Math in Living C O L O R !!

3.02 Polynomial Expressions and Equations

College Algebra: One Step at a Time, **Pages 352 - 360: #19, 26, 29, 32, 53, 58, 59, 60**

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See Section 3.02, with explanations, examples, and exercises, coming soon!

P. 354. #19. Find all real solutions for $x^3 + 12 = 4x + 3x^2$.

Solution: Start by setting the equation equal to zero, and arrange the terms in descending powers of x.

$$\begin{array}{rcl} x^3 \quad + \quad 12 &=& 4x + 3x^2 \\ -3x^2 - 4x \quad && -4x - 3x^2 \\ \hline \end{array}$$

$$x^3 - 3x^2 - 4x + 12 = 0$$

To solve this by algebraic methods, notice that it factors by grouping by grouping the first two and the last two terms.

$$x^3 - 3x^2 - 4x + 12 = 0$$

$$x^2(x-3) - 4(x-3) = 0$$

Factor the common factor of $(x-3)$:

$$(x-3)(x^2-4) = 0$$

Factor the difference of two squares:

$$(x-3)(x-2)(x+2) = 0$$

Solve for x: $x = 3, \; x = 2, \; x = -2$ There are three real solutions!

If you happen to have a graphing calculator with the program POLYSMLT, this works well on this exercise.

P. 355. #26. Find all real solutions for $x^3 + 2x^2 - 6x = 0$.

Solution: Factor the left side of the equation. Take out a factor of x.

$$x^3 + 2x^2 - 6x = 0$$

$$x(x^2 + 2x - 6) = 0$$

This gives you a solution of $x = 0$ and a trinomial $x^2 + 2x - 6$ that does NOT factor. You can use either the quadratic formula or the method of completing the square to solve this trinomial equation. Because the coefficient of x is even, completing the square is probably easier.

$$x^2 + 2x - 6 = 0$$

Add 6 to each side, and prepare to complete the square:

$$x^2 + 2x + \_\_\_ = 6 + \_\_\_$$

Take half of the 2 which is 1, and square which gives you $+1$, and add $+1$ to each side:

$$x^2 + 2x + 1 = 6 + 1$$

$$(x+1)^2 = 7$$

Take the square root of each side of the equation. Don't forget the $\pm$ signs!

$$x + 1 = \pm\sqrt{7}$$

Subtract 1 from each side:

$$x = -1 \pm \sqrt{7}$$

Final Answer: There are three real solutions: $x = 0$, $x = -1 \pm \sqrt{7}$

P. 355. #29. Find all solutions, both real and complex, for $x^4 = 8x^2 + 9$.

Solution: Set the equation equal to zero, and factor the resulting trinomial.

$$x^4 = 8x^2 + 9$$
$$x^4 - 8x^2 - 9 = 0$$
$$(x^2 - 9)(x^2 + 1) = 0$$

Set each factor equal to zero, and solve for x:

$$x^2 - 9 = 0 \quad \text{or} \quad x^2 + 1 = 0$$
$$x^2 = 9 \quad \text{or} \quad x^2 = -1$$

Take the square root of each side. Remember that $\sqrt{-1} = i$, and don't forget the $\pm$ signs.

$$x = \pm\, 3 \quad \text{or} \quad x = \pm\sqrt{-1}$$
$$x = \pm\, i$$

Final Answer: There are four solutions: $x = \pm\, 3,\quad x = \pm\, i$

P. 356. #32. Find all solutions, both real and complex, for $x^4 + 13x^2 + 36 = 0$.

Solution: Start by factoring the trinomial.

$$x^4 + 13x^2 + 36 = 0$$
$$(x^2 + 9)(x^2 + 4) = 0$$

Set each factor equal to zero, and solve for x:

$$x^2 + 9 = 0 \quad \text{or} \quad x^2 + 4 = 0$$
$$x^2 = -9 \quad \text{or} \quad x^2 = -4$$

Take the square root of each side. Remember that $\sqrt{-1} = i$, and don't forget the $\pm$ signs.

$$x = \pm\sqrt{-9} \quad \text{or} \quad x = \pm\sqrt{-4}$$
$$x = \pm\, 3i \quad \text{or} \quad x = \pm\, 2i$$

Final Answer: There are four solutions: $x = \pm\, 3i,\quad x = \pm\, 2i$

P. 360. #53. Find the equation whose roots are $x = -3 \pm 4\,i$.

Solution: In most exercises, you are given an equation and asked to solve it. Now, the process is reversed. You are given the solutions to an equation, and you are asked to find the equation. What you have to do in this exercise is to reverse the steps of a completing the square problem. If you need to see a completing the square problem, see #26, which is solved on this webpage.

Beginning with these two roots: $x = -3 \pm 4\,i$

Add +3 to each side: $x + 3 = \pm 4\,i$

Next, square both sides: $(x+3)^2 = (\pm 4i)^2$

$$x^2 + 6x + 9 = 16i^2$$

Remember that $i^2 = -1$: $x^2 + 6x + 9 = 16 \bullet (-1)$

$$x^2 + 6x + 9 = -16$$

Add +16 to each side $x^2 - 6x + 25 = 0$

Final Answer: Be sure to answer in the form of an equation!

You may want to check this by completing the square or by calculator methods (POLYSMLT).

P. 360. #58. Find the equation whose roots are $x = -3, \quad x = 3 \pm i\sqrt{2}$.

Solution: Begin the root $x = -3$, which came from the factor: $x + 3 = 0$

Next, consider at the last two roots: $x = 3 \pm i\sqrt{2}$

Subtract 3 from each side: $x - 3 = \pm i\sqrt{2}$

Next, square both sides: $(x-3)^2 = (\pm i\sqrt{2})^2$

$$x^2 - 6x + 9 = i^2 \bullet 2$$

$$x^2 - 6x + 9 = -2$$

$$x^2 - 6x + 11 = 0$$

Final Answer: Be sure to answer in the form of an equation!

$$(x+3)(x^2 - 6x + 11) = 0$$

P. 360. #59. Find the equation whose roots are $x = 3$, $x = -2$, $x = 4 \pm i$.

Solution: First consider the roots: $x = 3, \; x = -2$

These came from factors: $x - 3 = 0, \; x + 2 = 0$

$$(x-3)(x+2) = 0$$

which multiplied out becomes $x^2 - x - 6 = 0$

Now, consider the last two roots: $x = 4 \pm i$

Subtract 4 from each side: $x - 4 = \pm i$

Next, square both sides: $(x-4)^2 = (\pm i)^2$

$$x^2 - 8x + 16 = i^2$$

$$x^2 - 8x + 16 = -1$$

$$x^2 - 8x + 17 = 0$$

Final Answer: Be sure to answer in the form of an equation!

$$(x^2 - x - 6)(x^2 - 8x + 17) = 0$$

P. 360. #60. Find the equation whose roots are

$x = 2$, $x = -5$, $x = -3 \pm 5i$.

Solution: First consider at the roots: $x = 2, \; x = -5$

These came from factors: $x - 2 = 0, \; x + 5 = 0$

$$(x-2)(x+5) = 0$$

which multiplied out becomes $x^2 + 3x - 10 = 0$

Now, consider the last two roots: $x = -3 \pm 5i$

Add +3 to each side: $x + 3 = \pm 5i$

Next, square both sides: $(x+3)^2 = (\pm 5i)^2$

$$x^2 + 6x + 9 = 25i^2$$

$$x^2 + 6x + 9 = -25$$

$$x^2 + 6x + 9 + 25 = 0$$

$$x^2 + 6x + 34 = 0$$

Final Answer: Be sure to answer in the form of an equation!

$$(x^2 + 3x - 10)(x^2 + 6x + 34) = 0$$