Math in Living C O L O R !!

4.03 Properties of Logarithms

College Algebra: One Step at a Time.

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See Section 4.03, with explanations, examples, and exercises, coming soon!

Preliminary thoughts for problems 1 – 18

Remember the laws of logarithms:

I. $\log_b MN = \log_b M + \log_b N$

II. $\log_b \dfrac{M}{N} = \log_b M - \log_b N$

III. $\log_b M^N = N\log_b M$

P. 511 # 5. $\log_b \dfrac{x^4}{\sqrt{y}}$

Solution:

Remember that if you have a quotient, you **SUBTRACT** logarithms.

$$\log_b \frac{x^4}{\sqrt{y}} = \log_b x^4 - \log_b \sqrt{y}$$

Next, write $\sqrt{y}$ as an exponent $y^{\frac{1}{2}}$.

$$\log_b \frac{x^4}{\sqrt{y}} = \log_b x^4 - \log_b y^{\frac{1}{2}}$$

The last step is to bring down the exponents:

$\log_b x^4 = 4\log_b x$ and $\log_b y^{\frac{1}{2}} = \dfrac{1}{2}\log_b y$

This is the final answer: $4\log_b x - \dfrac{1}{2}\log_b y$.

P. 511 # 6. $\log_b \frac{\sqrt[3]{x}}{y^2}$

Solution:

Remember that if you have a quotient, you **SUBTRACT** logarithms.

$$\log_b \frac{\sqrt[3]{x}}{y^2} = \log_b \sqrt[3]{x} - \log_b y^2$$

Next, write $\sqrt[3]{x}$ as an exponent $x^{\frac{1}{3}}$.

$$\log_b \frac{\sqrt[3]{x}}{y^2} = \log_b x^{\frac{1}{3}} - \log_b y^2$$

The last step is to bring down the exponents:

$\log_b x^{\frac{1}{3}} = \frac{1}{3}\log_b x$ and $\log_b y^2 = 2\log_b y$

This is the final answer: $\frac{1}{3}\log_b x - 2\log_b y$.

P. 513 # 13. $\log_b \frac{\sqrt{a}}{x^3\sqrt{z}}$

Solution:

Remember that if you have factors in a product, you must **ADD** logarithms, and if you have a quotient, you **SUBTRACT** logarithms. It turns out that for ANY factor that is in the numerator you must **ADD** logarithms, and for any factor in the denominator you must **SUBTRACT** logarithms!

$$\log_b \frac{\sqrt{a}}{x^3\sqrt{z}} = \log_b \sqrt{a} - \log_b x^3 - \log_b \sqrt{z}$$

$$= \log_b a^{\frac{1}{2}} - \log_b x^3 - \log_b z^{\frac{1}{2}}$$

In each logarithm, bring down the exponents:

$$= \frac{1}{2}\log_b a - 3\log_b x - \frac{1}{2}\log_b z$$

P. 513 # 17. $\log_b \frac{1}{x\sqrt[3]{y}}$

Solution:

Remember that, as in the previous problems, if you have factors in a product, you must ADD logarithms, and if you have a quotient, you SUBTRACT logarithms. It turns out that for ANY factors that is in the numerator you must ADD logarithms, and for any factors in the denominator you must SUBTRACT logarithms!

$$\log_b \frac{1}{x\sqrt[3]{y}} = \log_b 1 - \log_b x - \log_b \sqrt[3]{y}$$

$$= \log_b 1 - \log_b x - \log_b y^{\frac{1}{3}}$$

In the first logarithm, $\log_b 1$ for any base b is 0, since $b^0 = 1$.

In the third logarithm, just bring down the exponent.

$$= 0 - \log_b x - \frac{1}{3}\log_b y$$

$$= -\log_b x - \frac{1}{3}\log_b y$$

P. 513 # 18. $\log_b \frac{1}{x^2\sqrt[3]{y^2}}$

Solution:

Remember that, as in the previous problems, if you have factors in a product, you must ADD logarithms, and if you have a quotient, you SUBTRACT logarithms. It turns out that for ANY factors that is in the numerator you must ADD logarithms, and for any factors in the denominator you must SUBTRACT logarithms!

$$\log_b \frac{1}{x^2\sqrt[3]{y^2}} = \log_b 1 - \log_b x^2 - \log_b \sqrt[3]{y^2}$$

$$= \log_b 1 - \log_b x^2 - \log_b y^{\frac{2}{3}}$$

In the first logarithm, $\log_b 1$ for any base b is 0, since $b^0 = 1$.

In the second and third logarithm, just bring down the exponent.

$$= 0 - 2\log_b x - \frac{2}{3}\log_b y$$

$$= -2\log_b x - \frac{2}{3}\log_b y$$

Preliminary thoughts for problems 19 – 32

Beginning with problem #19, the laws of logarithms are applied in reverse.

Whereas in the first few exercises, we wrote the laws this way:

$$\log_b MN = \log_b M + \log_b N$$

$$\log_b \frac{M}{N} = \log_b M - \log_b N$$

$$\log_b M^N = N \log_b M$$

Now, we are going to use them in this way is:

$$\log_b M + \log_b N = \log_b MN$$

$$\log_b M - \log_b N = \log_b \frac{M}{N}$$

$$N \log_b M = \log_b M^N$$

Before when you had the logarithm of a product, you would write this as the sum of two logarithms. Now, when you have the sum of two logarithms, you can write this as the product of a single logarithm. Before, when you had a quotient, you could write this as the difference of two logarithms. Now, when you have a difference of two logarithms, you can write this as the quotient of a single logarithm.

P. 514 # 23. $2\log_b x - 4\log_b y + \log_b z$

Solution:

Beginning with problem #19, the laws of logarithms were applied in reverse. In #1-19, the exponents were brought down to become coefficients. Now the coefficients will be moved up to become exponents.

$$\log_b x^2 - \log_b y^4 + \log_b z$$

Now, logarithms with positive coefficients become factors in the numerator of the fraction, and negative coefficients will become denominator factors. It may help to re-write the expression with the positive terms first, although this step is not really necessary if you understand it.

$$\log_b x^2 + \log_b z - \log_b y^4$$

$$\log_b \frac{x^2 z}{y^4}$$

P. 515 # 25. $3\log_b x - \frac{1}{2}\log_b y - 2\log_b z$

Solution:

Beginning with problem #19, the laws of logarithms were applied in reverse. In #1-19, the exponents were brought down to become coefficients. Now the coefficients will be moved up to become exponents.

$$\log_b x^3 - \log_b y^{\frac{1}{2}} - \log_b z^2$$
$$\log_b x^3 - \log_b \sqrt{y} - \log_b z^2$$

Now, logarithms with positive coefficients become factors in the numerator of the fraction, and negative coefficients will become denominator factors.

$$\log_b \frac{x^3}{\sqrt{y} \bullet z^2} \quad \text{or} \quad \log_b \frac{x^3}{z^2\sqrt{y}}$$

P. 515 # 28 $\log_b x - 2(\log_b y + 3\log_b z)$

Solution:

First, by distributive property, you can write this:

$$\log_b x - 2\log_b y - 6\log_b z$$

Now, the coefficients become exponents:

$$\log_b x - \log_b y^2 - \log_b z^6$$

Now, express as a single logarithm. Remember that terms with negative coefficients become factors in the denominator.

$$\log_b \frac{x}{y^2 z^6}$$

P. 515 # 29 $2\log_b x - 3(\log_b z - 2\log_b y)$

Solution:

First, by distributive property, you can write this:

$$2\log_b x - 3\log_b z + 6\log_b y$$

Now, the coefficients become exponents:

$$\log_b x^2 - \log_b z^3 + \log_b y^6$$

It may be helpful to re-write this with the positive terms first:

$$\log_b x^2 + \log_b y^6 - \log_b z^3$$

Now, express as a single logarithm. Remember that terms with negative coefficients become factors in the denominator.

$$\log_b \frac{x^2 y^6}{z^3}$$

P. 516 # 31 $\frac{1}{2}\log_b(5x+3) - 2\,[\,\log_b(x+2) + \log_b(3x+1)]$

Solution:

First, by distributive property, you can write this:

$$\frac{1}{2}\log_b(5x+3) - 2\,\log_b(x+2) - 2\log_b(3x+1)$$

Now, the coefficients become exponents:

$$\log_b(5x+3)^{\frac{1}{2}} - \log_b(x+2)^2 - \log_b(3x+1)^2$$

$$\log_b\sqrt{5x+3} - \log_b(x+2)^2 - \log_b(3x+1)^2$$

Now, express as a single logarithm. Remember that terms with negative coefficients become factors in the denominator.

$$\log_b \frac{\sqrt{5x+3}}{(x+2)^2(3x+1)^2}$$

P. 517 # 53. $\ln\ e^0$

Solution:

This is a "quickie"! This really means $\log_e e^0$, and since log base e is the inverse of e raised to the power, these are inverse functions of one another! Therefore, the answer is the power, which is 0.

Note: Since this is an "ln" problem, you can also use the calculator to calculate the answer using the LN button.

P. 517 # 54. $\ln\ 1$

Solution:

This is an even quicker "quickie"! Doesn't e^0 equal 1? Then #54 is exactly the same problem as #53. You can also use the calculator with the LN button, and the answer should also be 0.

P. 518 # 56. $\log_b \dfrac{a^3b^2}{c^4}$

Solution:

Remember that if you have factors in a product, you must ADD logarithms, and if you have a quotient, you SUBTRACT logarithms. It turns out that for ANY factor that is in the numerator you must ADD logarithms, and for any factor in the denominator you must SUBTRACT logarithms!

$$\log_b \frac{a^3b^2}{c^4} = \log_b a^3 + \log_b b^2 - \log_b c^4$$

In the second logarithm, you have the same base for the logarithm as the base number of the exponent, $\log_b b^2$, so the answer is the power, which is 2.

In the first and third logarithm, you have different base numbers, so the only thing you can do is bring down the exponent.

$$\log_b a^3 = 3\log_b a \text{ and } \log_b c^4 = 4\log_b c$$

This is the final answer: $3\log_b a + 2 - 4\log_b c$.

P. 518 # 57. $\log_5 \frac{2\sqrt{3}}{25}$

Solution:

Remember as before that if you have factors in a product, you must ADD logarithms, and if you have a quotient, you SUBTRACT logarithms. It turns out that for ANY factor that is in the numerator you must ADD logarithms, and for any factor in the denominator you must SUBTRACT logarithms!

$$\log_5 \frac{2\sqrt{3}}{25} = \log_5 2 + \log_5 \sqrt{3} - \log_5 25$$

$$= \log_5 2 + \log_5 3^{\frac{1}{2}} - \log_5 5^2$$

In the first logarithm, there is nothing you can do, at least for now. At the end of the next lesson, you will be able to find the value, but for now, just leave it as it is.

In the second logarithm, you have different base numbers, so the only thing you can do is bring down the exponent: $\log_5 3^{\frac{1}{2}} = \frac{1}{2}\log_5 3$.

In the third logarithm, you have the same base for the logarithm as the base number of the exponent, $\log_5 5^2$, so the answer is the power, which is 2.

This is the final answer: $\log_5 2 + \frac{1}{2}\log_5 3 - 2$.

P. 518 # 58. $\log_5 \frac{5\sqrt{6}}{7}$

Solution:

Remember that if you have factors in a product, you must ADD logarithms, and if you have a quotient, you SUBTRACT logarithms. It turns out that for ANY factor that is in the numerator you must ADD logarithms, and for any factor in the denominator you must SUBTRACT logarithms!

$$\log_5 \frac{5\sqrt{6}}{7} = \log_5 5 + \log_5 \sqrt{6} - \log_5 7$$

$$= \log_5 5^1 + \log_5 6^{\frac{1}{2}} - \log_5 7$$

In the first logarithm, you have the same base for the logarithm as the base number of the exponent, $\log_5 5^1$, so the answer is the power, which is 1.

In the second logarithm, you have different base numbers, so the only thing you can do is bring down the exponent. $\log_5 6^{\frac{1}{2}} = \frac{1}{2}\log_5 6$.

In the third logarithm, there is nothing you can do, at least for now. At the end of the next lesson, you will be able to find the value, but for now, just leave it as it is.

This is the final answer: $1 + \frac{1}{2}\log_5 6 - \log_5 7$.

P. 519 # 61. $\log_3 \frac{9\sqrt{3}}{2}$

Solution:

As in # 58, remember that for factors in the numerator you must ADD logarithms, and for factors in the denominator you must SUBTRACT logarithms!

$$\log_3 \frac{9\sqrt{3}}{2} = \log_3 9 + \log_3 \sqrt{3} - \log_3 2$$

$$= \log_3 3^2 + \log_3 3^{\frac{1}{2}} - \log_3 2$$

Notice that in the first two logarithms, you have the same base number as the base of the exponent, so as before, the answer is the power. The answers are 2 and $\frac{1}{2}$ respectively.

In the third logarithm, the base number is NOT the same, so you can't simplify it—at least not until the next section!!

$$= 2 + \frac{1}{2} - \log_3 2$$

The final answer is: $= 2.5 - \log_3 2$ or $\frac{5}{2} - \log_3 2$.

P. 519 # 63. $\log_7 7\sqrt{7}$

Solution:

Before beginning this problem, remember that logarithms and exponential functions with the same base numbers are inverses one of another. This is to say that $\log_b b^x = x$, where the base number is b>0 and not equal to 1. As examples,

$$\log_{10} 10^x = x, \quad \log_7 7^x = x, \quad \log_{14} 14^y = y, \text{ or } \ln e^{3x} = 3x.$$

Now, since this is log base 7, it would be nice if you could express the rest of the problem with a base number of 7. Remember that 7 is actually 7^1, $\sqrt{7}$ is actually $7^{\frac{1}{2}}$, and when you multiply with the same base number, by the law of logarithms, you add exponents! So,

$$\log_7 7\sqrt{7}$$

$\log_7 7^1 7^{\frac{1}{2}}$ (Remember that $1 + \frac{1}{2} = \frac{3}{2}$?)

$\log_7 7^{\frac{3}{2}} = \frac{3}{2}$, since the base numbers of the logarithm and the exponential are both 7.

P. 519 # 64. $\log_5 125\sqrt{5}$

Solution:

Continue with the same way of looking at logarithms in this next example. Notice that the base of the logarithm is 5, and that the numbers 125 and $\sqrt{5}$ can both be expressed as powers of 5.

$$\log_5 125\sqrt{5}$$

$\log_5 5^3 5^{\frac{1}{2}}$ (Remember that $3+\frac{1}{2}=\frac{7}{2}$?)

$\log_5 5^{\frac{7}{2}}=\frac{7}{2}$, since the base numbers of the logarithm and the exponential are both 5,

The final answer is the exponent, which is $\frac{7}{2}$.

P. 519 # 66. $\log_5 \frac{7\sqrt[3]{5}}{5}$

Solution:

As in previous problems, remember that for factors in the numerator you must ADD logarithms, and for factors in the denominator you must SUBTRACT logarithms!

$$\log_5 \frac{7\sqrt[3]{5}}{5} = \log_5 7 + \log_5 \sqrt[3]{5} - \log_5 5^1$$

$$= \log_5 7 + \log_5 5^{\frac{1}{3}} - \log_5 5^1$$

Notice that in the first logarithms, there is nothing to do, since you do NOT have the same base number as the base of the exponent. However, in the last two terms, you DO have the same base number as the base of the exponent, so the answer is the power. These exponents are $\frac{1}{3}$ and 1 respectively.

The final answer is: $\log_5 7+\frac{1}{3}-1$ or $\log_5 7-\frac{2}{3}$.