Math in Living C O L O R !!

3.10 Non Linear Systems of Equations

with TI 84 Graphing Calculator

College Algebra: One Step at a Time.

Pages 472-483: #12, 18, 23, 25, 29, 37, 39, 45, 47, 50, 51, 55, 63

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See Section 3.10, with explanations, examples, and exercises, coming soon!

P. 474. # 12. $y=-2x+10$

$y=x^2+3x-4$

Solution:

Since both equations are in the form of $y=$ __ , you can set

$y=y$, and substitute what each y equals:

$$-2x+10=x^2+3x-4$$

The result is a quadratic equation, with positive x squared term on the right side. Set this equal to zero by adding $2x-10$ to each side:

$$-2x+10+2x-10=x^2+3x-4+2x-10$$

$$0=x^2+5x-14$$

This just happens to factor:

$$0=(x+7)(x-2)$$

$$x=-7 \quad x=2$$

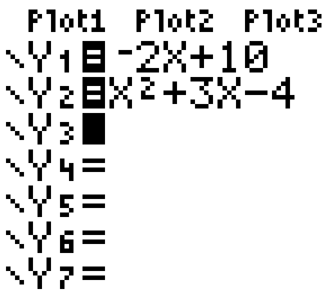
Choose the simplest equation and substitute the values of x in order to find the corresponding y values. In this case, $y=-2x+10$ is probably the easiest.

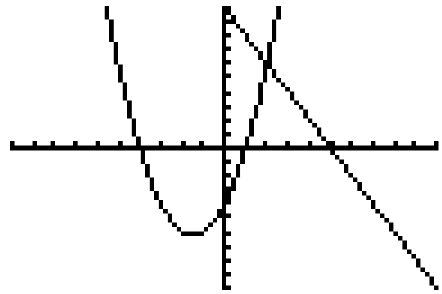
$y=-2x+10$	$y=-2x+10$
$y=-2(-7)+10$	$y=-2(2)+10$
$y=14+10$	$y=-4+10$
$y=24$	$y=6$

Final answer: (-7, 24) (2, 6)

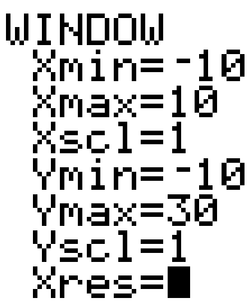
P. 474. # 12. Solution with TI 83/84 (or TI 85/86) Graphing Calculator

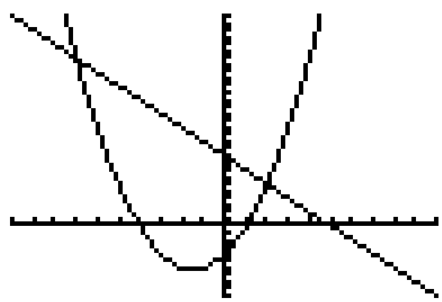
Since both equations are in the form $y = \_\_$, the graphs can easily be obtained on a graphing calculator. Let $y1 = -2x + 10$ and $y2 = x^2 + 3x - 4$. The equations should be entered as follows and the graphs in a standard window should look like this:



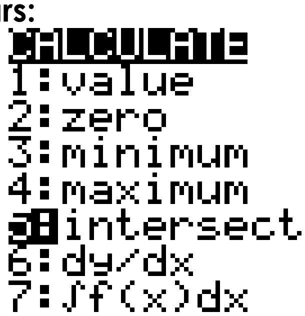


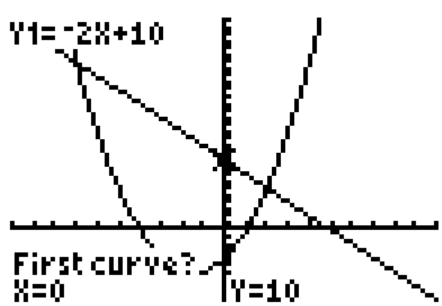
Notice that one of the points is clearly visible, and that there is another point of intersection that is above the graph as pictured here. It might be helpful to show the graph extended upwards beyond 10, let's say up to y = 30. To do this, press [WIND], press the down arrow to go down to [Ymax=] and change the 10 to 30, as shown below: Then press [GRAPH] and the graph should look like this:





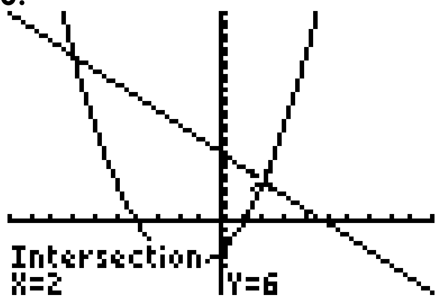
Now, you can see both points of intersection. To find the point(s) of intersection using a TI83/84, look for [2nd] [CALC], which is right above the [F4 (TRACE)] button. In the menu that appears and is shown below, notice [5: intersect]. Either press [5] or scroll down to it and press [ENTER], and the following screen appears:





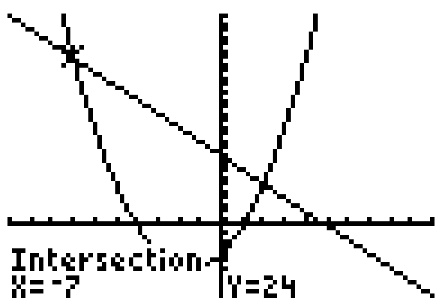
P. 474. # 12 continued.

The calculator is poised to go after the first point of intersection, but you need to give an indication of which point you want to find. It will actually find the point that is closest to the cursor, and the calculator is prompting you to give a point on the "First curve?". You should move the cursor close to the first point that you want to find. Let's find the one on the right side first. Move the cursor by pressing the right arrow until the cursor is "close" to the right point of intersection, and press [ENTER] [ENTER] [ENTER]. The screen should indicate the first point of intersection at x=2 and y=6:



For the TI 85/86, you can access this same process by pressing the following: [GRAPH] [MORE] [MATH] [MORE], and look for the function called [ISECT]. After pressing [ISECT], move the cursor towards the point of intersection, as explained for the TI 84 above, and as with the TI 84, press [ENTER] [ENTER] [ENTER].

Now back to the TI 84, after finding the first point of intersection, press [CLEAR], and begin again to find the second point of intersection: [2nd] [CALC] [5: intersection] . This time, press the left arrow key, and move the cursor "close" to the point of intersection on the left side. Press [ENTER] [ENTER] [ENTER]. The screen should indicate the second point of intersection at x=-7 and y=24:



The final answer consists of the two points: (2,6) and (-7, 24).

P. 474. # 18.

$$y = x^2 + 4x$$
$$y = 12 + 2x - x^2$$

Solution:

Since both equations are in the form of $y = \_\_$, you can set $y = y$, and substitute what each y equals:

$$x^2 + 4x = 12 + 2x - x^2$$

The result is a quadratic equation, with negative x squared term on the right side. Set this equal to zero by adding $-12 - 2x + x^2$ to each side:

$$x^2 + 4x - 12 - 2x + x^2 = 12 + 2x - x^2 - 12 - 2x + x^2$$
$$2x^2 + 2x - 12 = 0$$

This just happens to factor. Start with the common factor of 2.

$$2(x^2 + x - 6) = 0$$
$$2(x + 3)(x - 2) = 0$$
$$x = -3 \quad x = 2$$

Substitute these values of x into the simplest function, in this case, either will do:

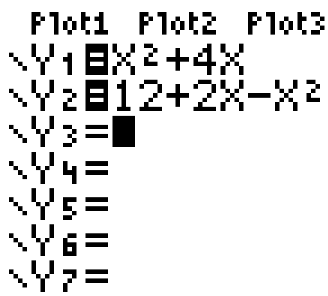
$y = x^2 + 4x$	$y = x^2 + 4x$
$y = (-3)^2 + 4(-3)$	$y = (2)^2 + 4(2)$
$y = 9 - 12$	$y = 4 + 8$
$y = -3$	$y = 12$

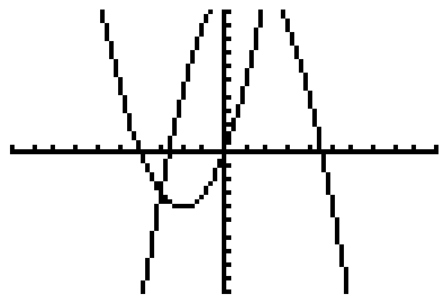
Final answer: (-3, -3) (2, 12)

Please see next page for Graphing Calculator Solution

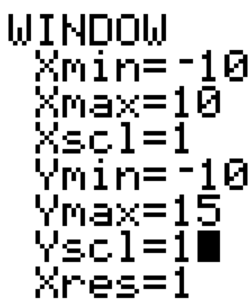
P. 474. #18 Graphing Calculator Solution with TI 83/84 or TI85/86

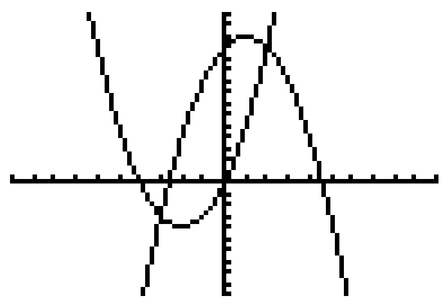
Since both equations are in the form $y = \_\_$, the graphs can easily be obtained on a graphing calculator. Let $y1 = x^2 + 4x$ and $y2 = 12 + 2x - x^2$. The equations should be entered as follows and the graphs in a standard window. If your window is NOT standardized, then press [F3 (ZOOM)], [6-ZStandard)]. It should look like this:



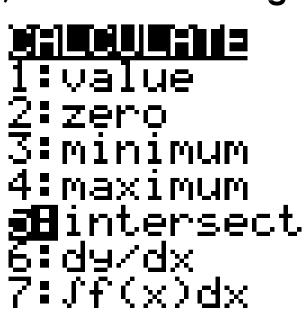


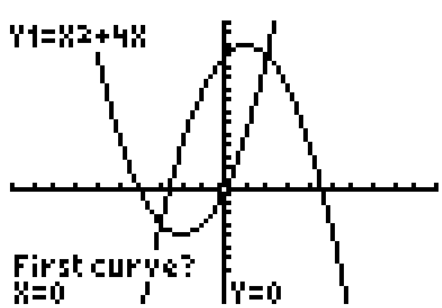
Oops! It looks like we cut off one of the points of intersection. Let's enlarge the window by pressing [WIND] , scroll down to [Ymax] and change it from 10 to 15.





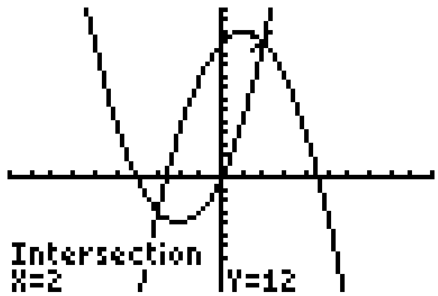
Next, to find the point(s) of intersection using a TI83/84, look for [2nd] [CALC], which is right above the [F4 (TRACE)] button. In the menu that appears and is shown below, notice [5: intersect]. Either press [5] or scroll down to it and press [ENTER], and the following screen appears:





P. 474. # 18 Graphing Calculator Solution continued.

The calculator is poised to go after the first point of intersection, but you need to give an indication of which point you want to find. It will actually find the point that is closest to the cursor, and the calculator is prompting you to give a point on the "First curve?". You should move the cursor close to the first point that you want to find. Let's find the one on the right side first. Move the cursor by pressing the right arrow until the cursor is "close" to the right point of intersection, and press [ENTER] [ENTER] [ENTER]. The screen should indicate the first point of intersection at x=2 and y=12:



For the TI 85/86, press the following keystrokes: [GRAPH] [MORE] [MATH] [MORE], and look for the function called [ISECT]. After pressing [ISECT], move the cursor towards the point of intersection, as explained for the TI 84 above, and as with the TI 84, press [ENTER] [ENTER] [ENTER].

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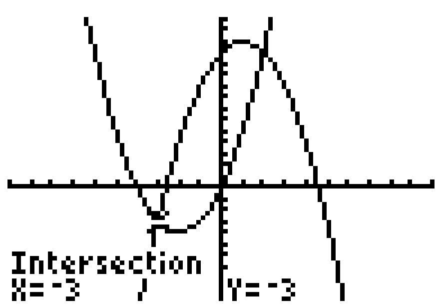
Now back to the TI84, after finding the first point of intersection, press [CLEAR], and begin again to find the second point of intersection: [2nd] [CALC] [5: intersection] . This time, press the left arrow key, and move the cursor "close" to the point of intersection on the left side. Press [ENTER] [ENTER] [ENTER]. The screen should indicate the second point of intersection at x=-3 and y=-3:



The final answer consists of the two points: (2,12) and (-3, -3).

P. 476. # 23.

$$xy = 21$$
$$y = 3x - 2$$

Solution:

The first step is to substitute the y of the second equation into the y in the first equation:

$$xy = 21$$
$$x(\qquad) = 21$$
$$x(3x - 2) = 21$$

The result is a quadratic equation. Set this equal to zero by subtracting 21 from each side:

$$3x^2 - 2x - 21 = 0$$

Factor (of course!!) the trinomial:

$$(3x + 7)(x - 3) = 0$$
$$3x = -7, \quad x = 3$$
$$x = -\frac{7}{3}$$

Substitute these values of x into the simplest function. In order to find y, the best way is use the equation $y = 3x - 2$.

$$x = -\frac{7}{3} \qquad\qquad x = 3$$
$$y = 3x - 2 \qquad\qquad y = 3x - 2$$
$$y = 3\left(-\frac{7}{3}\right) - 2 \qquad\qquad y = 3(3) - 2$$
$$y = -7 - 2 \qquad\qquad y = 9 - 2$$
$$y = -9 \qquad\qquad y = 7$$

Final answer: $\left(-\frac{7}{3}, -9\right)$ $(3, 7)$

Check: Substitute these values into the equation $xy = 21$. Essentially, this means to see if the product of each pair of numbers is 21. Since $-\frac{7}{3} \bullet -9 = 21$ and $3 \bullet 7 = 21$, the answers check!

P. 477. # 25.

$$xy = 30$$
$$2x - y = 7$$

Solution:

The first step is to solve for one variable in one of the two equations in terms of the other variable. It's probably easiest if you solve for y in the second equation:

$$2x - y = 7$$
$$\underline{-2x \qquad -2x}$$
$$-y = -2x + 7$$
$$y = 2x - 7$$

Substitute this in place of the y in the first equation:

$$xy = 30$$
$$x(\qquad) = 30$$
$$x(2x - 7) = 30$$

The result is a quadratic equation. Set this equal to zero by subtracting 30 from each side:

$$2x^2 - 7x - 30 = 0$$

Factor (of course!!) the trinomial:

$$(2x + 5)(x - 6) = 0$$
$$2x = -5, \quad x = 6$$
$$x = -\frac{5}{2}$$

Substitute these values of x into the simplest function. In order to find y, the best way is use the equation $y = 2x - 7$.

$$y = 2x - 7 \qquad\qquad y = 2x - 7$$
$$y = 2\left(-\frac{5}{2}\right) - 7 \qquad\qquad y = 2(6) - 7$$
$$y = -5 - 7 \qquad\qquad y = 12 - 7$$
$$y = -12 \qquad\qquad y = 5$$

Final answer: $\left(-\frac{5}{2}, -12\right)$ $(6, 5)$

Check: Substitute these values into the equation $xy = 30$. Essentially, this means to see if the product of each pair of numbers is 30. Since $-\frac{5}{2} \bullet -12 = 30$ and $6 \bullet 5 = 30$, the answers check!

P. 477. # 29.

$$x^2 + y^2 = 10$$
$$y = 2x - 5$$

Solution:

Since the second equation is in the form of $y = \_\_$ The first step is to substitute what y equals in the second equation back into the first equation:

$$x^2 + (\ \_\_\_\_\ )^2 = 10$$
$$x^2 + (2x - 5)^2 = 10$$

Remember, when you square a binomial, you square the first, take twice the product, and square the second.

$$x^2 + (4x^2 - 20x + 25) = 10$$

Combine like terms and set equal to zero:

$$5x^2 - 20x + 25 - 10 = 0$$
$$5x^2 - 20x + 15 = 0$$

Factor out the common factor of 5, see if the trinomial that results will factor. While it doesn't always factor, in this book it usually DOES!

$$5(x^2 - 4x + 3) = 0$$
$$5(x - 3)(x - 1) = 0$$
$$x = 3 \qquad x = 1$$

To find y, go back to the simplest equation. If one of the equations is written in the form of a function, like $y = \_\_$, use that equation to find y. Substitute both values of x:

$$y = 2x - 5 \qquad y = 2x - 5$$
$$y = 2(3) - 5 \qquad y = 2(1) - 5$$
$$y = 6 - 5 \qquad y = 2 - 5$$
$$y = 1 \qquad y = -3$$

Final answer: $(3,1)$ $(1, -3)$

P. 479. # 39.

$$4x^2 - 3y^2 = 4$$
$$y = x - 4$$

Solution:

Since the second equation is in the form of $y = \_\_$ The first step is to substitute what y equals in the second equation back into the first equation:

$$4x^2 - 3(\ \_\_\_\_\ )^2 = 4$$
$$4x^2 - 3(x - 4)^2 = 4$$

On the left side, be very careful. This takes two steps, since there is a -3 that is multiplied times the square of the binomial. First you must square the binomial, and second, you must multiply through by the -3. Don't try to do it all in one step! You probably aren't THAT good!! Remember, you square the first, take twice the product, and square the second.

$$4x^2 - 3(x^2 - 8x + 16) = 4$$
$$4x^2 - 3x^2 + 24x - 48 - 4 = 0$$
$$x^2 + 24x - 52 = 0$$

Do you think this trinomial will factor? No, it doesn't, and in real life it probably usually will NOT factor. But in this book is usually DOES!

$$(x + 26)(x - 2) = 0$$
$$x = -26, \quad x = 2$$

To find y, go back to the simplest equation. If one of the equations is written in the form of a function, like $y = \_\_$, use that equation to find y. Substitute both values of x:

$x = -26$	$x = 2$
$y = x - 4$	$y = x - 4$
$y = (-26) - 4$	$y = (2) - 4$
$y = -30$	$y = -2$

Final answer: $(-26, -30)$ $(2, -2)$

P. 479. # 37.

$$4x^2 - y^2 = -60$$
$$y = 3x - 5$$

Solution: Since the second equation is in the form of $y = \_\_$, the first step is to substitute what y equals in the second equation back into the first equation:

$$4x^2 - (\ \_\_\_\_\ )^2 = -60$$
$$4x^2 - (3x - 5)^2 = -60$$

On the left side, be very careful. This takes two steps, since there is a negative in front of the binomial squared. First you must square the binomial, and second, you must multiply through by the negative. Don't try to do it all in one step! You probably aren't THAT good!! Remember, you square the first, take twice the product, and square the second.

$$4x^2 - (9x^2 - 30x + 25) = -60$$
$$4x^2 - 9x^2 + 30x - 25 = -60$$
$$-5x^2 + 30x - 25 + 60 = 0$$
$$-5x^2 + 30x + 35 = 0$$

Factor out the common factor of -5, and hope that the trinomial that results will factor. It doesn't always factor. In fact, in real life, it usually will NOT factor. Perhaps a fault of this book is that it usually DOES!

$$-5(x^2 - 6x - 7) = 0$$
$$-5(x - 7)(x + 1) = 0$$
$$x = 7 \qquad x = -1$$

To find y, go back to the simplest equation. If one of the equations is written in the form of a function, like $y = \_\_$, use that equation to find y. Substitute both values of x:

$y = 3x - 5$	$y = 3x - 5$
$y = 3(7) - 5$	$y = 3(-1) - 5$
$y = 21 - 5$	$y = -3 - 5$
$y = 16$	$y = -8$

Final answer: (7,16) (-1, -8)

P. 479. # 39.

$$4x^2 - 3y^2 = 4$$
$$y = x - 4$$

Solution:

Since the second equation is in the form of $y = \_\_$ The first step is to substitute what y equals in the second equation back into the first equation:

$$4x^2 - 3(\ \_\_\_\_\ )^2 = 4$$
$$4x^2 - 3(x-4)^2 = 4$$

On the left side, be very careful. This takes two steps, since there is a -3 that is multiplied times the square of the binomial. First you must square the binomial, and second, you must multiply through by the -3. Don't try to do it all in one step! You probably aren't THAT good!! Remember, you square the first, take twice the product, and square the second.

$$4x^2 - 3(x^2 - 8x + 16) = 4$$
$$4x^2 - 3x^2 + 24x - 48 - 4 = 0$$
$$x^2 + 24x - 52 = 0$$

Do you think this trinomial will factor? No, it doesn't, and in real life it probably usually will NOT factor. But in this book is usually DOES!

$$(x+26)(x-2) = 0$$
$$x = -26, \quad x = 2$$

To find y, go back to the simplest equation. If one of the equations is written in the form of a function, like $y = \_\_$, use that equation to find y. Substitute both values of x:

$$y = x - 4 \qquad y = x - 4$$
$$y = (-26) - 4 \qquad y = (2) - 4$$
$$y = -30 \qquad y = -2$$

Final answer: $(-26, -30)$ $(2, -2)$

P. 480. # 45.

$$5x^2 + 3y^2 = 9$$
$$3x^2 + 2y^2 = 6$$

Solution:

It is a good idea to eliminate one of the variables in these equations, using the "elimination method" (also called the "addition method"). To eliminate the y variables, you can multiply both sides of the first equation times 2, and both sides of the second equation by -3, and add the two equations.

$$2 \bullet \left(5x^2 + 3y^2\right) = 2 \bullet (9)$$
$$-3 \bullet \left(x^2 + 2y^2\right) = -3 \bullet (16)$$

$$10x^2 + \ 6\,y^2 = 18$$
$$\underline{-9x^2 - 6y^2 = -18}$$

Add the equations: $x^2 = \ 0$

and solve for x: $x = \ 0$

Substitute back into either of the original equations:

$$5x^2 + 3y^2 = 9$$
$$5 \bullet 0^2 + 3y^2 = 9$$

and solve for y: $3y^2 = 9$

$$y^2 = 3$$
$$y = \pm\sqrt{3}$$

Final answer: $(0, \sqrt{3})$ $(0, -\sqrt{3})$

P. 480. # 47.

$$x^2 - y^2 = 16$$
$$x^2 + 2y^2 = 16$$

Solution:

It is a good idea to eliminate one of the variables in these equations, using the "elimination method" (also called the "addition method"). To do this, you can multiply both sides of the first equation times -1, and add the second equation.

$$-1(x^2 - y^2) = -1 \bullet 16$$
$$x^2 + 2y^2 = 16$$

$$-x^2 + y^2 = -16$$
$$x^2 + 2y^2 = 16$$

Add the equations: $3y^2 = 0$

and solve for y: $y^2 = 0$

$$y = 0$$

Substitute back into either of the original equations:

$$x^2 - y^2 = 16$$
$$x^2 - 0^2 = 16$$

and solve for x: $x^2 = 16$

$$x = \pm 4$$

Final answer: $(4, 0)$ $(-4, 0)$

P. 481. # 50.

$$4x^2 - 3y^2 = 16$$
$$2x^2 + y^2 = 18$$

Solution:

It is a good idea to eliminate one of the variables in these equations, using the "elimination method" (also called the "addition method"). To eliminate the x variables, you can multiply both sides of the second equation times -2, or to eliminate the y variables, you can multiply both sides of the second equation times, and add the two equations. Let's multiply the second equation times -2.

$$4x^2 - 3y^2 = 16$$
$$-2 \bullet \left(2x^2 + y^2\right) = -2 \bullet (18)$$

$$4x^2 - 3y^2 = 16$$
$$\underline{-4x^2 - 2y^2 = -36}$$

Add the equations: $-5y^2 = -20$

and solve for y: $y^2 = 4$

$$y = \pm 2$$

$$y = 2 \qquad\qquad y = -2$$

Substitute each of these values back into either of the original equations. The second one looks simpler to me:

$$2x^2 + y^2 = 18 \qquad\qquad 2x^2 + y^2 = 18$$
$$2x^2 + (2)^2 = 18 \qquad\qquad 2x^2 + (-2)^2 = 18$$
$$2x^2 + 4 = 18 \qquad\qquad 2x^2 + 4 = 18$$
$$2x^2 = 14 \qquad\qquad 2x^2 = 14$$
$$x^2 = 7 \qquad\qquad x^2 = 7$$
$$x = \pm\sqrt{7} \qquad\qquad x = \pm\sqrt{7}$$

For each of the y values, there are TWO solutions for x, which gives you a total of four solutions:

Final answer: $(\sqrt{7}, 2)$ $(-\sqrt{7}, 2)$ and $(\sqrt{7}, -2)$ $(-\sqrt{7}, -2)$

P. 481. # 51.

$$x^2 - 2xy + y^2 = 49$$
$$y = 3x - 5$$

Solution:

Since the second equation is in the form of $y = \_\_$ The first step is to substitute what y equals in the second equation back into the first equation:

$$x^2 - 2x(\quad) + (\quad)^2 = 49$$
$$x^2 - 2x(3x-5) + (3x-5)^2 = 49$$

Remember, when you square the binomial, you square the first, take twice the product, and square the second. This might also be a good time to set the equation equal to zero.

$$x^2 - 6x^2 + 10x + (9x^2 - 30x + 25) - 49 = 0$$

Combine like terms:

$$4x^2 - 20x - 24 = 0$$

Factor out the common factor of -4, and hope that the trinomial that results will factor.

$$4(x^2 - 5x - 6) = 0$$
$$4(x-6)(x+1) = 0$$
$$x = 6 \qquad x = -1$$

To find y, go back to the simplest equation. If one of the equations is written in the form of a function, like $y = \_\_$, use that equation to find y. Substitute both values of x:

$x = 6$	$x = -1$
$y = 3x - 5$	$y = 3x - 5$
$y = 3(6) - 5$	$y = 3(-1) - 5$
$y = 18 - 5$	$y = -3 - 5$
$y = 13$	$y = -8$

Final answer: $(6, 13)$ $(-1, -8)$

P. 482. # 55.

$$x^2 + xy + y^2 = 21$$
$$2x - y = 7$$

Solution:

Neither equation is set up for substitution since neither equation is in the form of $x = \_\_$ or $y = \_\_$. However, it would not be difficult to solve for y in the second equation, by adding y and subtracting 7 from both sides of the equation.

$$2x - 7 = y \quad \text{or} \quad y = 2x - 7$$

Next, substitute this y equals from the second equation back into the first equation:

$$x^2 + xy + y^2 = 21$$
$$x^2 + x(\_\_\_\_) + (\_\_\_\_)^2 = 21$$
$$x^2 + x(2x - 7) + (2x - 7)^2 = 21$$
$$x^2 + x(2x - 7) + (2x - 7)^2 = 21$$

Remember, when you square the binomial, you square the first, take twice the product, and square the second. At the same time, set the equation equal to zero.

$$x^2 + 2x^2 - 7x + (4x^2 - 28x + 49) - 21 = 0$$

Combine like terms:

$$7x^2 - 35x + 28 = 0$$

Factor out the common factor of 7, and hope that the trinomial that results will factor.

$$7(x^2 - 5x + 4) = 0$$
$$7(x - 4)(x - 1) = 0$$
$$x = 4 \qquad x = 1$$

To find y, go back to the simplest equation. If one of the equations is written in the form of a function, like $y = \_\_$, use that equation to find y. Substitute both values of x:

$x = 4$	$x = 1$
$y = 2x - 7$	$y = 2x - 7$
$y = 2(4) - 7$	$y = 2(1) - 7$
$y = 8 - 7$	$y = 2 - 7$
$y = 1$	$y = -5$

Solution : $(4, 1) \quad (1, -5)$

P. 483. # 63.

$$x^2 y = -4$$
$$y = x^2 - 5$$

Solution: Since the second equation is written in the form of $y = \_\_$, it is set up for substitution of this y into the first equation, and solve for x.

$$x^2 y = -4$$
$$x^2(\quad) = -4$$
$$x^2\left(x^2 - 5\right) = -4$$
$$x^4 - 5x^2 + 4 = 0$$

This factors!! (Is anyone surprised??)

$$(x^2 - 4)(x^2 - 1) = 0$$
$$(x - 2)(x + 2)(x - 1)(x + 1) = 0$$
$$x = 2, \quad x = -2, \quad x = 1, \quad x = -1$$

Next, you have to substitute each of these values back into the simplest equation (I recommend the second one, but either will do!):

$y = x^2 - 5$	$y = x^2 - 5$	$y = x^2 - 5$	$y = x^2 - 5$
$y = (2)^2 - 5$	$y = (-2)^2 - 5$	$y = (1)^2 - 5$	$y = (-1)^2 - 5$
$y = 4 - 5$	$y = 4 - 5$	$y = 1 - 5$	$y = 1 - 5$
$y = -1$	$y = -1$	$y = -4$	$y = -4$

Final Answer : $(2, -1)$ $(-2, -1)$ $(1, -4)$ $(-1, -4)$.