Math in Living C O L O R !!

2.10 Inverse Functions

College Algebra: One Step at a Time, **Pages 324 - 332: #5, 6, 7, 11, 14**

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See Section 2.10, with explanations, examples, and exercises, coming soon!

P. 326. # 5. Show that $f(x)$ and $f^{-1}(x)$ are inverse functions of one another.

$$f(x)=\frac{x+4}{x} \qquad f^{-1}(x)=\frac{4}{x-1}$$

Solution:

First find $f\left[f^{-1}(x)\right]$.

$$f[Junk]=\frac{(Junk)+4}{(Junk)}$$

$$f\left[f^{-1}(x)\right]=\frac{\left(f^{-1}(x)\right)+4}{\left(f^{-1}(x)\right)}$$

$$f\left[f^{-1}(x)\right] = f\left[\frac{4}{x-1}\right]=\frac{\left(\frac{4}{x-1}\right)+4}{\left(\frac{4}{x-1}\right)}$$

This is a complex fraction, which can be simplified by a couple of different methods. I think it works nicely if you multiply numerator and denominator by the LCD which is $x-1$. It may help to think of this as $\frac{(x-1)}{1}$.

$$f\left[f^{-1}(x)\right]=\frac{\frac{(x-1)}{1}\cdot\left[\left(\frac{4}{x-1}\right)+4\right]}{\frac{(x-1)}{1}\cdot\left[\left(\frac{4}{x-1}\right)\right]}$$

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P. 326: #5 continued.

$$f\left[f^{-1}(x)\right]=\frac{\left[\frac{(x-1)}{1}\bullet\left(\frac{4}{x-1}\right)+\frac{(x-1)}{1}\bullet 4\right]}{\left[\frac{(x-1)}{1}\bullet\left(\frac{4}{x-1}\right)\right]}$$

$$f\left[f^{-1}(x)\right]=\frac{[4+4(x-1)]}{[4]}$$

$$f\left[f^{-1}(x)\right]=\frac{[4+4x-4]}{[4]}$$

$$f\left[f^{-1}(x)\right]=\frac{4x}{4}=x$$

For the second part, you must find $f^{-1}[f(x)]$

Recall that $f(x)=\frac{x+4}{x}$ $\quad f^{-1}(x)=\frac{4}{x-1}$

Now $f^{-1}[Junk]=\frac{4}{(Junk)-1}$

$$f^{-1}[f(x)]=\frac{4}{(f(x))-1}$$

$$f^{-1}\left[\frac{x+4}{x}\right]=\frac{4}{\left(\frac{x+4}{x}\right)-1}$$

$$f^{-1}\left[\frac{x+4}{x}\right]=\frac{4}{\left(\frac{x+4}{x}\right)-1}$$

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P. 326: #5 continued.

This is also a complex fraction, which can be simplified by a couple of different methods. Again, I think it works nicely if you multiply numerator and denominator by the LCD which is $\frac{x}{1}$.

$$f^{-1}[f(x)] = \frac{\frac{x}{1} \bullet}{\frac{x}{1} \bullet} \frac{4}{\left[\left(\frac{x+4}{x}\right) - 1\right]}$$

$$f^{-1}[f(x)] = \frac{\frac{x}{1} \bullet 4}{\left[\frac{x}{1} \bullet \left(\frac{x+4}{x}\right) - \frac{x}{1} \bullet 1\right]}$$

$$f^{-1}[f(x)] = \frac{4x}{[x+4-x]}$$

$$f^{-1}[f(x)] = \frac{4x}{4} = x$$

P. 327. # 6. Show that $f(x)$ and $f^{-1}(x)$ are inverse functions of one another.

$$f(x)=\frac{x}{x-4} \qquad\qquad f^{-1}(x)=\frac{4x}{x-1}$$

Solution:

First find $f\left[f^{-1}(x)\right]$.

$$f[Junk]=\frac{(Junk)}{(Junk)-4}$$

$$f[f^{-1}(x)]=\frac{(f^{-1}(x))}{(f^{-1}(x))-4}$$

$$f[f^{-1}(x)]=\frac{\left(\frac{4x}{x-1}\right)}{\left(\frac{4x}{x-1}\right)-4}$$

This is a complex fraction, which can be simplified by a couple of different methods. I think it works nicely if you multiply numerator and denominator by the LCD which is $x-1$. It may help to think of this as $\frac{(x-1)}{1}$.

$$f\left[f^{-1}(x)\right]=\frac{\frac{(x-1)}{1}\bullet}{\frac{(x-1)}{1}\bullet}\frac{\left[\left(\frac{4x}{x-1}\right)\right]}{\left[\left(\frac{4x}{x-1}\right)-4\right]}$$

$$f\left[f^{-1}(x)\right]=\frac{\left[\frac{\cancel{(x-1)}}{1}\bullet\left(\frac{4x}{\cancel{x-1}}\right)\right]}{\left[\frac{\cancel{(x-1)}}{1}\bullet\left(\frac{4x}{\cancel{x-1}}\right)-4\bullet\frac{(x-1)}{1}\right]}$$

$$f\left[f^{-1}(x)\right]=\frac{[4x]}{[4x-4x+4]}$$

$$f\left[f^{-1}(x)\right]=\frac{4x}{4}=x$$

For the second part, you must find $f^{-1}[f(x)]$

Recall that $f(x)=\frac{x}{x-4}$ $f^{-1}(x)=\frac{4x}{x-1}$

Now $f^{-1}[Junk]=\frac{4(Junk)}{(Junk)-1}$

$$f^{-1}[f(x)]=\frac{4(f(x))}{(f(x))-1}$$

$$f^{-1}[f(x)]=\frac{4\left(\frac{x}{x-4}\right)}{\left(\frac{x}{x-4}\right)-1}$$

This is also a complex fraction, which can be simplified by a couple of different methods. Again, I think it works nicely if you multiply numerator and denominator by the LCD which is $\frac{x-4}{1}$.

$$f^{-1}[f(x)]=\frac{\frac{x-4}{1}\bullet}{\frac{x-4}{1}\bullet}\frac{4\left(\frac{x}{x-4}\right)}{\left(\frac{x}{x-4}\right)-1}$$

$$f^{-1}[f(x)]=\frac{\frac{x-4}{1}\bullet 4\left(\frac{x}{x-4}\right)}{\frac{x-4}{1}\bullet\left(\frac{x}{x-4}\right)-1\bullet\frac{x-4}{1}}$$

$$f^{-1}[f(x)]=\frac{4x}{x-x+4}$$

$$f^{-1}[f(x)]=\frac{4x}{4}=x$$

P. 327. # 7. Show that $f(x)$ and $f^{-1}(x)$ are inverse functions of one another.

$$f(x)=\frac{x^3-8}{4} \qquad f^{-1}(x)=\sqrt[3]{4x+8}$$

Solution:

First find $f\left[f^{-1}(x)\right]$.

$$f[Junk]=\frac{(Junk)^3-8}{4}$$

$$f[f^{-1}(x)]=\frac{(f^{-1}(x))^3-8}{4}$$

$$f[f^{-1}(x)]=\frac{(\sqrt[3]{4x+8})^3-8}{4}$$

$$f[f^{-1}(x)]=\frac{4x+8-8}{4}$$

$$f\left[f^{-1}(x)\right]=\frac{4x}{4}=x$$

For the second part, you must find $f^{-1}\left[f(x)\right]$

Recall that $f(x)=\frac{x^3-8}{4} \qquad f^{-1}(x)=\sqrt[3]{4x+8}$

Now $f^{-1}\left[Junk\right]=\sqrt[3]{4\left(Junk\right)+8}$

$$f^{-1}\left[f(x)\right]=\sqrt[3]{4f(x)+8}$$

$$f^{-1}\left[f(x)\right]=\sqrt[3]{4\left(\frac{x^3-8}{4}\right)+8}$$

$$f^{-1}\left[f(x)\right]=\sqrt[3]{\cancel{4}\left(\frac{x^3-8}{\cancel{4}}\right)+8}$$

$$f^{-1}\left[f(x)\right]=\sqrt[3]{x^3-8+8}$$

$$f^{-1}\left[f(x)\right]=\sqrt[3]{x^3}=x$$

Page 328. # 11. Find $f^{-1}(x)$ for $f(x)=\frac{5x+3}{2}$

Solution: Let $y=\frac{5x+3}{2}$

Step 1: Interchange the x and y: $x=\frac{5y+3}{2}$

Step 2: Solve for y!

Multiply both sides by 2: $2x=5y+3$

Subtract 3 $2x-3=5y$

Divide both sides by 5: $\frac{2x-3}{5}=y$

Therefore, $f^{-1}(x)=\frac{2x-3}{5}$

Page 329. # 14. Find $f^{-1}(x)$ for $y=\frac{3x+4}{5x}$

Solution: Let $y=\frac{3x+4}{5x}$

Step 1: Interchange the x and y: $x=\frac{3y+4}{5y}$

Step 2: Solve for y!

Multiply both sides by 5y: $5xy=3y+4$

Get y terms on left side: $5xy-3y=4$

Factor out the y: $y(5x-3)=4$

Divide both sides by 5x-3: $y=\frac{4}{5x-3}$

Therefore, $f^{-1}(x)=\frac{4}{5x-3}$