sqrt{20x+4} = 24$$

4. Next, undo multiplication by 3. Divide by 3:

$$\sqrt{20x+4} = 8$$

3. Next, undo square root. Square both sides:

$$\left(\sqrt{20x+4}\right)^2 = 8^2$$

$$20x+4 = 64$$

2. Subtract 4 from each side:

$$20x = 60$$

1. Divide by 20:

$$x = 3$$

Now, since you SQUARED both sides of the equation, you must check the solution to make sure the answer is legitimate. Substitute

$$\frac{3\sqrt{20x+4}}{4} = 6$$

$$\frac{3\sqrt{20 \cdot 3 + 4}}{4} = 6$$

$$\frac{3\sqrt{64}}{4} = 6$$

$$\frac{3 \cdot 8}{4} = 6$$

$$\frac{24}{4} = 6$$

Final answer: $x = 3$

$6 = 6$ It checks!