Math in Living C O L O R !!

3.03 Polynomial Functions

College Algebra: One Step at a Time, **Pages 361 - 377: #3, 9, 25, 31, 34, 36, 37, 39, 40**

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See Section 3.03, with explanations, examples, and exercises, coming soon!

P. 365. #3. $y = x^2 - 5x + 6$

Solution: Begin with the y-intercept (where x=0) and ALL the x-intercepts (where y=0, which is where the graph crosses the x-axis).

y-intercept (x=0): $y = 0^2 - 5 \bullet 0 + 6$

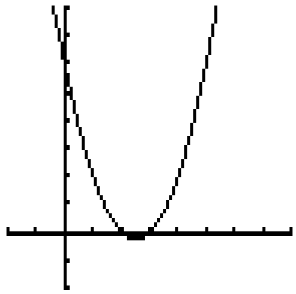
$y = 6$

x-intercept (y=0): $0 = x^2 - 5x + 6$

$0 = (x-2)(x-3)$

$x = 2 \quad x = 3$

Since the degree of the polynomial is 2, the graph opens UP on both sides. Since the zeros are at $x = 2 \quad x = 3$, which are each of multiplicity 1, the graph passes through each of these zeros. The graph should look like this:



P. 367. # 9. $y = x(x-1)(x-2)(x+3)$

Solution: Start by finding the y-intercept (where x=0) and ALL the x-intercepts (where y=0, which is where the graph crosses the x-axis).

y-intercept (x=0): $y = 0(0-1)(0-2)(0+3)$

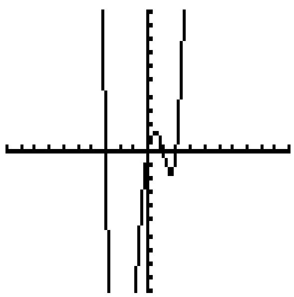
$y = 0$

x-intercept (y=0): $y = x(x-1)(x-2)(x+3)$

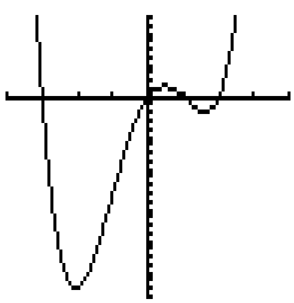
$0 = x(x-1)(x-2)(x+3)$

$x = 0,\ \ x = 1,\ x = 2,\ x = -3$

Since the degree of the polynomial is 4, the graph opens UP on both sides. Since the zeros are at $x = 0,\ \ x = 1,\ x = 2,\ x = -3$, which are each of multiplicity 1, the graph passes through each of these zeros. Of course, you have no way of knowing how far the graph goes up or down between zeros, but the calculator should give you some idea. The graph in a standard window should look like this:



For a more complete picture of this graph, you might want to experiment with the window of the calculator. If you set the x window for the interval [-4, 4] and y window [-25,10], the graph looks like this:



P. 372. #25. $y = -x^2(x-1)^2(x+3)^2(x-4)$

Solution: Start by finding the degree of the polynomial function, which is the highest power of x when it is written in expanded form. The degree of the polynomial is 7, and the leading coefficient is negative, so the graph opens DOWN on the right and UP on the left side of the graph. Also, find the y-intercept (where x=0) and ALL the x-intercepts (where y=0, which is where the graph crosses the x-axis).

y-intercept (x=0): $y = -0^2(0-1)^2(0+3)^2(0-4)$

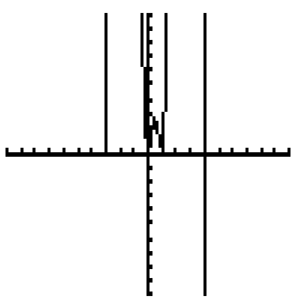
$y = 0$

x-intercept (y=0): $y = -x^2(x-1)^2(x+3)^2(x-4)$

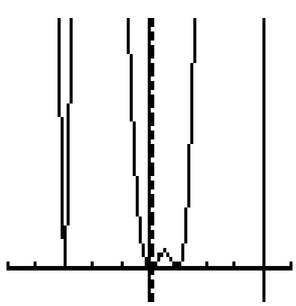
$0 = -x^2(x-1)^2(x+3)^2(x-4)$

$x = 0$ (mult 2), $x = 1$(mult 2), $x = 3$(mult 2), $x = 4$(mult 1)

Since the degree of the polynomial is 4, the graph opens UP on both sides. Since the zeros are at $x = 0,\ x = 1,\ x = 3$, which are each of multiplicity 2, the graph bounces at these roots, but passes through the root at $x = 4$. The graph in a standard window is not helpful at all:



For a more complete picture of this graph, you might want to experiment with the window of the calculator. Any way you look at it, this one will be hard to show on the calculator. If you set the x window for the interval [-5, 5] and y window [-5,40], the graph looks like this:



IMPORTANT NOTE: In Exercises 31 – 34, the critical step in finding all roots (zeros) is to FACTOR by GROUPING—you must group the first two and the last two terms!

P. 374. #31. $y = x^3 - x^2 - 9x + 9$

Solution: Start by finding the y-intercept (where x=0) and ALL the x-intercepts (where y=0, which is where the graph crosses the x-axis).

y-intercept (x=0): $y = 0^3 - 0^2 - 9 \bullet 0 + 9$

$y = 9$

x-intercept (y=0): $y = x^3 - x^2 - 9x + 9$

$0 = x^3 - x^2 - 9x + 9$

The critical step here is to factor the right side, by grouping:

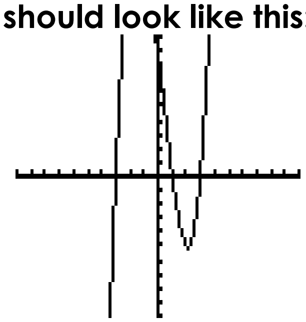
$0 = x^2(x-1) - 9(x-1)$

$0 = (x-1)(x^2-9)$

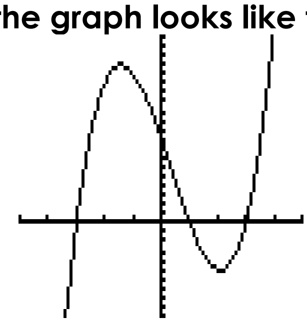
$0 = (x-1)(x-3)(x+3)$

$x = 1$, $x = 3$, $x = -3$ (each of mult 1)

Since the degree of the polynomial is 3, the graph opens UP on the right side and DOWN on the left side. Since the zeros are at $x = 1$, $x = 3$, $x = -3$, which are each of multiplicity 1, the graph passes through each of these zeros. Of course, you have no way of knowing how far the graph goes up or down between zeros, but the calculator should give you some idea. The graph in a standard window should look like this:



For a more complete picture of this graph, you might want to experiment with the window of the calculator. If you set the x window for the interval [-5, 5] and y window [-10,20], the graph looks like this:



P. 375. #34. $y = x^3 + 2x^2 - 4x - 8$

Solution: Start by finding the y-intercept (where x=0) and ALL the x-intercepts (where y=0, which is where the graph crosses the x-axis).

y-intercept (x=0): $y = 0^3 + 2 \bullet 0^2 - 4 \bullet 0 - 8$

$y = -8$

x-intercept (y=0): $y = x^3 + 2x^2 - 4x - 8$

$0 = x^3 + 2x^2 - 4x - 8$

The critical step here is to factor the right side, by grouping:

$$0 = x^2(x+2) - 4(x+2)$$

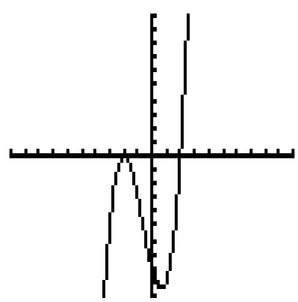
$$0 = (x+2)(x^2 - 4)$$

$$0 = (x+2)(x-2)(x+2)$$

$$0 = (x+2)^2(x-2)$$

$x = -2$ (mult 2), $x = 2$ (mult 1)

Since the degree of the polynomial is 3, the graph opens UP on the right side and DOWN on the left side. Since the zero at $x = -2$ is of even multiplicity, the graph bounces at $x = -2$, and at $x = 2$ where there is odd multiplicity, the graph passes through the zero. Of course, you have no way of knowing how far the graph goes up or down between zeros, but the calculator should give you some idea. The graph in a standard window should look like this:



P. 376. #36. $y = x^4 - 18x^2 + 81$

Solution: Start by finding the y-intercept (where x=0) and ALL the x-intercepts (where y=0, which is where the graph crosses the x-axis).

y-intercept (x=0): $y = 0^4 - 18 \bullet 0^2 + 81$

$$y = 81$$

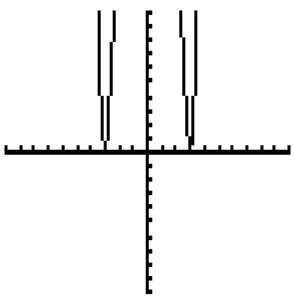
x-intercept (y=0): $y = x^4 - 18x^2 + 81$

$$0 = (x^2 - 9)(x^2 - 9)$$

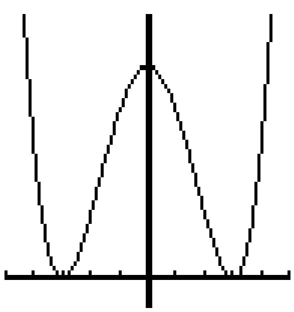
$$0 = (x-3)(x+3)(x-3)(x+3)$$

$$x = 3 \text{ (mult 2)}, \quad x = -3 \text{ (mult 2)}$$

Since the degree of the polynomial is 4, the graph opens UP on both sides. Since the zeros are at $x = 3,\ x = -3$, which are each of multiplicity 2, the graph bounces off each of these zeros. Of course, you have no way of knowing how far the graph goes up or down between zeros, but the calculator should give you some idea. The graph in a standard window should look like this:



For a more complete picture of this graph, you might want to experiment with the window of the calculator. If you set the x window for the interval [-5, 5] and y window [-10,100], the graph looks like this:



P. 376. #37. $y = x^6 - 13x^4 + 36x^2$

Solution: Start by finding the y-intercept (where x=0) and ALL the x-intercepts (where y=0, which is where the graph crosses the x-axis).

y-intercept (x=0): $y = 0^6 - 13 \bullet 0^4 + 36 \bullet 0^2$

$y = 0$

x-intercept (y=0): $y = x^6 - 13x^4 + 36x^2$

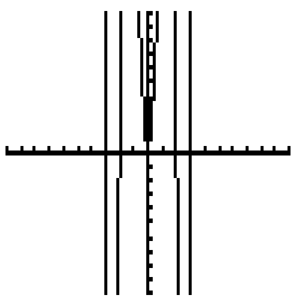
$y = x^2(x^4 - 13x^2 + 36)$

$0 = x^2(x^2 - 9)(x^2 - 4)$

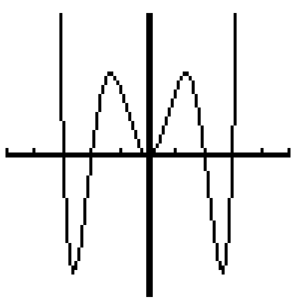
$0 = x^2(x-3)(x+3)(x-2)(x+2)$

$x = 0$ (mult 2), $x = 3$ (mult 1), $x = -3$ (mult 1), $x = 2$ (mult 1), $x = -2$ (mult 1)

Since the degree of the polynomial is 6, the graph opens UP on both sides. Since the zeros are at $x = 0$ (mult 2) the graph bounces at $x = 0$. At $x = 3,\ x = -3,\ x = 2,\ x = -2$, which are each of multiplicity 1, the graph passes through these zeros. Of course, you have no way of knowing how far the graph goes up or down between zeros, but the calculator should give you some idea. The graph in a standard window should look like this:



For a more complete picture of this graph, you might want to experiment with the window of the calculator. If you set the x window for the interval [-5, 5] and y window [-50,50], the graph looks like this:



P. 376. #39. $y = x^5 + 2x^4 - 4x^3 - 8x^2$

Solution: Start by finding the y-intercept (where x=0) and ALL the x-intercepts (where y=0, which is where the graph crosses the x-axis).

y-intercept (x=0): $y = 0^5 + 2 \bullet 0^4 - 4 \bullet 0^3 - 8 \bullet 0^2$

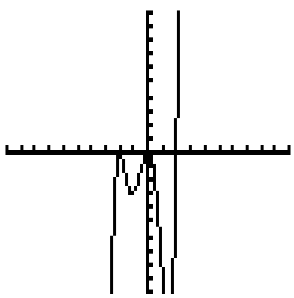
$y = 0$

x-intercept (y=0): $y = x^2[x^3 + 2x^2 - 4x - 8]$

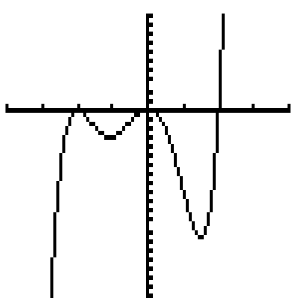
$$0 = x^2[x^2(x+2) - 4(x+2)]$$
$$0 = x^2[(x+2)(x^2 - 4)]$$
$$0 = x^2[(x+2)(x-2)(x+2)]$$
$$0 = x^2(x-2)(x+2)^2$$

$x = 0$ (mult 2), $x = 2$ (mult 1), $x = -2$ (mult 2)

Since the degree of the polynomial is 5, the graph opens UP on the right and down on the left side. Since the zeros are at $x = 0$ (mult 2) and -2 (mult 2) the graph bounces at $x = 0$ and -2. At $x = 2$, the graph passes through the zero since it is of odd multiplicity. The graph in a standard window should look like this:



For a more complete picture of this graph, you might want to experiment with the window of the calculator. If you set the x window for the interval [-4, 4] and y window [-20,10], the graph looks like this:



P. 376. #40. $y = x^7 - 8x^5 + 16x^3$

Solution: Start by finding the y-intercept (where x=0) and ALL the x-intercepts (where y=0, which is where the graph crosses the x-axis).

y-intercept (x=0): $y = 0^7 - 8 \bullet 0^5 + 16 \bullet 0^2$

$y = 0$

x-intercept (y=0): $y = x^7 - 8x^5 + 16x^3$

$0 = x^3(x^4 - 8x^2 + 16)$

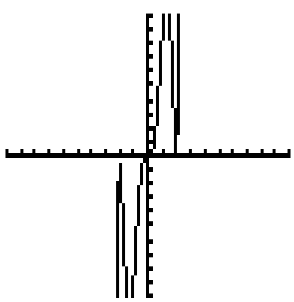
$0 = x^3(x^2 - 4)(x^2 - 4)$

$0 = x^3(x - 2)(x + 2)(x - 2)(x + 2)$

$0 = x^3(x - 2)^2(x + 2)^2$

$x = 0$ (mult 3), $x = 2$ (mult 2), $x = -2$ (mult 2)

Since the degree of the polynomial is 7, the graph opens UP on the right and down on the left side. Since the zeros are at $x = 0$ (mult 3) the graph passes through this zero, and it bounces $x = 2$ and -2. The graph in a standard window should look like this:



For a more complete picture of this graph, you might want to experiment with the window of the calculator. If you set the x window for the interval [-4, 4] and y window [-20,20], the graph looks like this: