# Math in Living C O L O R !!
## 2.01  Linear Graphs, Slope, and the Equation of a Line

*College Algebra: One Step at a Time,*  Page 209-211:  #18, 20, 21, 22, 24, 27

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See Section 2.01, with explanations, examples, and exercises, coming soon!

## P. 210.  # 18.  Find the equation of the line through $(-2,\ 5)$ and $(-8,\ 9)$.

## Solution:

In order to find the equation of any line, you must have a point (from which to start!) and a slope (a direction in which to go!).  In this case, you are given two points.  The first step is to find the slope between these two points.

Remember the formula for the slope between two points

$$m = \frac{RISE}{RUN} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{9 - 5}{-8 - (-2)}$$

$$m = \frac{4}{-6}$$

The slope of the given line is  $m = -\frac{2}{3}$ .

Now find the equation of a line with $m = -\frac{2}{3}$  passing through either of the given points.  It doesn't matter which point you use.  Let's use the first point  $(-2,\ 5)$.

Start with the formula: $y = mx + b$ , where $m = -\frac{2}{3}, \quad x = -2,\ y = 5$.

$$5 = -\frac{2}{3} \bullet -2 + b$$

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# P. 210: #18 continued.

$$5 = -\frac{2}{3} \bullet -2 + b$$

**To clear the fraction, multiply by the denominator which is** $3$.

$$3 \bullet 5 = 3 \bullet -\frac{2}{3} \bullet -2 + 3 \bullet b$$

**Divide out the** $3$: $\quad 15 = 4 + 3b$

**Subtract** $4$: $\quad 11 = 3b$

**Divide by** $3$: $\quad \dfrac{11}{3} = b$

$$b = \frac{11}{3}$$

**Be sure to answer the question! Find the equation of the line**

$$y = mx + b$$

$$y = -\frac{2}{3}x + \frac{11}{3}$$

**Check your answer. Substitute the value of the other point** $x = -8$ **to see if** $y=9$.

$$y = -\frac{2}{3}x + \frac{11}{3}$$

$$y = -\frac{2}{3} \bullet (-8) + \frac{11}{3}$$

$$y = \frac{16}{3} + \frac{11}{3}$$

$$y = \frac{27}{3} = 9 \quad \text{It checks!!}$$

**Final answer:** $\quad y = mx + b$

$$y = -\frac{2}{3}x + \frac{11}{3}$$

**P. 210.  # 20.**  Find the equation of the line through $(-3,5)$ and parallel to $y = -7x - 1$.

## Solution:

In order to find the equation of any line, you must have a point (from which to start!) and a slope (a direction in which to go!).  In this case, you are given a point, but instead of being given the slope of the line, you are given the equation of a given line that is parallel to it.  Since the lines are parallel, they have the same slope!!

The slope of the given line is $m = -7$ , so the slope of any line parallel to this line is also $m = -7$.   Now find the equation of a line with $m = -7$ passing through $(-3,5)$.

Start with the formula: $y = mx + b$ , where $m = -7$,   $x = -3,\ y = 5$.

$$5 = -7 \bullet -3 + b$$
$$5 = 21 + b$$
$$b = -16$$

Be sure to answer the question!  Find the equation of the line
$$y = mx + b$$
$$y = -7x - 16$$

Check your answer, be substituting $x = -3$ to see if $y = 5$
$$y = -7x - 16$$
$$y = -7 \bullet (-3) - 16$$
$$y = 21 - 16$$
$$y = 5 \text{ It checks!!}$$

**Final answer:**      $y = -7x - 16$

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**P. 210. # 21.** Find the equation of the line through $(4,-5)$ and parallel

to $y = \dfrac{3}{4}x + 2$.

## Solution:

In order to find the equation of any line, you must have a point (from which to start!) and a slope (a direction in which to go!). In this case, you are given a point, but instead of being given the slope of the line, you are given the equation of a given line that is parallel to it. Since the lines are parallel, they have the same slope!! The slope of the given line is $m = \dfrac{3}{4}$, so the slope of any

line parallel to this line is also $m = \dfrac{3}{4}$. Now find the equation of a line with

$m = \dfrac{3}{4}$ passing through $(4,-5)$.

Start with the formula: $y = mx + b$, where $m = \dfrac{3}{4}$, $x = 4$, $y = -5$.

$$-5 = \frac{3}{4} \bullet 4 + b$$

In this problem, you might want to multiply both sides of the equation by the denominator which is 4, and if you do it will be correct! However, notice that the 4 in the denominator divides out with the other 4 in the product, and the result is just 3. Isn't this easier?

$$-5 = 3 + b$$
$$b = -8$$

To answer the question, find the equation of the line
$$y = mx + b$$

$$y = \frac{3}{4}x - 8$$

Check your answer, be substituting $x = 4$ to see if $y = -5$

$$y = \frac{3}{4}x - 8$$

$$y = \frac{3}{4} \bullet (4) - 8$$

$$y = 3 - 8$$

$$y = -5 \text{ It checks!!}$$

**Final answer:** $\quad y = \dfrac{3}{4}x - 8$

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**P. 210. #22.** Find the equation of the line through $(-3,5)$ and perpendicular to $y = -7x - 1$.

## Solution:

In order to find the equation of any line, you must have a point (from which to start!) and a slope (a direction in which to go!). In this case, you are given a point, but instead of being given the slope of the line, you are given the equation of a given line that is perpendicular to it. Since the lines are perpendicular one slope must be the negative reciprocal of the other! The slope of the given line is $m = -7$, so the slope of a line perpendicular to this line is $m = \dfrac{1}{7}$. Now find the equation of a line with $m = \dfrac{1}{7}$ passing through $(-3,5)$.

Start with the formula: $y = mx + b$, where $m = \dfrac{1}{7}$, $x = -3$, $y = 5$.

$$5 = \frac{1}{7} \bullet -3 + b$$

To clear the fraction, multiply by the denominator which is $7$.

$$7 \bullet 5 = 7 \bullet \frac{1}{7} \bullet -3 + 7 \bullet b$$

Divide out the $7$: $\qquad 35 = -3 + 7b$

Add $+3$: $\qquad\qquad 38 = 7b$

Divide by $7$: $\qquad \dfrac{38}{7} = b \quad$ or $\quad b = \dfrac{38}{7}$

Be sure to answer the question! Find the equation of the line $y = mx + b$

$$y = \frac{1}{7}x + \frac{38}{7}$$

Check your answer, be substituting $x = -3$ to see if $y = 5$

$$y = \frac{1}{7}x + \frac{38}{7}$$

$$y = \frac{1}{7} \bullet (-3) + \frac{38}{7}$$

$$y = \frac{-3}{7} + \frac{38}{7}$$

$$y = \frac{35}{7} = 5 \text{ It checks!!}$$

**Final answer:** $\qquad y = \dfrac{1}{7}x + \dfrac{38}{7}$

**P. 211. #24.** Find the equation of the line through $(-3,5)$ and perpendicular to $7x - 4y = 1$.

## Solution:

In order to find the equation of any line, you must have a point (from which to start!) and a slope (a direction in which to go!). In this case, you are given a point, but instead of being given the slope of the line, you are given the equation of a given line. Your line must be perpendicular to this given line, which means that the given line has a slope which is the negative reciprocal of the slope of the line you need to find.

Find the slope of the given line by solving for y in terms of x.
$$7x - 4y = 1$$

Add $-7x$ to each side of the equation:
$$-4y = -7x + 1$$

Divide both sides by -4:
$$\frac{-4y}{-4} = \frac{-7x}{-4} + \frac{1}{-4}$$
$$y = \frac{7}{4}x - \frac{1}{4}$$

The slope of the given line is $m = \dfrac{7}{4}$, so the slope of a line perpendicular to this line is also $m = -\dfrac{4}{7}$. Now find the equation of a line with $m = -\dfrac{4}{7}$ passing through $(-3,5)$. Start with the formula: $y = mx + b$, where $m = -\dfrac{4}{7}$, $x = -3$, $y = 5$.

$$5 = \frac{-4}{7} \bullet -3 + b$$

To clear the fraction, multiply by the denominator which is $7$.
$$7 \bullet 5 = 7 \bullet \frac{-4}{7} \bullet -3 + 7 \bullet b$$

Divide out the $7$:     $35 = 12 + 7b$

Subtract $12$:     $23 = 7b$

Divide by $7$:     $\dfrac{23}{7} = b$  or  $b = \dfrac{23}{7}$

Be sure to answer the question! Write the equation of the line $y = mx + b$

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$$y = -\frac{4}{7}x + \frac{23}{7}$$

# P. 210: #24 continued.

Check your answer, be substituting $x = -3$ to see if $y=5$

$$y = -\frac{4}{7}x + \frac{23}{7}$$

$$y = -\frac{4}{7} \bullet (-3) + \frac{23}{7}$$

$$y = \frac{12}{7} + \frac{23}{7}$$

$$y = \frac{35}{7} = 5 \text{ It checks!!}$$

**Final answer:** $\quad y = -\frac{4}{7}x + \frac{23}{7}$

# P. 211.  # 27.

Find the equation of the perpendicular bisector of the line segment between the two given points:  $(3, -6)$  and  $(-5, 4)$.

## Solution:

It isn't required, but it might help to draw a sketch of these two points, and draw the line segment between them.  You can start by finding the slope of this line segment.  Remember the formula for the slope between two points

$$m = \frac{RISE}{RUN} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{4 - (-6)}{-5 - 3}$$

$$m = \frac{10}{-8} = \frac{-5}{4}$$

The slope of this line segment is  $m = \dfrac{-5}{4}$

Now, you need to find the perpendicular bisector of this line segment.  The perpendicular bisector will be a line that passes through the midpoint of these two points, and it will be perpendicular to the line segment.   Do you remember how to find the midpoint between two points?  It's like the average of the x coordinates and the average of the y coordinates. So you add the x coordinates together and divide by 2, and add the y coordinates and divide by 2:

$$\text{Midpoint} = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

**To be continued next page!!**

$$\text{Midpoint} = \left( \frac{3 + (-5)}{2}, \frac{(-6) + 4}{2} \right)$$

$$= \left( \frac{-2}{2}, \frac{-2}{2} \right)$$

$$= (-1, -1)$$

Next, you know that the perpendicular bisector of this line segment will be perpendicular to the line segment. Since the slope of the line segment is already known, $m = \dfrac{-5}{4}$, the slope of the perpendicular bisector will be the negative reciprocal of the slope of the line segment, which will be $m_\perp = \dfrac{4}{5}$.

Here is a place to be careful. In previous problems in which you were trying to find the equation of a line between two points, it didn't matter which point you used in the formula. In this case there are three points, and at first glance you might think that, like before, one point will work as well as any other point. However, look at the sketch of the two given points and the midpoint. All three points lie on the line segment, but how many of these points actually lie on the perpendicular bisector of the segment? Answer: only the midpoint! Therefore, you must use the midpoint. Do NOT use the end points of the line segment because these points are NOT on the perpendicular bisector!!

So you must now find the equation of a line with $m_\perp = \dfrac{4}{5}$ passing through $(-1, -1)$. Start with the formula: $y = mx + b$, where $m = \dfrac{4}{5}$, $x = -1$, $y = -1$.

$$-1 = \frac{4}{5} \bullet -1 + b$$

Multiply both sides of the equation by 5:

$$5 \bullet -1 = 5 \bullet \frac{4}{5} \bullet -1 + 5 \bullet b$$

$$-5 = -4 + 5b$$

$$-1 = 5b$$

$$b = -\frac{1}{5}$$

Now, write the equation of the line $y = mx + b$: $\quad y = \dfrac{4}{5}x - \dfrac{1}{5}$

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# P. 211: #27 continued.

Check your answer, be substituting $x = -1$ to see if $y = -1$

$$y = \frac{4}{5}x - \frac{1}{5}$$

$$y = \frac{4}{5} \bullet (-1) - \frac{1}{5}$$

$$y = -\frac{4}{5} - \frac{1}{5}$$

$$y = -\frac{5}{5} = -1 \text{ It checks!!}$$

**Final answer:** $\qquad y = \frac{4}{5}x - \frac{1}{5}$

# To be used in #28 or 29.

To clear the fraction, multiply by the denominator which is $7$.

$$7 \bullet 5 = 7 \bullet \frac{-4}{7} \bullet -3 + 7 \bullet b$$

Divide out the $7$: $\qquad 35 = 12 + 7b$

Subtract $12$: $\qquad 23 = 7b$

Divide by $7$: $\qquad \dfrac{23}{7} = b$

$$b = \frac{23}{7}$$

Be sure to answer the question! Find the equation of the line $y = mx + b$

$$y = -\frac{4}{7}x + \frac{23}{7}$$

Check your answer, be substituting $x = -3$ to see if $y = 5$

$$y = -\frac{4}{7}x + \frac{23}{7}$$

$$y = -\frac{4}{7} \bullet (-3) + \frac{23}{7}$$

$$y = \frac{12}{7} + \frac{23}{7}$$

$$y = \frac{35}{7} = 5 \text{ It checks!!}$$

**Final answer:** $\qquad y = -\frac{4}{7}x + \frac{23}{7}$

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