Math in Living C O L O R !!

1.08 Quadratic Equations

College Algebra: One Step at a Time. Pages 122-124: #7
Pages 125 -129: #11, 12

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See Section 1.08, with explanations, examples, and exercises, coming soon!

When solving a quadratic equation, there are several methods to consider. Some problems can be solved best by one method, and other problems are best solved by other methods.

Methods of Solving Quadratic Equations
1. Factoring 2. Quadratic Formula 3. Completing the Square 4. Calculator Methods by Graphing 5. Calculator POLYSMLT

Of course, the easiest way to solve any quadratic equation is the method of factoring--that is, of course, IF the equation can be factored. Not all quadratic equations can be factored.

The most reliable method is the quadratic formula:

If $ax^2 + bx + c = 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

The quadratic formula ALWAYS works!

However, while the quadratic formula can be used to solve any problem, the completing the square method is sometimes easier. Specifically, in order to solve by the completing the square method, the coefficient of x^2 must be 1 in order for "half and square" to work. Secondly, the completing the square method works very nicely when the x coefficient is an even number (so taking "half and square" doesn't end up with horrible fractions to deal with!).

Summary for Completing the Square

for $ax^2 + bx + c = 0$

1. Recommended as the best method whenever a=1 and b = even number!
2. Express the equation with variable terms on the left and the number term on the right side of the equation, in the form $x^2 + bx + \_\_\_\_ = -c + \_\_\_\_$.
3. Take half of the x coefficient, and square, in order to decide what number to add to each side of the equation to complete the square. That is, "half, and square".
4. Express as a binomial squared equal to a number.
5. Take the square root of each side of the equation—don't forget the $\pm$.
6. Solve for x.

Completing the Square

P. 124. #7. Solve for x by completing the square: $x^2 - 10x - 15 = 0$.

Solution: $x^2 - 10x - 15 = 0$

Set up for completing the square by adding 15 to each side:

$$x^2 - 10x = 15$$

$$x^2 - 10x + \_\_\_ = 15 + \_\_\_$$

Next, take "half and square":

$$x^2 - 10x + 25 = 15 + 25$$
$$(x-5)^2 = 40$$

Square root both sides:

$$\sqrt{(x-5)^2} = \sqrt{40}$$

$$x - 5 = \pm\sqrt{40}$$

Simplify $\sqrt{40} = 2\sqrt{10}$: $\qquad x - 5 = \pm\, 2\sqrt{10}$

Add $+5$ to each side: $\qquad x = 5 \pm 2\sqrt{10}$

You may want to check your answer by using calculator methods, in particular a program on the TI83/84 or TI85/86 called POLYSMLT (Polynomials and Simultaneous Equations). If you have a graphing calculator, see the instructions on the next page.

NOTE: If you do NOT have POLYSMLT on your TI 83+ or TI 84 calculator in the APPS menu of your calculator, it can be easily installed for you if you know someone who has it on their calculator! It can probably be downloaded on-line.

Graphing Calculator

P. 124. #7. Solve for x using POLYSMLT: $x^2 - 10x - 15 = 0$.

Solution with POLYSMLT (TI 83+ or TI 84):

[APPS]
[POLYSMLT]
[ENTER] [ENTER] [ENTER]
Degree = [2] (This is the highest power of x in the equation!)
Now, enter the coefficients, followed by [ENTER]
[1] [ENTER]
[-10] [ENTER]
[-15] [ENTER]
[F5, SOLVE]
Answer: x = -1.32455532; x = 11.32455532

Now calculate, and compare to the calculator values from POLYSMLT:

$x = 5 + 2\sqrt{(10)}$ = 11.32455532

$x = 5 - 2\sqrt{(10)}$ = -1.32455532

If you have a TI 86/85, the program you need is built into the calculator at [2nd] [POLY].

Solution with POLYSMLT (TI 86 or TI 85)

[2nd] [POLY]
Order = [2] (This is the highest power of x in the equation!)
Now, enter the coefficients, followed by [ENTER]
[1] [ENTER]
[-10] [ENTER]
[-15] [ENTER]
[F5, SOLVE]
Answer: x = -1.32455532; x = 11.32455532
Now calculate, and compare to the calculator values from [2nd] [POLY]:

$x = 5 + 2\sqrt{(10)}$ = 11.32455532

$x = 5 - 2\sqrt{(10)}$ = -1.32455532

Quadratic Formula

P. 129: #11. $2x(2-x)=3$

Solution: $4x-2x^2=3$

This is a quadratic equation, so the first step is to set it equal to zero, preferably with a positive x^2 term. This can be done by moving everything to the right side, by adding $+2x^2$ and $-4x$ to each side.

$$4x-2x^2+2x^2-4x=3+2x^2-4x$$

$$0=2x^2-4x+3$$

This quadratic equation cannot be solved by factoring, so in this case, the quadratic formula is the best choice of methods.

Remember that the solution to the quadratic equation: $ax^2+bx+c=0$

is given by the formula: $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

In the equation: $2x^2-4x+3=0$

$$a=2,\quad b=-4,\quad c=3$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4\bullet 2\bullet 3}}{2\bullet 2}$$

$$x=\frac{4\pm\sqrt{16-24}}{4}$$

$$x=\frac{4\pm\sqrt{-8}}{4}$$

$$x=\frac{4\pm\sqrt{-4}\bullet\sqrt{2}}{4}$$

$$x=\frac{4\pm 2i\sqrt{2}}{4}$$

Factor out the common factor of 2 in the numerator, and reduce the fraction.

$$x=\frac{2(2\pm i\sqrt{2})}{4}$$

Final answer: $x=\dfrac{2\pm i\sqrt{2}}{2}$

P. 129: #12. $2x(x+2)=-5$

Solution: The first step is to multiply out the parentheses, and notice that it is a quadratic equation. When you see this, you might as well go ahead and set it equal to zero by adding +5 to each side of the equation.

$$2x^2+4x+5=0$$

This quadratic equation cannot be solved by factoring, so the quadratic formula is the best method.

Remember that the solution to the quadratic equation: $ax^2+bx+c=0$

is given by the formula: $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

In the equation: $2x^2+4x+5=0$

$$a=2,\quad \text{b=4},\quad c=5$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(4)\pm\sqrt{(4)^2-4\bullet 2\bullet 5}}{2\bullet 2}$$

$$x=\frac{-4\pm\sqrt{16-40}}{4}$$

$$x=\frac{-4\pm\sqrt{-24}}{4}$$

$$x=\frac{-4\pm\sqrt{-4}\bullet\sqrt{6}}{4}$$

$$x=\frac{-4\pm 2i\sqrt{6}}{4}$$

Factor out the common factor of 2 in the numerator, and reduce the fraction.

$$x=\frac{2(-2\pm i\sqrt{6})}{4}$$

Final answer: $x=\dfrac{-2\pm i\sqrt{6}}{2}$