Math in Living C O L O R !!

4.04 Part II
Solving Logarithmic Equations

College Algebra: One Step at a Time.
Page 524-530: #23, Extra Prob, 27, 29, 33, 34, 35, 36, Extra Prob

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See Section 4.04, with explanations, examples, and exercises, coming soon!

See Math in Living C O L O R, Section 4.04 Part I, coming soon!

P. 525 # 23. $\log_{10}(x) + \log_{10}(x-15) = 2$

Solution: Since all the logarithms are already on the same side of the equation, you should begin by combining the logarithm terms on the left side into a single logarithm, using the first law of logarithms. Remember, when you have a sum of logarithms, you get the log of a product.

$$\log_{10} x(x-15) = 2$$

Now convert this logarithmic form to exponential form, using the basic definition of logarithms.

$\log_{10} x(x-15) = 2$ means that $10^2 = x(x-15)$

$$100 = x^2 - 15x$$

Solve as a quadratic equation. $0 = x^2 - 15x - 100$

This factors into: $0 = (x-20)(x+5)$

$$x = 20 \text{ or } x = -5$$

Of these answers, $x = -5$ must be rejected, since it results in a log of a negative in the original problem. The final answer is $x = 20$.

Extra Problem from algebra.com.

$$\log_4(x) \;+\; \log_4(x+60)=4$$

Solution: Since all the logarithms are already on the same side of the equation, you should begin by combining the logarithm terms on the left side into a single logarithm, using the first law of logarithms. Remember, when you have a sum of logarithms, you get the log of a product.

$$\log_4 x(x+60)=4$$

Now according to the definition of logarithms.

$$\log_4 x(x+60)=4 \quad \text{means that} \quad 4^4 = x(x+60)$$

$$256 = x^2+60x$$

Solve as a quadratic equation: $0 = x^2+60x-256$

This factors into: $0=(x-4)(x+64)$

$$x=4 \;\text{ or }\; x=-64$$

Of these answers, $x=-64$ must be rejected, since it results in a log of a negative in the original problem. The final answer is $x=4$.

Also it checks! You can substitute $x=4$ into the original equation:

$$\log_4(x) \;+\; \log_4(x+60)=4$$

$$\log_4(4) \;+\; \log_4(64)=4$$

$$1 \;+\; 3 \;=\; 4$$

P. 526 # 27. $\log_5(x+4) = \log_5(x)+1$

Solution: You should begin by getting all the logarithm terms on one side, and any non-log terms on the other side. In this case, subtract $\log_5(x)$ from each side of the equation.

$$\log_5(x+4) - \log_5(x) = 1$$

By the second law of logarithms, the difference of two logarithms can be expressed as a quotient:

$$\log_5\left(\frac{x+4}{x}\right) = 1$$

Now, by the definition of logarithms.

$$\log_5\left(\frac{x+4}{x}\right) = 1 \quad \text{means that} \quad 5^1 = \frac{x+4}{x}$$

$$\frac{5}{1} = \frac{x+4}{x}$$

Either cross multiply, or multiply both sides of the equation by the LCD

$$5x = 1(x+4)$$

Solve as a linear equation:

$$5x = x+4$$

$$4x = 4$$

Final answer:

$$x = 1$$

Check:

This answer is acceptable, since when substituted into the original equation, it always results in the logs of positive numbers. It can also be checked by substitution into the original equation.

$$\log_5(x+4) = \log_5(x)+1$$

$$\log_5(1+4) = \log_5(1)+1$$

$$\log_5(5) = \log_5(1)+1$$

$$1 = 0 + 1$$ It checks!

P. 527 # 29. $\log_3(x-2) = \log_3(x+2)-2$

Solution: You should begin by getting all the logarithm terms on one side, and any non-log terms on the other side. In this case, subtract $\log_3(x+2)$ from each side of the equation. This leaves:

$$\log_3(x-2) - \log_3(x+2) = -2$$

By the second law of logarithms, the difference of two logarithms can be expressed as a quotient:

$$\log_3\frac{(x-2)}{(x+2)} = -2$$

Now convert this logarithmic form to exponential form, by the definition of logarithms.

$$\log_3\frac{(x-2)}{(x+2)} = -2 \quad \text{means that} \quad 3^{-2} = \frac{(x-2)}{(x+2)}$$

$$\frac{1}{9} = \frac{(x-2)}{(x+2)}$$

Either cross multiply, or multiply both sides of the equation by the LCD:

$$1(x+2) = 9(x-2)$$

Solve as a linear equation:

$$x+2 = 9x-18$$

$$-8x = -20$$

$$\frac{-8x}{-8} = \frac{-20}{-8}$$

$$x = \frac{-20}{-8}$$

$$x = \frac{5}{2} \text{ or } 2.5$$

This answer is acceptable, since when substituted into the original equation, it always results in the logs of positive numbers.

P. 528 # 33. $\log_b(x-6) - \log_b(x) = \log_b(x-4)$

Solution: In previous exercises, you were told to get all the log terms on one side and the non-log terms on the other side. However, in this case, there are no non-log terms, so there is no need to do this step. Use the second law of logarithms to make the log of a quotient on the left.

$$\log_b \frac{(x-6)}{(x)} = \log_b(x-4)$$

This is a good place to use my own "This Equals That" Theorem that I previously mentioned in #3? My "This Equals That" applies to logarithms as well as exponents:

This Equals That Theorem

$$\textit{If} \quad \log_b \text{this} = \log_b \text{that}, \quad \textit{then} \quad \text{this} = \text{that}!$$

$$\textit{If} \quad \log_b \frac{(x-6)}{(x)} = \log_b(x-4), \quad \textit{then} \quad \frac{(x-6)}{(x)} = (x-4) \quad .$$

Now, you may either cross multiply, or multiply both sides of the equation by the LCD—the result is the same:

$$\frac{x-6}{x} = \frac{x-4}{1}$$

$$x-6 = x(x-4)$$

Solve as a quadratic equation.

$$x-6 = x^2-4x$$

Set the equation equal to zero:

$$\begin{array}{rcl} x-6 & = & x^2-4x \\ -x+6 & & -x+6 \\ \hline \end{array}$$

$$0 = x^2-5x+6$$

Factor the trinomial:

$$0 = (x-3)(x-2)$$

Set each factor equal to zero:

$$x = 3 \text{ or } x = 2$$

Now, you must check these answers by substituting them back into the original equation to make sure that the answers do not give the log of a negative. As it turns out, BOTH of these answers result in logs of negatives, so neither answer is acceptable. Both must be rejected! Therefore the final answer is NO SOLUTION!

P. 528 # 34. $\log_b(x^2+3) - \log_b(x) = \log_b(x+7) - \log_b 2$

Solution: As in the previous example, there are no non-log terms, so there is no need to get all the logs on one side of the equation. It is a good idea in this case to use the second law of logarithms to write a single logarithm on the left and right side of the equation.

$$\log_b \frac{(x^2+3)}{(x)} = \log_b \frac{(x+7)}{2}$$

Next, remember my "This Equals That" Theorem from the previous exercise? Well, here it is again:

This Equals That Theorem

If $\log_b \text{this} = \log_b \text{that}$, *then* this = that!

If $\log_b \frac{(x^2+3)}{(x)} = \log_b \frac{(x+7)}{2}$, *then* $\frac{(x^2+3)}{(x)} = \frac{(x+7)}{2}$

Now, you may either cross multiply, or multiply both sides of the equation by the LCD—it gives you the same result:

$$\frac{(x^2+3)}{(x)} = \frac{(x+7)}{2}$$

$$2(x^2+3) = x(x+7)$$

Solve as a quadratic equation. $2x^2+6 = x^2+7x$

Set the equation equal to zero. $x^2-7x+6 = 0$

Factor the trinomial: $(x-6)(x-1) = 0$

Set each factor equal to zero: $x = 6$ or $x = 1$ **Final answer!**

Both answers are acceptable, since neither answer results in a log of a negative.

P. 528 # 35. $\log_b(x-2) + \log_b(x+2) = \log_b(3) + \log_b x$

Solution: As in the previous examples, there are no non-log terms, so there is no need to get all the logs on one side of the equation. It is a good idea in this case to use the first law of logarithms to get a single make the log of a product on the left and also on the right side of the equation.

$$\log_b(x-2)(x+2) = \log_b(3 \bullet x)$$

Next, remember the "This Equals That" Theorem from the previous exercises? Well, here it is again:

This Equals That Theorem

If $\log_b \text{this} = \log_b \text{that}$, *then* this = that!

If $\log_b(x-2)(x+2) = \log_b(3 \bullet x)$, *then* $(x-2)(x+2) = 3 \bullet x$

Solve as a quadratic equation $\qquad (x-2)(x+2) = 3 \bullet x$

$$x^2 - 4 = 3x$$

Set equal to zero. $\qquad x^2 - 3x - 4 = 0$

Factor the trinomial: $\qquad (x-4)(x+1) = 0$

Set each factor equal to zero: $\qquad x = 4 \text{ or } x = -1$

The first answer $x = 4$, is acceptable, but you must reject the $x = -1$ since it results in a log of a negative.

P. 528 # 36. $\log_b(x+4) + \log_b(x+2) = \log_b(4) + \log_b(x+8)$

Solution: As in the previous examples, there are no non-log terms, so there is no need to get all the logs on one side of the equation. It is a good idea in this case to use the first law of logarithms to make the log of a product on the left and also on the right side of the equation.

$$\log_b(x+4)(x+2) = \log_b 4(x+8)$$

Next, remember the This Equals That Theorem from the previous exercises? Well, here it is again:

This Equals That Theorem

$$\textit{If} \quad \log_b \text{this} = \log_b \text{that}, \quad \textit{then} \quad \text{this} = \text{that!}$$

If $\log_b(x+4)(x+2) = \log_b 4(x+8)$, then $(x+4)(x+2) = 4(x+8)$

Solve as a quadratic equation $(x+4)(x+2) = 4(x+8)$

$$x^2+6x+8 = 4x+32$$

Set equal to zero. $x^2+2x-24 = 0$

Factor the trinomial: $(x+6)(x-4) = 0$

Set each factor equal to zero: $x=-6$ or $x=4$

The first answer $x=-6$ must rejected since it results in a log of a negative (actually it results in two negatives, as it most always does!). However, second answer $x=4$ is acceptable. This is the final answer.

Extra Problem from Keith.

Find the exact value, and also calculate the answer to the nearest millionth.

$$\ln(8-5x) = 3$$

Solution: **This really means** $\log_e(8-5x) = 3$ **which means** $e^3 = 8-5x$

Now solve for x: $e^3 = 8-5x$

$$5x = 8-e^3$$

$$x = \frac{8-e^3}{5} \qquad \textbf{Exact value.}$$

$$x \approx -2.417107 \qquad \textbf{Approximate value to nearest millionth.}$$