Math in Living C O L O R !!

4.01 Definition of Logarithms

College Algebra: One Step at a Time. **Page 493-501: #18, 60, 67-69.**

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See Section 4.01, with explanations, examples, and exercises, coming soon!

P. 497 # 18. $\log_3 \sqrt[4]{3} =$

Solution: There are two ways to solve this problem depending upon how familiar you have become with logarithms. In the first method, set the logarithm equal to x, and translate this from logarithmic notation to exponential notation:

$$\log_3 \sqrt[4]{3} = x \quad \text{means} \quad 3^x = \sqrt[4]{3}$$

Next, translate from radical form to exponential form.

$$3^x = \sqrt[4]{3}$$

$$3^x = 3^{\frac{1}{4}}$$

Now, since the base numbers are the same, the exponents must be equal, so

$$x = \frac{1}{4}$$

As a second method, as you become a bit more familiar with logarithms, there is a short-cut. Just convert from radical to exponential notation:

$$\log_3 \sqrt[4]{3}$$

$$\log_3 3^{\frac{1}{4}}$$

Now, since the base of the logarithm is the same as the base of the exponent, and since a logarithm is really "the exponent", then the answer is "the exponent." The "log base 3" is the inverse of the operation of "raising 3 to the power," so the answer is the "power", which is $\frac{1}{4}$

P. 519 # 60. $\log_b 9 = -\frac{2}{3}$

Solution: Translate this from logarithmic notation to exponential notation

$$\log_b 9 = -\frac{2}{3} \quad \text{means} \quad b^{-\frac{2}{3}} = 9$$

Notice that this equation has a base number of b which is raised to a power. It would be nice to end up with b raised to the 1 power. In order to do this, you could raise both sides to the $-\frac{3}{2}$ power, which when you **multiply exponents**, will give you what you need! It looks like this:

$$\left(b^{-\frac{2}{3}}\right)^{-\frac{3}{2}} = (9)^{-\frac{3}{2}}$$

$$b = (9)^{-\frac{3}{2}}$$

Now, do you remember how to simplify a negative fractional exponent? Remember that the denominator gives you the index of the radical, and the numerator gives you the exponent. In this case, take the square root of 9, and raise to the -3 power.

$$b = (9)^{-\frac{3}{2}}$$

$$b = \left(\sqrt{9}\right)^{-3}$$

$$b = 3^{-3} = \frac{1}{27}$$

You can also calculate fractional exponents with the calculator, but remember to place parentheses around the exponent!

P. 501 # 67. $\log_{10} 0.1 = x$

 # 68. $\log_{10} 0.01 = x$

 # 69. $\log_{10} 0.001 = x$

Solution: Since this is a log base 10 problem, you can solve it with a calculator. Just use the LOG button. The answers above are: -1, -2, and -3.

Just for fun (math is fun, isn't it??), do the following problems with log base 10, both with and without the calculator, and see what you get for answers:

$\log_{10} 10 =$

$\log_{10} 100 =$

$\log_{10} 1000 =$

$\log_{10} 10,000 =$

$\log_{10} 1,000,000 =$

$\log_{10} 0.01 =$

$\log_{10} 0.001 =$

$\log_{10} 0.000001 =$

$\log_{10} 1 =$

$\log_{10} 0 =$

The answer should be as follows: 1, 2, 3, 4, 6, -2, -3, -6, 0.

And, of course, $\log_{10} 0$ is Undefined!!