

Math in Living C O L O R !!

3.07 Systems of Equations (2x2) and Inequalities

College Algebra: One Step at a Time, Pages 430-437: 2 Extra,#13,16,18,19, 20, 25,Extra

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See Section 3.07, with explanations, examples, and exercises, coming soon!

In each problem involving inequalities, there are three steps. **First**, you must get the line in place, by graphing the corresponding equation. **Second**, you must decide whether the line should be included or not--that is, should it be a dotted or solid line. **Third**, you must decide whether to shade above or below the line. In later problems, there is a **fourth** step, in which, when there are two or more inequalities, you must decide whether to shade the **Union** or the **Intersection** of the areas.

Step 1: Graph the Line!

(Use methods of previous sections!)

Step 2: Dotted ($<$ or $>$) or Solid (\leq or \geq)!

Step 3: Shade ABOVE or BELOW!

+ $Y > \geq$ Shade ABOVE the Line!

+ $Y < \leq$ Shade BELOW the Line!

**NOTE: You MUST have a positive Y coefficient!!
If you have a negative Y coefficient,
this REVERSES the RULE!!**

NOTE: Unfortunately, in the format of this website, I have not learned how to make a dotted line. I will have to ask YOU to make the lines dotted that have either a $<$ or $>$ symbol.

Example 1. $3x + 2y > -12$

Solution:

Step 1: Graph the line: $3x + 2y = -12$

Since this is in standard form, find the x and y intercepts.

If $x = 0$, then $2y = -12$

$$y = -6$$

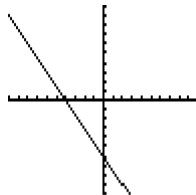
If $y = 0$, then $3x = -12$

$$x = -4$$

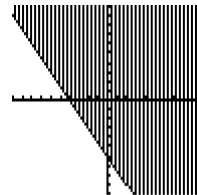
Step 2: Graph this line with a dotted line.

(You will have to do this for me, since I don't know how to graph a dotted line in this format).

$$3x + 2y = -12$$



$$3x + 2y > -12$$



(Dotted Line!)

Step 3: Shade above the line. Don't forget to make this line dotted!!

Example 2. $-2x + y \leq 8$

Solution:

Step 1: Graph the line: $-2x + y = 8$

Since this is in standard form, find the x and y intercepts.

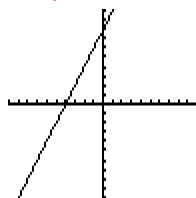
If $x = 0$, then $y = 8$

If $y = 0$, then $-2x = 8$

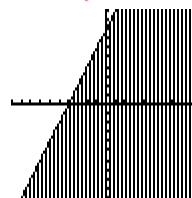
$$x = -4$$

Step 2: Graph this line with a solid line.

$$-2x + y = 8$$



$$-2x + y \leq 8$$



(Solid Line!)

Step 3: Shade below the line. Don't forget to make this line solid!!

P. 433 #13. $2x - 3y \geq -12$
 $-y \leq 4x - 8$

Solution:

Preliminary Step: Multiply both sides of the second equation by -1:

$$2x - 3y \geq -12$$

$$y \geq -4x + 8$$

Step 1: Graph the first line: $2x - 3y = -12$

Since the first line is in standard form, find the x and y intercepts.

If $x = 0$, then $-3y = -12$

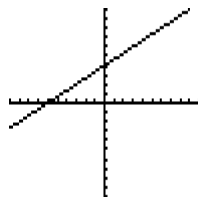
$$y = 4$$

If $y = 0$, then $2x = -12$

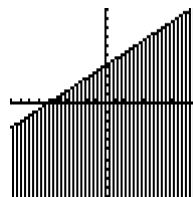
$$x = -6$$

Step 2: Graph this line with a solid line.

$$2x - 3y = -12$$



$$2x - 3y \geq -12$$



(Solid Line!)

Step 3: Since this is a negative y coefficient, the rule is reversed.
 Shade below the line! Don't forget to make this line solid!!

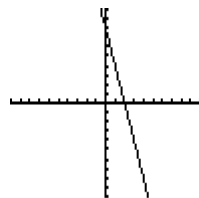
Now, do the same process with the second inequality, using the same xy axes:

Step 1: Graph the second line: $y = -4x + 8$

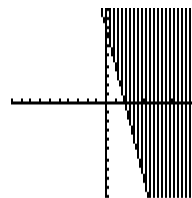
Since the second line is in slope intercept form, find the y intercept and the slope. The y intercept is 8, and the slope is -4.

Step 2: Graph this line with a solid line.

$$y = -4x + 8$$



$$y \geq -4x + 8$$



(Solid Line!)

Step 3: Shade above this line! Don't forget to make this line solid!!

Continued on next page

P. 433 #13 continued.

Step 4: Shade the **UNION** of the two regions. This includes EVERYTHING that was shaded in either of the two graphs. It should look like this:



Final Answer: You must shade everything that is shaded on this graph !

P. 433 #16. $4x - y > -8$
 $x + 2y < -4$

Solution:

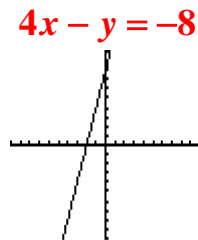
Step 1: Graph the first line: $4x - y = -8$

Since the first line is in standard form, find the x and y-intercepts.

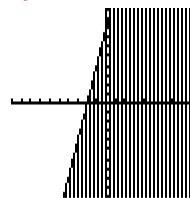
If $x = 0$, then $-y = -8$
 $y = 8$

If $y = 0$, then $4x = -8$
 $x = -2$

Step 2: Graph this line with a dotted line.



$4x - y > -8$



(Dotted Line!)

Step 3: Since this is a negative y-coefficient, the rule is reversed. Shade below the line! Don't forget to make this line dotted!!

Continued on next page

P. 433 #16 continued.

Now, do the same process with the second inequality, using the same xy axes:

Step 1: Graph the second line: $x + 2y = -4$

Since the second line is in standard form, find the x and y-intercepts.

$$\text{If } x = 0, \text{ then } 2y = -4$$

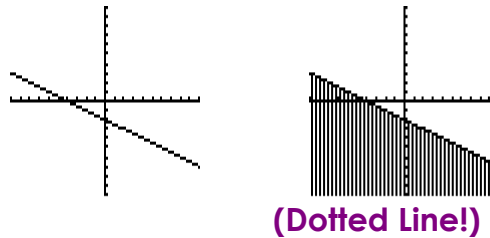
$$y = -2$$

$$\text{If } y = 0, \text{ then } x = -4$$

Step 2: Graph this line with a dotted line.

$$x + 2y = -4$$

$$x + 2y < -4$$



Step 3: Shade below this line! Don't forget to make this line dotted!!

Step 4: Shade the **INTERSECTION** of the two regions.

This includes **ONLY** the region common to both, the cross-shaded area **ONLY**. It should look like this:



Final Answer: You must shade **ONLY** the cross-shaded region on this graph!

The solution is the lower right corner of the graph **ONLY**!

Also, don't forget to draw dotted lines!!

P. 433 #18. $y \geq -4x + 4$
 $y \leq 2x - 4$

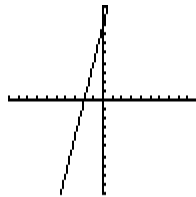
Solution:

Step 1: Graph the first line: $y = -4x + 4$

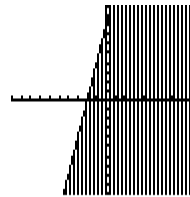
Since the first line is in slope intercept form, find the slope and y intercept. The y-intercept is 4 and the slope is -4.

Step 2: Graph this line with a solid line.

$y = -4x + 4$



$y \geq -4x + 4$



(Solid Line!)

Step 3: Shade above the line! Don't forget to make this line solid!!

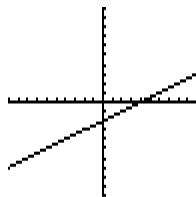
Now, do the same process with the second inequality, using the same xy axes:

Step 1: Graph the second line: $y = 2x - 4$

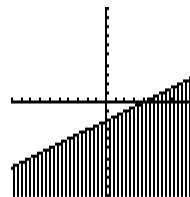
Since the first line is in slope intercept form, find the y intercept and the slope. The y intercept is -4, and the slope is 2.

Step 2: Graph this line with a solid line.

$y = 2x - 4$



$y \leq 2x - 4$



(Solid Line!)

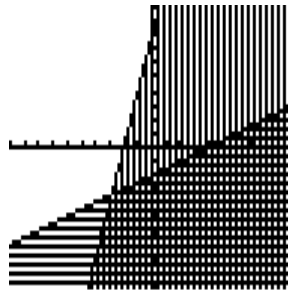
Step 3: Shade below this line! Don't forget to make this line solid!!

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P. 433 #18 continued.

Step 4: Shade the **INTERSECTION** of the two regions.

This includes **ONLY** the region common to both, the cross-shaded area **ONLY**. It should look like this:



Final Answer: You must shade **ONLY** the cross-shaded region on this graph!
(The solution is the lower right corner of the graph **ONLY**!)

P. 434 #19. $3x - 2y \leq 12$

$$x \geq 0$$

$$y \leq 0$$

Solution:

Step 1: Graph the first line: $3x - 2y = 12$

Since the first line is in standard form, find the x and y-intercepts.

$$\text{If } x = 0, \text{ then } -2y = 12$$

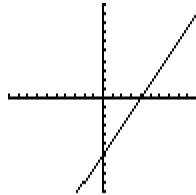
$$y = -6$$

$$\text{If } y = 0, \text{ then } 3x = 12$$

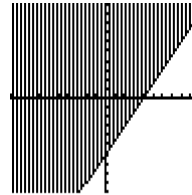
$$x = 4$$

Step 2: Graph this line with a solid line.

$$3x - 2y = 12$$



$$3x - 2y \leq 12$$



(Solid Line!)

Step 3: Since this is a negative y-coefficient, the rule is reversed.

Shade above the line! Don't forget to make this line solid!!

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P. 433 #19 continued.

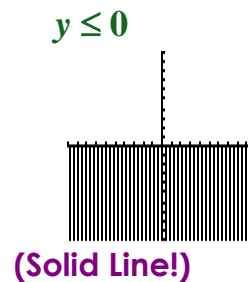
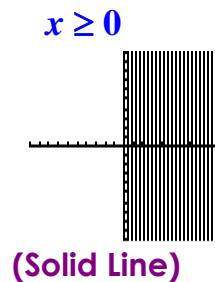
Now, do the same process with the second and third inequality (use the same xy axes as the first):

Graph the second line: $x = 0$

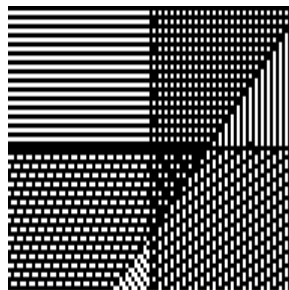
This is a vertical line, the y-axis; make it a solid line; shade to the right.

Graph the third line: $y = 0$

This is a horizontal line, the x-axis; make it a solid line; shade below.



Step 4: Shade the **INTERSECTION** of the three regions.
This includes **ONLY** the region common to all three,
the triple-shaded area **ONLY**. It should look like this:



Final Answer: You must shade **ONLY** the triple-shaded region on this graph!
The solution is the triangular shaped region in the graph **ONLY**!
Also, don't forget to draw solid lines!!

P. 434 #20. $x + 3y > -6$

$x < 2$

$y < 3$

Solution:

Step 1: Graph the first line: $x + 3y = -6$

Since the first line is in standard form, find the x and y-intercepts.

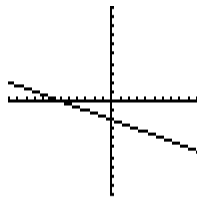
If $x = 0$, then $3y = -6$

$y = -2$

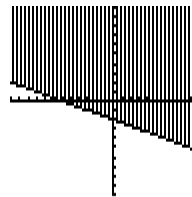
If $y = 0$, then $x = -6$

Step 2: Graph this line with a dotted line.

$x + 3y = -6$



$x + 3y > -6$



(Dotted Line!)

Step 3: This is a positive y-coefficient, with a “>” symbol.

Shade above the line! Don't forget to make this line dotted!!

Now, do the same process with the second and third inequality, using the same xy axes.

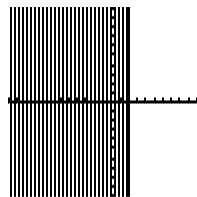
Graph the second line: $x = 2$

This is a vertical line, 2 units to the right of the y-axis, make it a dotted line, and shade to the left.

Graph the third line: $y = 3$

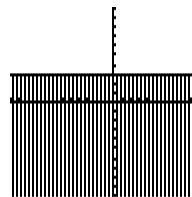
This is a horizontal line, up 3 units above the x-axis, make it a dotted line, and shade below the line.

$x < 2$



(Dotted Line)

$y < 3$



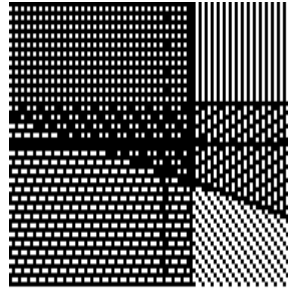
(Dotted Line!)

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P. 434 #20 continued.

Step 4: Shade the **INTERSECTION** of the three regions.

This includes **ONLY** the region common to all three, the triple-shaded area **ONLY**. It should look like this:



Final Answer: You must shade **ONLY** the triple-shaded region on this graph!
The solution is the triangular shaped region in the graph **ONLY**!
Don't forget to draw dotted lines!!

P. 436 #25.

$$x + 2y < -4$$
$$x - 2y > -4$$
$$y < -3x + 2$$

Solution:

Step 1: Graph the **first line:** $x + 2y = -4$

Since the first line is in standard form, find the x and y-intercepts.

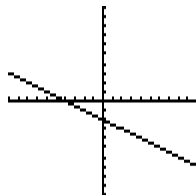
If $x = 0$, then $2y = -4$

$$y = -2$$

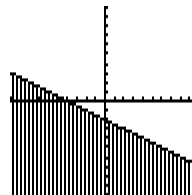
If $y = 0$, then $x = -4$

Step 2: Graph this line with a dotted line.

$$x + 2y = -4$$



$$x + 2y < -4$$



(Dotted Line!)

Step 3: This is a positive y-coefficient, with "<".

Shade below the line! Don't forget to make this line dotted!!

Continued on next page

P. 436 #25 continued.

Now, do the same process with the **second inequality**, using the same xy axes:

Step 1: Graph the second line: $x - 2y > -4$

Since the second line is in standard form, find the x and y-intercepts.

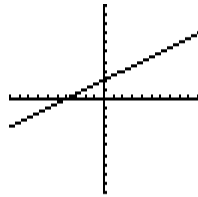
$$\text{If } x = 0, \text{ then } -2y = -4$$

$$y = 2$$

$$\text{If } y = 0, \text{ then } x = -4$$

Step 2: Graph this line with a dotted line.

$$x - 2y = -4$$



$$x - 2y > -4$$



(Dotted Line!)

Step 3: This is a negative y-coefficient, with “>”, so the rule is REVERSED! Shade below the line! Don't forget to make this line dotted!!

Now, do the same process with the **third inequality** using the same xy axes:

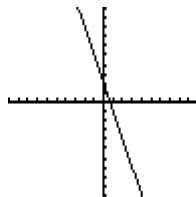
Step 1: Graph the **third line**: $y = -3x + 2$

Since the first line is in slope intercept form, find the slope and y intercept.

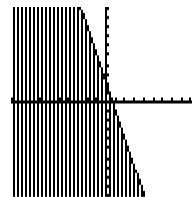
The y-intercept is 2 and the slope is -3.

Step 2: Graph this line with a solid line.

$$y = -3x + 2$$



$$y < -3x + 2$$



(Dotted Line!)

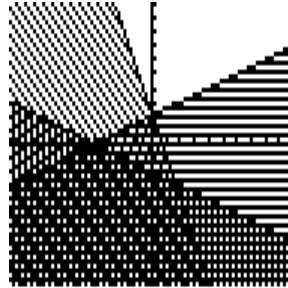
Step 3: Shade below the line! Don't forget to make this line solid!!

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P. 436 #25 continued.

Step 4: Shade the **INTERSECTION** of the three regions.

This includes **ONLY** the region common to all three, the triple-shaded area **ONLY**. It should look like this:



Final Answer: You must shade **ONLY** the triple-shaded region on this graph!
The solution is the infinite region, bounded by dotted lines, in the lower left of the graph!

EXTRA PROBLEM: Find the **INTERSECTION** of the regions.

$$3x - y < 2$$

$$x + y > 2$$

Solution:

Step 1: Graph the first line: $3x - y = 2$

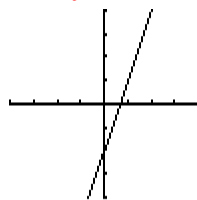
Since the first line is in standard form, find the x and y-intercepts.

$$\begin{aligned} \text{If } x = 0, \text{ then } -y &= 2 \\ y &= -2 \end{aligned}$$

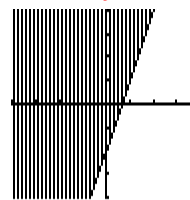
$$\begin{aligned} \text{If } y = 0, \text{ then } 3x &= 2 \\ x &= 2/3 \end{aligned}$$

Step 2: Graph this line with a dotted line.

$$3x - y = 2$$



$$3x - y < 2$$



(Dotted Line!)

Step 3: Since this is a negative y-coefficient, the rule is reversed.
Shade above the line! Don't forget to make this line dotted!!

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Extra Problem continued.

Now, do the same process with the second inequality using the same xy axes:

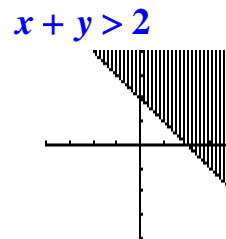
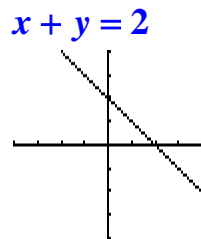
Step 1: Graph the second line: $x + y = 2$

Since the second line is in standard form, find the x and y-intercepts.

If $x = 0$, then $y = 2$

If $y = 0$, then $x = 2$

Step 2: Graph this line with a dotted line.

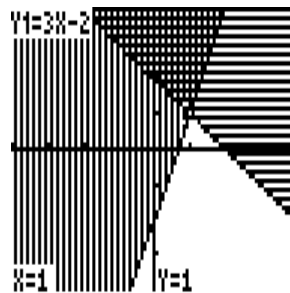


(Dotted Line!)

Step 3: Shade above this line! Don't forget to make this line dotted!!

Step 4: Shade the **INTERSECTION** of the two regions.

This includes **ONLY** the region common to both, the cross-shaded area **ONLY**. It should look like this:



Final Answer: You must shade **ONLY** the cross-shaded region on this graph!
The solution is the triangular-shaped region, bounded by dotted lines, in the upper middle of the graph!