**SHOW ALL WORK ON THIS TEST OR ON SEPARATE PAPER.  Circle answers.**
**TURN IN ALL WORKSHEETS.  CALCULATORS ARE PERMITTED ON THIS TEST.**

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$
$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

**In 1 - 6, factor <u>completely</u>:**

**1.**  $3x^2 - 3x - 18$

**2.**  $81x^4 - 16$

**3.**  $(3x + y)^2 + 7(3x + y) - 18$

**4.**  $x^3 + 5x^2 - 9x - 45$

**5.**  $5x^2 - 22x + 21$

**6.**  $x^3 y^6 - 8x^3$

**In 7 - 8, solve for $x$:**

**7.**  $x^2 + 2x = 8$

**8.**  $x^3 - 4x^2 = -4x$

**In 9 - 12,  perform the indicated operations (add, subtract, multiply, or divide.)**
**REDUCE ALL FRACTIONS COMPLETELY!**

**9.**  $\dfrac{x^3 - 27}{x^2 - 9}$

**10.**  $\dfrac{x^2 - 8xy + 16y^2}{x^2 - 3xy - 10y^2} \cdot \dfrac{x^2 - 4y^2}{x^2 - 5xy + 4y^2}$

11. $\dfrac{7}{5xy^2} + \dfrac{8}{45x^4y^3}$

12. $\dfrac{5}{x^2 - 10x + 25} - \dfrac{3}{x^2 - 5x}$

13. If 5 pounds of dog food cost $3.29, how much should it cost for a 12 pound bag?

14. y varies directly as the square of z and inversely as the cube of x.  If y = 20 when z = 2 and x = 3, find y when z = 3 and x = 2.

In 15 - 18, solve for $x$:

15. $C = \dfrac{5}{9}(x - 32)$

16. $\dfrac{ax + b}{x} = c$

17. $\dfrac{4}{x} = \dfrac{x + 2}{2}$

18. $\dfrac{x}{x - 1} + \dfrac{2}{x - 5} = \dfrac{-4}{(x - 5)(x - 1)}$

**In 19 - 20, divide:**

**19.** $\dfrac{40x^3 + 8x^2 - 12}{8x^2}$

**20.  Use synthetic division:** $\dfrac{x^3 - 2x^2 + 10x - 4}{x - 2}$

**In 21 - 24, simplify the complex fractions (any method you prefer) :**

**21.** $\dfrac{1 - \dfrac{5}{x} + \dfrac{6}{x^2}}{1 - \dfrac{3}{x} + \dfrac{2}{x^2}}$

**22.** $\dfrac{1 - \dfrac{2}{x+2}}{1 + \dfrac{2}{x-2}}$

**23.** $(x^{-2} - y^{-2})^{-1}$

**24.** $\dfrac{1 + (2x)^{-1}}{2x + (2x)^{-2}}$

25.  Under what conditions is Method II an advantage (a good shortcut) over Method I?

1. $3x^2-3x-18$
$=3(x^2-x-6)$
$=3(x-3)(x+2)$

2. $81x^4-16$
$=(9x^2-4)(9x^2+4)$
$=(3x-2)(3x+2)(9x^2+4)$

3. $(3x+y)^2+7(3x+y)-18$
$=[(3x+y)+9][(3x+y)-2]$
$=(3x+y+9)(3x+y-2)$

4. $x^3+5x^2-9x-45$
$=x^2(x+5)-9(x+5)$
$=(x+5)(x^2-9)$
$=(x+5)(x-3)(x+3)$

5. $5x^2-22x+21$
$(5x-7)(x-3)$

6. $x^3y^6-8x^3$
$=x^3(y^6-8)$
$=x^3(y^2-2)(y^4+2y^2+4)$

7. $x^2+2x=8$
$x^2+2x-8=0$
$(x+4)(x-2)=0$
$x=-4 \quad x=2$

8. $x^3-4x^2=-4x$
$x^3-4x^2+4x=0$
$x(x^2-4x+4)=0$
$x(x-2)(x-2)=0$
$x=0 \quad x=2$

9. $\dfrac{x^3-27}{x^2-9}$
$\dfrac{(x-3)(x^2+3x+9)}{(x-3)(x+3)}$
$\dfrac{x^2+3x+9}{x+3}$

10. $\dfrac{x^2-8xy+16y^2}{x^2-3xy-10y^2} \cdot \dfrac{x^2-4y^2}{x^2-5xy+4y^2}$
$\dfrac{(x-4y)(x-4y)}{(x-5y)(x+2y)} \cdot \dfrac{(x-2y)(x+2y)}{(x-4y)(x-y)}$
$\dfrac{(x-4y)(x-2y)}{(x-5y)(x-y)}$

11. $\dfrac{7}{5xy^2}+\dfrac{8}{45x^4y^3}$
$\dfrac{7}{5xy^2}\cdot\dfrac{9x^3y}{9x^3y}+\dfrac{8}{45x^4y^3}$
$\dfrac{63x^3y+8}{45x^4y^3}$

12. $\dfrac{5}{(x-5)}\dfrac{x}{x} - \dfrac{3}{x(x-5)}\dfrac{(x-5)}{(x-5)}$
$\dfrac{5x-3x+15}{x(x-5)^2}$
$\dfrac{2x+15}{x(x-5)^2}$

13. $\dfrac{5}{3.29}=\dfrac{12}{x}$
$5x=12(3.29)$
$x=\dfrac{12(3.29)}{5}=7.90$

14. $y=\dfrac{kz^2}{x^3}$ $\quad y=\dfrac{135\cdot2^2}{x^3}$
$20=\dfrac{kz^2}{3^3}$ $\quad =\dfrac{135\cdot3^2}{2^3}$
$4k=20\cdot27 \quad =151,875$
$k=135 \quad$ or $\dfrac{1215}{8}$

15. $C=\dfrac{5}{9}(x-32)$
$\dfrac{9}{5}C=\dfrac{9}{5}\dfrac{5}{9}(x-32)$
$\dfrac{9}{5}C=x-32$
$\phantom{\dfrac{9}{5}C}+32 \quad +32$
$\dfrac{9}{5}C+32=x$

16. $\dfrac{ax+b}{x}=C$
$ax+b=cx$
$\underline{-cx \quad -cx}$
$ax-cx=-b$
$x(a-c)=-b$
$x=\dfrac{-b}{a-c}$ or $\dfrac{b}{c-a}$

17. $\dfrac{4}{x}=\dfrac{x+2}{2}$
$x^2+2x=8$
$x^2+2x-8=0$
$(x+4)(x-2)=0$
$x=-4 \quad x=2$

18. $\dfrac{x}{x-1}+\dfrac{2}{x-5}=\dfrac{-4}{(x-5)(x-1)}$
$x(x-5)+2(x-1)=-4$
$x^2-5x+2x-2=-4$
$x^2-3x+2=0$
$(x-2)(x-1)=0$
$x=2 \quad x=1 \quad x\neq5$ or $1$
Reject.

19. $\dfrac{40x^3+8x^2-12}{8x^2}$
$=\dfrac{40x^3}{8x^2}+\dfrac{8x^2}{8x^2}-\dfrac{12}{8x^2}$
$=5x+1-\dfrac{3}{2x^2}$

20. $2|\begin{array}{rrrr} 1 & -2 & 10 & -4 \\ & 2 & 0 & 20 \\ \hline 1 & 0 & 10 & 16 \end{array}$
$x^2+10+\dfrac{16}{x-2}$

21. $\dfrac{x^2\left(1-\dfrac{5}{x}+\dfrac{6}{x^2}\right)}{x^2\left(1-\dfrac{3}{x}+\dfrac{2}{x^2}\right)}$
$\dfrac{x^2-5x+6}{x^2-3x+2}$
$\dfrac{(x-2)(x-3)}{(x-2)(x-1)}=\dfrac{x-3}{x-1}$

22. $\left(1-\dfrac{2}{x+2}\right)\div\left(1+\dfrac{2}{x-2}\right)$
$=\dfrac{(x+2)-2}{x+2}\div\dfrac{(x-2)+2}{x-2}$
$=\dfrac{x}{x+2}\cdot\dfrac{x-2}{x}$
$=\dfrac{x-2}{x+2}$

23. $(x^{-2}-y^{-2})^{-1}$
$=\left(\dfrac{1}{x^2}-\dfrac{1}{y^2}\right)^{-1}$
$=\left(\dfrac{y^2-x^2}{x^2y^2}\right)^{-1}$
$=\dfrac{x^2y^2}{y^2-x^2}$

24. $\dfrac{1+\dfrac{1}{2x}}{2x+\dfrac{1}{4x^2}}$
$=\left(1+\dfrac{1}{2x}\right)\div\left(2x+\dfrac{1}{4x^2}\right)$
$=\dfrac{2x+1}{2x}\div\dfrac{8x^3+1}{4x^2}$
$=\dfrac{2x+1}{2x}\cdot\dfrac{4x^2}{(2x+1)(4x^2-2x+1)}$
$=\dfrac{2x}{4x^2-2x+1}$