Math in Living C O L O R !!

1.11 Solving Inequalities

by Graphing Calculator Methods

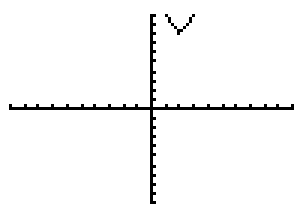
College Algebra: One Step at a Time, **Pages 155 - 163: #5, 11, 23, 29**
Pages 164 - 172: #17, Extra Problems

Dr. Robert J. Rapalje, Retired
Central Florida, USA

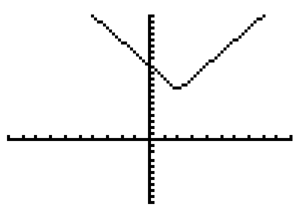
See Section 1.11, with explanations, examples, and exercises, coming soon!

P. 163. #5. Solve for x. Give answers in interval notation. $|2x-4|<-8$.

Solution: First, set the inequality to zero by adding $+8$ to each side of the inequality: $|2x-4|+8<0$, and graph $Y_1=|2x-4|+8<0$ with a graphing calculator. Since the inequality is $Y_1<0$, you will be looking for values of x where the graph is BELOW the x axis. The graph in the standard window [-10,10] of a graphing calculator should look like this:



In order to see the graph better, you may want to extend the y window of the graph to [-10,20].



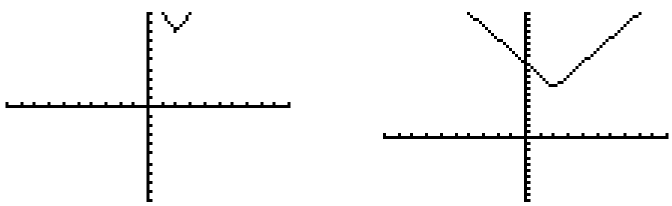
Notice that this graph is always above and NEVER below the x-axis. Therefore, there is NO SOLUTION.

P. 164. #11. Solve for x. Give answers in interval notation. $|2x-4|>-8$.

Solution: First, compare this exercise to #5 from page 163, which has already been solved above. Except for the inequality sign, it is the same! As in the previous exercise, by adding $+8$ to each side of the inequality: $|2x-4|+8<0$, and graph

$Y_1=|2x-4|+8$ with a graphing calculator.

The graph is exactly the same as in Exercise #5! However, since the inequality is $Y_1>0$, you will be looking for values of x where the graph is **ABOVE the x axis**. The graph in the standard window [-10,10] is given on the left, and an extended window for y from [-10,20] is given on the right:



Notice that this graph is ALWAYS above the x-axis. Therefore, the solution is **ALL REAL VALUES of x.**

$(-\infty,\infty)$

P. 163. #23. Solve for x. Give answers in interval notation. $\left|\frac{3x+2}{2}\right|\geq 4$.

Solution: First, set the inequality to zero by adding -4 to each side of the inequality.

Graph with a graphing calculator $Y_1=\left|\left(\frac{(3x+2)}{2}\right)\right|-4\geq 0$.

When you enter this in the calculator, you will need double parentheses. Be sure that you have parentheses around the numerator of the fraction and also

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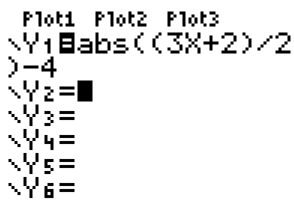
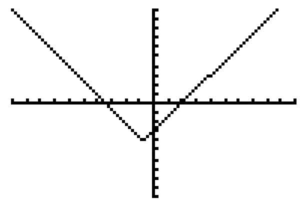
P. 163. #23 continued.

around the quantity that is within the absolute value. When you enter the equation, the equation and the graph should look like this:



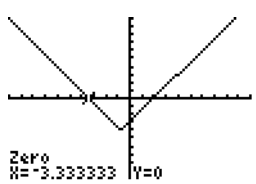


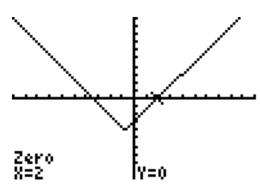
Since the inequality is $Y_1 \geq 0$, you will be looking for values of x, where the graph is ON or ABOVE the x axis. As you can see from the graph, there are two intervals in which the graph is ABOVE the x-axis, but you need to find the x-intercepts (zeros) of the graph. You can do this with algebraic methods (see "Algebraic Method of Finding Endpoints" at the end of this exercise!), or you can use the "zeros method" for the TI 83/84. (For the TI 85/86, it is called the "root method"!).

You can probably tell that the right endpoint is x=2, or you can get it from the TABLE function of the calculator. However, the left endpoint is NOT that obvious. For the TI 83/84, this is the method:

Type [2nd] [F4 (CALC)] [2 (zero)]

The calculator now wants a "Left Bound", so give the calculator any value that is to the left of what you think the x-intercept will be. That is, give it -4 or -5 or -6, and press [ENTER]. Next, the calculator wants a "Right Bound", so give it anything larger than this particular x-intercept, like -2 or -1 or 0. Be sure that the number you choose is something less than 2. Then press [ENTER]. The calculator will then ask for a "Guess?", to which you can just press [ENTER] again. Using this procedure, and using appropriate Left and Right Bound values for x = 2, the calculator will give you the following zeros (roots):





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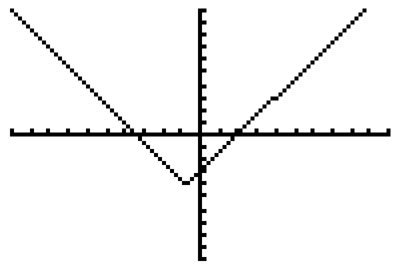
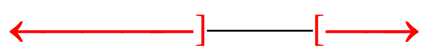
P. 163. #23 continued.

Of course, $x = -3.333\ldots$ or $-3\frac{1}{3}$ or $-\frac{10}{3}$. Notice that this graph is always above the x-axis when x is to the left of $x = -\frac{10}{3}$ or to the right of $x = 2$. The graph on a number line and the interval notation should look like this:





Final answer:

$$(-\infty, -\frac{10}{3}] \cup [2, \infty)$$

Algebraic Method of Finding Endpoints

Weaknesses of the graphing calculator method of solving problems include being sure that the window of your calculator is large (or small!) enough to be sure that you have ALL of the endpoints and being able to find their exact value. Solving for the endpoints algebraically is often necessary to overcome this. Besides, algebraic methods are often easier than calculator methods. To find the endpoints for $\left|\frac{3x+2}{2}\right| \geq 4$, simply change the inequality to an equation, and solve.

$$\left|\frac{3x+2}{2}\right| \geq 4$$

$$\left|\frac{3x+2}{2}\right| = 4$$

There are TWO solutions for endpoints:

$$\frac{3x+2}{2} = 4 \qquad\qquad \frac{3x+2}{2} = -4$$

Multiply both sides of each by the denominator which is 2:

Continued next page

P. 163. #23 continued.

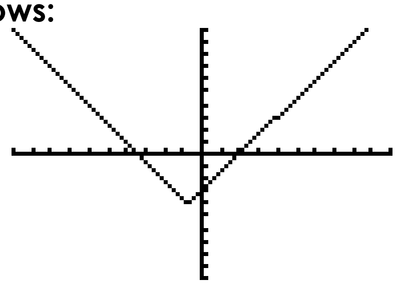
$$\cancel{2} \bullet \frac{3x+2}{\cancel{2}} = 2 \bullet 4 \qquad \cancel{2} \bullet \frac{3x+2}{\cancel{2}} = 2 \bullet -4$$

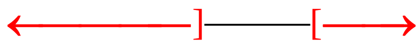
$$3x+2=8 \qquad 3x+2=-8$$

$$3x=6 \qquad 3x=-10$$

$$x=2 \qquad x=\frac{-10}{3}$$

With these as the endpoints, the solution from the graphing calculator is ON or ABOVE the x-axis as follows:



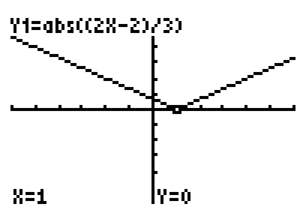


$$(-\infty,-\frac{10}{3}]\cup[2,\infty)$$

P. 163. #29.

Solve for x. Give answers in interval notation. $\left|\frac{2x-2}{3}\right|>0$.

Solution: First, graph with a graphing calculator $Y_1=\left|\frac{2x-2}{3}\right|$. Since the inequality is $Y_1>0$, you will be looking for values of x, where the graph is above the x axis.



Notice that this graph is always above the x-axis, except at the value of x=1. The solution is therefore all values except 1. The graph on a number line and the interval notation should look like this:

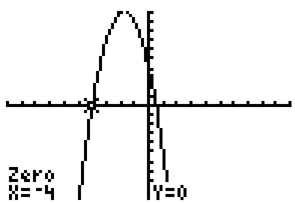
$$(-\infty,1)\cup(1,\infty)$$

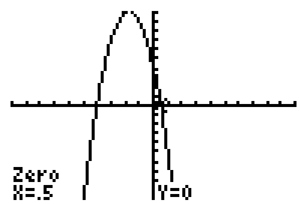
P. 168. #17. **Solve for x. Give answers in interval notation.**

$$2(2-x^2)\le 7x\,.$$

Solution: **Start by setting the inequality to zero:** $2(2-x^2)-7x\le 0$, **and graph** $y1\le 2(2-x^2)-7x$. **Since the inequality is** $y1\le 0$, **you will be looking for values of x where the graph of** $y1=2(2-x^2)-7x$ **is BELOW the x-axis, and INCLUDE the endpoints. Notice that you don't even need to multiply it out. If you do that's fine, but it's not necessary.**

The graph should look like this:

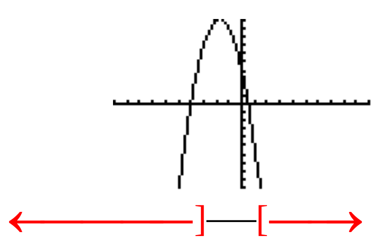




$$Y_1=2(2-x^2)-7x$$

The graph has roots at x=-4 and between 0 and 1. You can find the exact value of the root either by algebra methods (set the equation equal to zero and solve for x), or using the graphing calculator. For a TI 83 or TI 84, you can use [2nd] [CALC] and then look for [2: zero] . Select a left bound of 0 or -1, and a right bound of 1 or 2, and press [ENTER] [ENTER]. The calculator should show the screen given above.

Remember, the solution is the set of all values of x, for which the graph is BELOW the x-axis, and INCLUDES the endpoints. That is, on or below the x axis. The graph on a number line and interval notation will be:



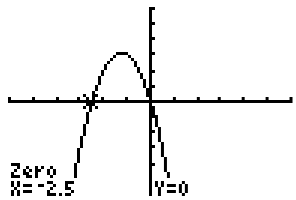
$$(-\infty,-4]\cup[\frac{1}{2},\infty)$$

P. 168. Similar to #17. Solve for x. Give interval notation. $2(x-x^2)\le 7x$.

Solution: Set the inequality to zero: $2(x-x^2)-7x\le 0$, and graph

$$Y_1 = 2(x-x^2)-7x\le 0$$

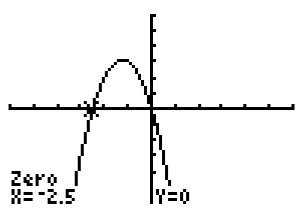
Notice that you don't even need to multiply it out. If you do that's fine, but it's not necessary. Since the inequality is $Y_1 \le 0$, you will be looking for values of x, where the graph is on or below the x axis. The graph should look like this:

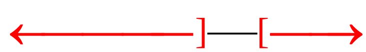


$$Y_1 = 2(x-x^2)-7x$$

The graph has roots at x=0 and between -3 and -2. You can find the exact value of the root either by algebra methods (set the equation equal to zero and solve for x), or using the graphing calculator. For a TI 83 or TI 84, you can use [2nd] [CALC] and then look for [2: zero] . Select a left bound of -3 or -4, and a right bound of -1 or -2, and press [ENTER] [ENTER]. The calculator should show the screen given above.

The solution is the set of all values of x, for which the graph is on or below the x axis. The graph on a number line and interval notation will be:





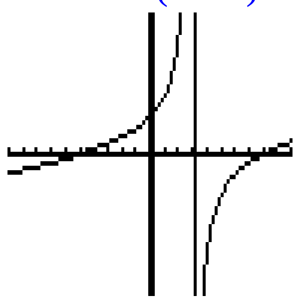
$$(-\infty,-2.5]\cup[0,\infty)$$

Extra Problem (by Aimee). Solve for x. Give answer in interval notation.

$$\frac{(x-8)(x+5)}{(x-3)} \geq 0.$$

Solution: Start by drawing the graph of

$$Y1 \;=\; \frac{(x-8)(x+5)}{(x-3)} \;\geq\; 0$$



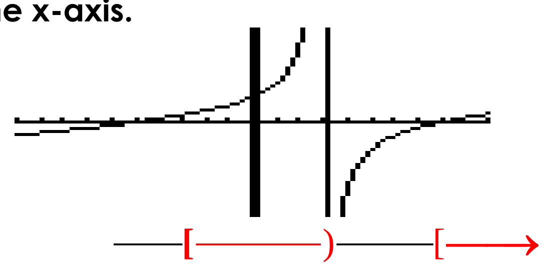
Notice that the roots (or zeros) of this function are at x=8 and x=-5, and there is an asymptote at x=3. Since the inequality is $Y_1 \geq 0$, you will be looking for values of x, where the graph is on or above the x axis.

The graph has roots at x=-5 and 8. The vertical line at x=3 is not really a part of the graph, but it is an asymptote, a line that the graph approaches by never actually touches.

Since there are two roots (zeros) and one asymptote, this gives you three endpoints on the number line, and four intervals to consider for your solution. You must select the intervals that are ON or ABOVE the x-axis.

In the first interval, from –infinity to -5, the graph is below the x-axis.
In the second interval, from -5 to 3 the graph is above the x-axis.
In the third interval, from 3 to 8, the graph is below the x-axis.
In the fourth interval, from 8 to infinity, the graph is above the x-axis.

Therefore the solution consists of the second and fourth interval, where the graph is above the x-axis, including the endpoints at x=-5 and x=8, since these are points that are ON the x-axis.



$$[\,-5\,,\;3) \;\cup\; [8,\infty)$$