Math in Living C O L O R !!

3.06 Polynomial and Fractional Inequalities

by Graphing Calculator Methods

College Algebra: One Step at a Time, **Pages 413 - 424: 29, 3 Extra Problems**

Dr. Robert J. Rapalje, Retired
Central Florida, USA

See Section 3.06, with explanations, examples, and exercises, coming soon!

SUMMARY

I. Find roots and asymptotes graphically.
Set inequality to zero, and graph y1=_____

--AND/OR--

II. Find the roots and asymptotes algebraically.

A. Asymptotes--Set denominators $\neq$ 0.

B. Roots--Change inequality to equation. Solve for x.

III. Examine each interval, whether above/below x axis.

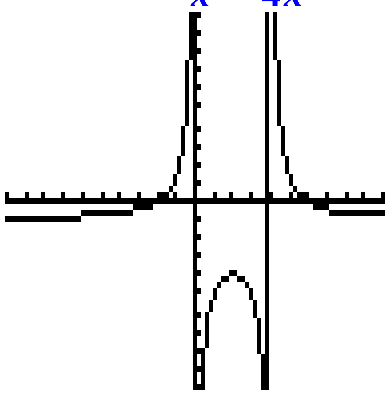
P. 423 #29. Solve for x. (Give the answer in interval notation.)

$$\frac{12}{x^2-4x}>1.$$

Solution: First, you must set the inequality to zero by subtracting 1 from each side of the inequality.

$$\frac{12}{x^2-4x}-1>0$$

Next, draw the graph of $Y1=\frac{12}{x^2-4x}-1$



Notice that the roots (or zeros) of this function are at x=-2 and x=6, and there are asymptotes at x=0 and x=4. These zeros and asymptotes can and will be obtained algebraically. Since the inequality is $Y_1>0$, you will be looking for values of x, where the graph is above the x axis.

Since there are two roots (zeros) and two asymptotes, this gives you four endpoints on the number line, and five intervals to consider for your solution. You must select the intervals that are ABOVE the x-axis.

In the first interval, from –infinity to -2, the graph is below the x-axis.
In the second interval, from -2 to 0 the graph is above the x-axis.
In the third interval, from 0 to 4, the graph is below the x-axis.
In the fourth interval, from 4 to 6, the graph is above the x-axis.
In the fifth interval, from 6 to infinity, the graph is below the x-axis.

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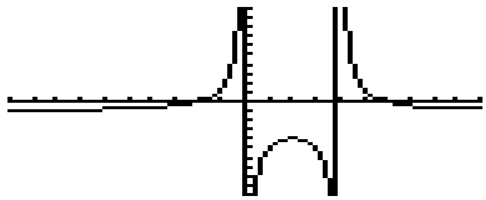
P. 423 #29 continued. $\frac{12}{x^2-4x} > 1$.

Therefore the solution consists of the second and fourth interval, where the graph is above the x-axis (endpoints are NOT included!).



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$(-2,0) \cup (4,\ 6)$

Finding the Roots and Asymptotes Algebraically

A. Asymptotes--Set denominators $\neq$ 0.

Solve $x^2 - 4x \neq 0$

$x(x-4) \neq 0$

$x \neq 0, \quad x \neq 4$

B. Roots--Change inequality to equation and solve for x.

Solve $\frac{12}{x^2-4x} = 1$. Multiply both sides by the LCD = $x^2 - 4x$.

$12 = x^2 - 4x$

$0 = x^2 - 4x - 12$

$0 = (x-6)(x+2)$

$x = 6, \quad x = -2$

Final answer (using the endpoints and the calculator graph from above):

————————(——)———(——)——————

$(-2,0) \cup (4,\ 6)$

Extra Problem #1 (by Vera). **Solve for x:** (Give interval notation.)

$$\frac{5}{4-2y} < \frac{3}{2}.$$

Solution: First, you must set the inequality to zero by subtracting $\frac{3}{2}$ from each side of the inequality and find the endpoints for the inequality.

$$\frac{5}{4-2y} - \frac{3}{2} < 0$$

Next, find the endpoints by finding Asymptotes and Roots.

A. Asymptotes: Find any values of y that make a denominator zero.

Solve

$$4-2y \neq 0$$
$$4 \neq 2y$$
$$y \neq 2$$

B. Roots: Solve the EQUATION:

$$\frac{5}{4-2y} - \frac{3}{2} = 0$$
$$\frac{5}{4-2y} = \frac{3}{2}$$
$$5 \bullet 2 = 3(4-2y)$$
$$10 = 12 - 6y$$
$$6y = 2$$
$$y = \frac{1}{3}$$

So the endpoints are $y = \frac{1}{3}$ and $y \neq 2$.

Next, use the graphing calculator to draw the graph of

$$Y1 = \frac{5}{4-2y} - \frac{3}{2}$$

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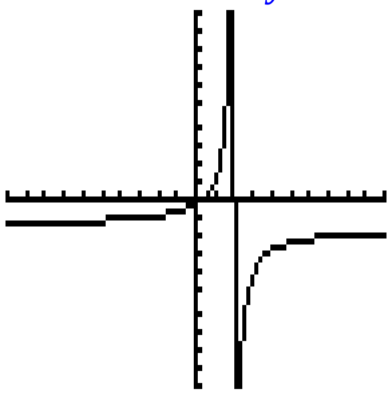
Extra Problem #1 (by Vera) continued.

$$Y1 = \frac{5}{4-2y} - \frac{3}{2}$$



Since the inequality is $Y_1 < 0$, you will be looking for values of x, where the graph is BELOW the x axis. You must select the intervals that are BELOW the x-axis.

In the first interval, from –infinity to $\frac{1}{3}$, the graph is below the x-axis.

In the second interval, from $\frac{1}{3}$ to 2, the graph is above the x-axis.

In the third interval, from 2 to infinity, the graph is below the x-axis.

Therefore the solution consists of the first and third interval, where the graph is below the x-axis (endpoints are NOT included!).

$$\longleftarrow) \text{———} (\longrightarrow$$

Final answer: $(-\infty, \frac{1}{3}) \cup (2, \infty)$

Extra Problem #2 (by Vera). Solve for x. (Give interval notation.)

$$\frac{3}{5y+3} < \frac{13}{10}.$$

Solution: First, you must set the inequality to zero by subtracting $\frac{13}{10}$ from each side of the inequality and find the endpoints for the inequality.

$$\frac{3}{5y+3} - \frac{13}{10} < 0$$

Next, find the endpoints by finding Asymptotes and Roots.

A. Asymptotes: Find any values of y that make a denominator zero.

Solve

$$5y+3 \neq 0$$
$$5y \neq -3$$
$$y \neq -\frac{3}{5}$$

B. Roots: Solve the EQUATION:

$$\frac{3}{5y+3} - \frac{13}{10} = 0$$
$$\frac{3}{5y+3} = \frac{13}{10}$$
$$3 \bullet 10 = 13(5y+3)$$
$$30 = 65y+39$$
$$-9 = 65y$$
$$y = -\frac{9}{65}$$

So the endpoints are $y = -\frac{9}{65}$ and $y \neq -\frac{3}{5}$.

Next, draw the graph of

$$Y1 = \frac{3}{5y+3} - \frac{13}{10}$$

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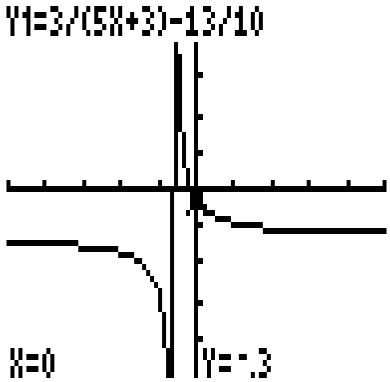
Extra Problem #2 (by Vera) continued.

$$Y1 = \frac{3}{5y+3} - \frac{13}{10}$$



Since the inequality is $Y_1 < 0$, you will be looking for values of x, where the graph is BELOW the x axis. You must select the intervals that are BELOW the x-axis.

In the first interval, from –infinity to $-\frac{3}{5}$, the graph is below the x-axis.

In the second interval, from $-\frac{3}{5}$ to $-\frac{9}{65}$, the graph is above the x-axis.

In the third interval, from $-\frac{9}{65}$ to infinity, the graph is below the x-axis.

Therefore the solution consists of the first and third interval, where the graph is below the x-axis (endpoints are NOT included!).

$\longleftarrow)$———$(\longrightarrow$

Final answer: $(-\infty, -\frac{3}{5}) \cup (-\frac{9}{65}, \infty)$

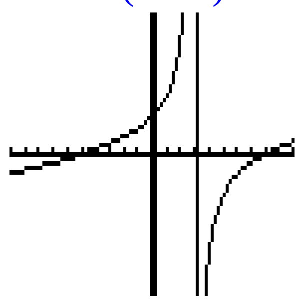
Extra Problem #3 (by Aimee).

Solve for x. Give the answer in interval notation.

$$\frac{(x-8)(x+5)}{(x-3)} \geq 0.$$

Solution: Use the graphing calculator to graph

$$Y1 = \frac{(x-8)(x+5)}{(x-3)}$$



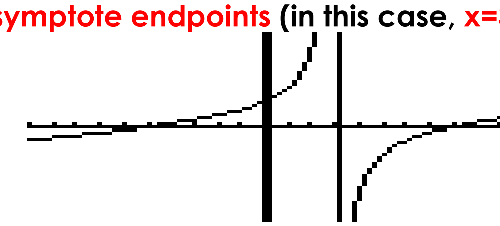
Notice that the roots (or zeros) of this function are at x=8 and x=-5, and there is an asymptote at x=3. Since the inequality is $Y_1 \geq 0$, you will be looking for values of x, where the graph is on or above the x axis.

The graph has roots at x=-5 and 8. The vertical line at x=3 is not really a part of the graph, but it is an asymptote, a line that the graph approaches by never actually touches.

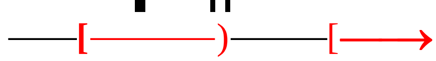
Since there are two roots (zeros) and one asymptote, this gives you three endpoints on the number line, and four intervals to consider for your solution. You must select the intervals that are ON or ABOVE the x-axis.

In the first interval, from –infinity to -5, the graph is below the x-axis.
In the second interval, from -5 to 3 the graph is above the x-axis.
In the third interval, from 3 to 8, the graph is below the x-axis.
In the fourth interval, from 8 to infinity, the graph is above the x-axis.

Therefore the solution consists of the second and fourth interval, where the graph is above the x-axis, including the endpoints at x=-5 and x=8, since these are points that are ON the x-axis. Asymptote endpoints (in this case, x=3) are NEVER included!!



Final answer:



$$[-5,\ 3) \cup [8,\infty)$$