Math in Living C O L O R !!

2.04 The Circle

College Algebra: One Step at a Time, **Page 234 - 241: #18, 19, 26, 28, 30, 39**

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See Section 2.04, with explanations, examples, and exercises, coming soon!

Circle Summary

$$(x-h)^2+(y-k)^2=r^2$$

represents the equation of a circle with center at $(h,\ k)$ and of radius r

$$x^2+y^2+ax+by+c=0$$

also represents a circle (or a point or no solution!)

P. 237 # 18. $x^2+y^2+8x-6y=39$

Solution: Before you ever start the problem, you know that this looks like a CIRCLE! The best way to find the center and radius is by completing the square. Since you already have coefficients of $1\ x^2$ and $1\ y^2$, you should begin by re-writing the equation with the x terms together and y terms together, leaving a space to complete the square:

$$x^2+8x+\_\_\_\_+y^2-6y+\_\_\_\_=39+\_\_\_\_+\_\_\_\_$$

When you complete the square, remember that you take half of the coefficient, and square. Half of 8 is 4 and 4^2 is 16. Half of -6 is -3, and $(-3)^2$ is 9.

$$x^2+8x+\_\_\_\_+y^2-6y+\_\_\_\_=39+\_\_\_\_+\_\_\_\_$$

$$x^2+8x+16 \quad + \quad y^2-6y+9 \quad = \quad 39+16+9$$

$$(x+4)^2+(y-3)^2=64$$

This is a circle with center at $(-4,\ 3)$ and with radius $r=8$.

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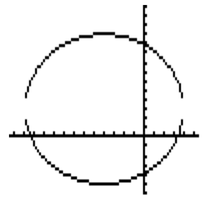
P. 237: #18 continued.

This is a circle with center at $(-4,\ 3)$ and with radius $r=8$. To graph this circle, start at the origin and count 4 units to the left, then up 3 units. This is the center of the circle. Next, the radius is $r=8$ so measure 8 units in each direction from the center. The graph should look like this (connect the halves of the circle!).



Circle with center at $(-4,\ 3)$ and with radius $r=8$.

P. 238. # 19 $x^2+y^2-12x+8y-48=0$

Solution: As in #18, this is a completing the square problem. Begin by re-writing the equation with the x terms together and y terms together, leaving a space to complete the square. Also, you should add +48 to each side of the equation:

$$x^2-12x+\_\_\_\_+y^2+8y+\_\_\_\_=48+\_\_\_\_+\_\_\_\_$$

When you complete the square, remember that you take half of the coefficient, and square. Half of -12 is -6 and $(-6)^2$ is 36. Half of 8 is 4, and 4^2 is 16.

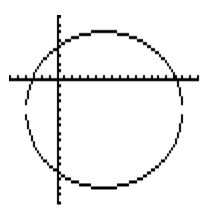
$$x^2-12x+\_\_\_\_+y^2+8y+\_\_\_\_=48+\_\_\_\_+\_\_\_\_$$

$$x^2-12x+36 \quad + \quad y^2+8y+16 \quad = \quad 48+36+16$$

$$(x-6)^2+(y+4)^2=100$$

This is a circle with center at $(6,\ -4)$ and with radius $r=10$.

To graph this circle, start at the origin and count 6 units to the right, then down 4 units. This is the center of the circle. Now, the radius is $r=10$ so measure 10 units in each direction from the center. The graph should look like this.



Circle with center at $(6,\ -4)$ and with radius $r=10$.

P. 238. # 26. $2x^2+2y^2+4x-3y=3$

Solution: Before you ever start the problem, you know that this looks like a CIRCLE! The most effective way to find the center and radius is by completing the square. However, before you can complete the square, you need to have coefficients of $1\ x^2$ and $1\ y^2$. The best way to accomplish this is to divide both sides of the equation by 2.

$$2x^2+2y^2+4x-3y=3$$

$$x^2+y^2+2x-\frac{3}{2}y=\frac{3}{2}$$

Rearrange the terms with the *x* terms together, and the *y* terms together, and leave spaces to complete the square for each variable.

$$x^2+y^2+2x-\frac{3}{2}y=\frac{3}{2}$$

$$x^2+2x+\_\_\_\_+y^2-\frac{3}{2}y+\_\_\_\_=\frac{3}{2}+\_\_\_\_+\_\_\_\_$$

When you complete the square, remember that you take half of the coefficient, and square. Half of 2 is 1 and 1^2 is 1. Half of $-\frac{3}{2}$ is $-\frac{3}{4}$, and $\left(-\frac{3}{4}\right)^2$ is $\frac{9}{16}$.

$$x^2+2x+\_\_\_\_+y^2-\frac{3}{2}y+\_\_\_\_=\frac{3}{2}+\_\_\_\_+\_\_\_\_$$

$$x^2+2x+1+y^2-\frac{3}{2}y+\frac{9}{16}=\frac{3}{2}+1+\frac{9}{16}$$

$$(x+1)^2+\left(y-\frac{3}{4}\right)^2=\frac{3}{2}\bullet\frac{8}{8}+1\bullet\frac{16}{16}+\frac{9}{16}$$

$$(x+1)^2+\left(y-\frac{3}{4}\right)^2=\frac{24}{16}+\frac{16}{16}+\frac{9}{16}$$

$$(x+1)^2+\left(y-\frac{3}{4}\right)^2=\frac{49}{16}$$

This is a circle with center at $\left(-1\ ,\ \frac{3}{4}\right)$ and with radius $r=\frac{7}{4}$.

P. 238. # 28. $4x^2+4y^2-20x-12y=-25$

Solution: Before you ever start the problem, you know that this looks like a CIRCLE! The most effective way to find the center and radius is by completing the square. However, before you can complete the square, you need to have coefficients of $1\ x^2$ and $1\ y^2$. The best way to accomplish this is to divide both sides of the equation by 4.

$$4x^2+4y^2-20x-12y=-25$$

$$x^2+y^2-5x-3y=-\frac{25}{4}$$

Rearrange the terms with the x terms together, and the *y* terms together, and leave spaces to complete the square for each variable.

$$x^2+y^2-5x-3y=-\frac{25}{4}$$

$$x^2-5x+\_\_\_\_+y^2-3y+\_\_\_\_=-\frac{25}{4}+\_\_\_\_+\_\_\_\_$$

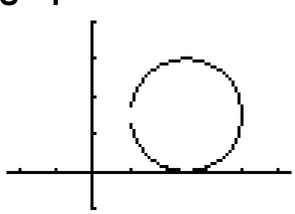
When you complete the square, remember to take half of the coefficient, and square. Half of -5 is $-\frac{5}{2}$ and $\left(-\frac{5}{2}\right)^2$ is $\frac{25}{4}$. Half of -3 is $-\frac{3}{2}$, and $\left(-\frac{3}{2}\right)^2$ is $\frac{9}{4}$.

$$x^2-5x+\_\_\_\_+y^2-3y+\_\_\_\_=-\frac{25}{4}+\_\_\_\_+\_\_\_\_$$

$$x^2-5x+\frac{25}{4}+y^2-3y+\frac{9}{4}=-\frac{25}{4}+\frac{25}{4}+\frac{9}{4}$$

$$\left(x-\frac{5}{2}\right)^2+\left(y-\frac{3}{2}\right)^2=\frac{9}{4}$$

This is a circle with center at $\left(\frac{5}{2},\ \frac{3}{2}\right)$ and with radius $r=\frac{3}{2}$. To graph this circle, start at the origin and count 2.5 units to the right, then up 1.5 units. This is the center of the circle. Now, the radius is $r=\frac{3}{2}$ or 1.5 so measure 1.5 units in each direction from the center. The graph should look like this.



Circle with center at $\left(\frac{5}{2},\ \frac{3}{2}\right)$ and with radius $r=\frac{3}{2}$.

P. 239. # 30. $4x^2+4y^2-20x+4y=-24$

Solution: Before you ever start the problem, you know that this looks like a CIRCLE! The most effective way to find the center and radius is by completing the square. However, before you can complete the square, you need to have coefficients of $1\ x^2$ and $1\ y^2$. The best way to accomplish this is to divide both sides of the equation by 4.

$$4x^2+4y^2-20x+4y=-24$$

$$x^2+y^2-5x+1y=-6$$

Rearrange the terms with the x terms together, and the *y* terms together, and leave spaces to complete the square for each variable.

$$x^2+y^2-5x+1y=-6$$

$$x^2-5x+\_\_\_\_+y^2+1y+\_\_\_\_=-6+\_\_\_\_+\_\_\_\_$$

When you complete the square, remember that you take half of the coefficient, and square. Half of -5 is $-\frac{5}{2}$ and $\left(-\frac{5}{2}\right)^2$ is $\frac{25}{4}$. Half of 1 is $\frac{1}{2}$, and $\left(\frac{1}{2}\right)^2$ is $\frac{1}{4}$.

$$x^2-5x+\_\_\_\_+y^2+1y+\_\_\_\_=-6+\_\_\_\_+\_\_\_\_$$

$$x^2-5x+\frac{25}{4}+y^2+1y+\frac{1}{4}=-\frac{24}{4}+\frac{25}{4}+\frac{1}{4}$$

$$\left(x-\frac{5}{2}\right)^2+\left(y+\frac{1}{2}\right)^2=\frac{2}{4}$$

This is a circle with center at $\left(\frac{5}{2},\ -\frac{1}{2}\right)$ and with radius $r^2=\frac{2}{4}$, so $r=\frac{\sqrt{2}}{2}$.

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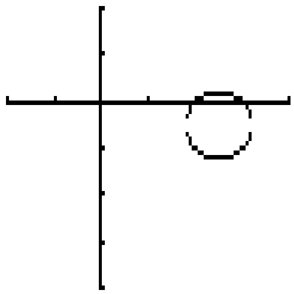
P. 239: #30 continued.

This is a circle with center at $\left(\frac{5}{2}, -\frac{1}{2}\right)$ and with radius $r^2 = \frac{2}{4}$, so $r = \frac{\sqrt{2}}{2}$.

To graph this circle, start at the origin and count 2.5 units to the right, then down 0.5 units. This is the center of the circle. Now, the radius is $r = \frac{\sqrt{2}}{2}$ or approximately 0.7. Measure about .7 units (less than 1 unit) in each direction from the center. The graph should look like this.



Circle with center at $\left(\frac{5}{2}, -\frac{1}{2}\right)$ and with radius $r = \frac{\sqrt{2}}{2}$.

P. 238. # 39. Find the equation of a circle with center at $(4\ ,\ -2)$ and passing through $(1\ ,\ 2)$.

Solution: In order to find the equation of a circle, you must know the center and the radius of the circle. In this case, you are given the center, but not the radius of the circle. However, if you know the center and a point that is ON the circle, then the distance between these two points is the radius. You must find the distance between the two given points by the distance formula.

Distance Formula

The distance between two points (x_1, y_1) and (x_2, y_2)

is given by the formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Now, the distance between the two points given in this problem, and therefore the radius of the circle, is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Radius $= \sqrt{(1-4)^2 + (2+2)^2}$

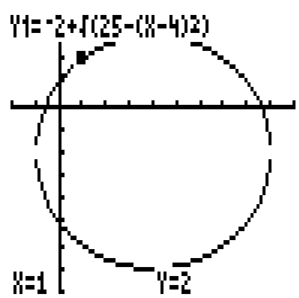
Radius $= \sqrt{(-3)^2 + (4)^2}$

Radius $= \sqrt{9+16}$

Radius $= \sqrt{25} = 5$

Therefore the equation of the circle with center at $(4\ ,\ -2)$ and of radius $\sqrt{25} = 5$ is given by $(x-4)^2 + (y+2)^2 = 25$.

As a check (no extra charge!!), you might want to sketch the graph, and see if it does indeed pass through the point $(1\ ,\ 2)$. As you can see, it does!!



Final Answer: $(x-4)^2 + (y+2)^2 = 25$