TRIGONOMETRY EXAM 3A

Show all work as necessary. Calculators, Trig Sheet, Unit Circle are allowed. Formulas on back cover of text are allowed.
Problems 1–4, 7 each
5–24, 4 each. One problem to be "free."

Prove the trig identities, as in class, from one side to the other:

1. $\tan x + \cot x = \sec x \csc x$

2. $\frac{1}{1-\sin x} + \frac{1}{1+\sin x} = 2\sec^2 x$

3. $\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2 = 1 + \sin x$

4. $\cos 3x = 4\cos^3 x - 3\cos x$

In 5–12, answer in exact form:

5. $\cos(\text{Arccos } x)$ 6. $\text{Cos}^{-1}\left(\sin\frac{3\pi}{2}\right)$ 7. $\sin(2\,\text{Sin}^{-1} 1)$

8. $\tan(2\,\text{Tan}^{-1} 1)$ 9. $\text{Tan}^{-1}\left(\sin\frac{5\pi}{3}\right)$ 10. $\text{Cos}^{-1}\left(\sin\frac{5\pi}{3}\right)$

11. $\cot\left(\text{Sin}^{-1}\frac{2}{3}\right)$ 12. $\sin\left(\text{Cos}^{-1}\left(\sin\frac{\pi}{6}\right)\right)$

In 13–15, answer in radians correct to four decimal places:

13. $\text{Sin}^{-1}\left(\frac{4+\sqrt{5}}{17}\right)$ 14. $\text{Sec}^{-1}\left(\frac{1+\sqrt{5}}{3}\right)$ 15. $\sin\left(\frac{1}{2}\text{Sec}^{-1} 2.576\right)$

16. Find in exact form: $\sin\left(2\,\text{Sin}^{-1}\frac{1}{4}\right)$

In 17–19, let $\tan\theta = -\frac{5}{12}$, where $90° < \theta < 180°$. Answer in exact form:

17. $\cos\theta$ 18. ~~$\tan 2\theta$~~ 19. $\sin\frac{\theta}{2}$

In 20–22, let $\sin\theta = .3642$, where $90° < \theta < 180°$. Find values to 4 decimal places.

20. $\cos\theta$ 21. $\cos 2\theta$ 22. $\cot\frac{\theta}{4}$.

23. If $\sin\theta = -3/5$, $180° < \theta < 270°$, find $\cos\frac{\theta}{2}$.

24. $\cos\left(\text{Arccos}\frac{2}{5} + \text{Arccos}\frac{3}{5}\right) =$

TRIG EXAM 3A solutions:

1. $\tan x + \cot x = \sec x \csc x$

Proof:

$$LHS = \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{\sin^2 x + \cos^2 x}{\cos x \sin x} = \frac{1}{\cos x \sin x} = \sec x \csc x = RHS$$

2. $\frac{1}{1-\sin x} + \frac{1}{1+\sin x} = 2\sec^2 x$

Proof:

$$LHS = \frac{1 + \cancel{\sin x} + 1 - \cancel{\sin x}}{(1-\sin x)(1+\sin x)} = \frac{2}{1-\sin^2 x} = \frac{2}{\cos^2 x} = 2\sec^2 x = RHS.$$

3. $\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2 = 1 + \sin x$

Proof:

$$LHS = \sin^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2} + \cos^2\frac{x}{2}$$
$$= \underbrace{\sin^2\frac{x}{2} + \cos^2\frac{x}{2}}_{} + 2\sin\frac{x}{2}\cos\frac{x}{2}$$
$$= 1 + \sin 2\left(\frac{x}{2}\right)$$
$$= 1 + \sin x = RHS.$$

4. $\cos 3x = \cos(2x + x)$

$$= \cos 2x \cos x - \sin 2x \sin x$$
$$= (2\cos^2 x - 1)\cos x - 2\sin x \cos x \sin x$$
$$= 2\cos^3 x - \cos x - 2\cos x(1-\cos^2 x)$$
$$= 2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x$$
$$= 4\cos^3 x - 3\cos x = RHS.$$

5. $\cos(\text{Arccos}\, x) = x$

6. $\text{Cos}^{-1}\left(\sin\frac{3\pi}{2}\right) = \text{Cos}^{-1}(-1) = \pi$

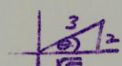
7. $\sin\left(2\,\text{Sin}^{-1} 1\right) = \sin 2\left(\frac{\pi}{2}\right) = \sin \pi = 0$

8. $\tan\left(2\,\text{Tan}^{-1} 1\right) = \tan\left(2\cdot\frac{\pi}{4}\right) = \tan\frac{\pi}{2} =$ undef.

9. $\text{Tan}^{-1}\left(\sin\frac{5\pi}{3}\right) = \text{Tan}^{-1}\left(-\frac{\sqrt{3}}{2}\right) = ??$ (Sorry!)

10. $\text{Cos}^{-1}\left(\sin\frac{5\pi}{3}\right) = \text{Cos}^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$

11. $\cot\left(\text{Sin}^{-1}\frac{2}{3}\right) = \cot\theta = \frac{\sqrt{5}}{2}$



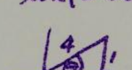
12. $\sin\left(\text{Cos}^{-1}\left(\sin\frac{\pi}{6}\right)\right) = \sin\left(\text{Cos}^{-1}\frac{1}{2}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

13. $\frac{4+\sqrt{5}}{17}$, Inv. Sin → .3756

14. $\frac{1+\sqrt{5}}{3}$, $\frac{1}{x}$, Inv. Cos → .3843

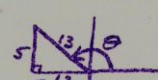
15 2.576, $\frac{1}{x}$, Inv. Cos, ÷2, =, sin → .5531

16. $\sin\left(2\sin^{-1}\frac{1}{4}\right) = \sin 2\theta$



$$= 2\sin\theta\cos\theta = 2\cdot\frac{1}{4}\cdot\frac{\sqrt{15}}{4} = \frac{\sqrt{15}}{8}$$

17. $\tan\theta = -\frac{5}{12}$, $90° < \theta < 180°$ $\quad 45° < \frac{\theta}{2} < 90°$



$\cos\theta = -\frac{12}{13}$

18. $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\cdot\left(-\frac{5}{12}\right)}{1-\frac{25}{144}} = -\frac{5}{6} \div \frac{119}{144} = -\frac{5}{6}\cdot\frac{144}{119} = -\frac{120}{119}$

19. $\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$

$\left(\frac{\theta}{2} \text{ in QI}\right) = +\sqrt{\frac{1-(-12/13)}{2}} = \sqrt{\frac{25/13}{2}} = \frac{5}{\sqrt{26}}$

20. $\sin\theta = .3642$ QII.

$\theta = 21.3584$ (Calculator QI)

$\theta = 158.6416$ QII

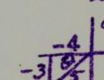
$\cos\theta = -.9313$

21. $\cos 2\theta = 1 - 2\sin^2\theta$ or $2\cos^2\theta - 1$ $= .7347$

22. $\cot\frac{\theta}{4} = \cot\frac{158.6416}{4} = 1.2062$

23. $\sin\theta = -3/5$ $\quad 180 < \theta < 270°$ QIII

$\cos\theta = -4/5$ $\quad 90° < \frac{\theta}{2} < 135°$ QII.



$\cos\frac{\theta}{2} = -\sqrt{\frac{1+\cos\theta}{2}} = -\sqrt{\frac{1-4/5}{2}} = \frac{-1}{\sqrt{10}}$

24. $\cos\left(\text{Arccos}\,^{a_1}\!\!2/5 + \text{Arccos}\,^{a_2}\!\!3/5\right)$

$= \cos a_1 \cos a_2 - \sin a_1 \sin a_2$

$= \frac{2}{5}\cdot\frac{3}{5} - \frac{\sqrt{21}}{5}\,\frac{4}{5}$

$= \frac{6-4\sqrt{21}}{25}$