

TRIGONOMETRY EXAM 3C

NAME \_\_\_\_\_

(NO CALCULATORS)

PROVE ANY 3 OF THE FOLLOWING WORKING FROM ONE SIDE TO OTHER.

1.  $\frac{1}{1+\sin X} + \frac{1}{1-\sin X} = 2 \sec^2 X$

2.  $\tan X + \cot X = \sec X \csc X$

3.  $\cot X = \frac{1 + \cos 2X}{\sin 2X}$

4.  $\sin 3X = 3 \sin X - 4 \sin^3 X$

[Let  $\sin 3X = \sin(2X+X)$ . Use sum formula]

Separate paper please!

You may do "omitted" problem for extra credit. 5 points

SOLVE FOR ALL  $0 \leq X \leq 2\pi$ . Use Table III if necessary.

5.  $2 \cos^3 X - \cos X = 0$

6.  $2 \sin^4 X + 3 \cos^2 X = 2$

7.  $\tan^2 2X + 3 = 4 \tan 2X$

Separate paper please!

SHORT ANSWER

8.  $\cos^2 2X - \sin^2 2X =$  \_\_\_\_\_

12.  $\frac{16 \tan 16X}{1 - \tan^2 16X} =$  \_\_\_\_\_

9.  $\cos^2 2X + \sin^2 2X =$  \_\_\_\_\_

13.  $16 \sin 16X \cos 16X =$  \_\_\_\_\_

10.  $1 - \cos^2 2X =$  \_\_\_\_\_

14.  $\sin 2X \cos 4X + \sin 4X \cos 2X =$  \_\_\_\_\_

11.  $4 - 8 \cos^2 2X =$  \_\_\_\_\_

Use TABLE III (Be careful of signs!)

15 a)  $\sin 52^\circ 10' =$  \_\_\_\_\_

e)  $\csc 1110^\circ =$  \_\_\_\_\_

b)  $\sin 100^\circ =$  \_\_\_\_\_

f)  $\sec(-34^\circ 30') =$  \_\_\_\_\_

c)  $\cos 100^\circ =$  \_\_\_\_\_

g)  $\arctan(.5930) =$  \_\_\_\_\_

d)  $\cot 650^\circ 40' =$  \_\_\_\_\_

h)  $\arccos(-.9936) =$  \_\_\_\_\_

16. If the terminal side of  $\alpha$  contains  $(12, -5)$ , then find:

a)  $\cos \alpha =$  \_\_\_\_\_, b)  $\sin \alpha =$  \_\_\_\_\_, c)  $\tan \alpha =$  \_\_\_\_\_, d)  $\cot \alpha =$  \_\_\_\_\_, e)  $\sec \alpha =$  \_\_\_\_\_

f)  $\csc \alpha =$  \_\_\_\_\_

17. If  $\sin X = \frac{12}{13}$  and  $\tan X < 0$ , find  $\cos 2X$  and  $\sin 2X$ .

18. Find the linear velocity of the tip of the minute hand of a clock if the length of the hand is 1 ft.

TRIG EXAM 3C Solutions

Not guaranteed! Rush job!

$$1. \frac{1}{1+\sin x} + \frac{1}{1-\sin x} = \frac{1-\sin x + 1+\sin x}{(1+\sin x)(1-\sin x)}$$

$$= \frac{2}{1-\sin^2 x}$$

$$= \frac{2}{\cos^2 x} = 2 \sec^2 x$$

$$2. \tan x + \cot x = \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$$

$$= \frac{\sin^2 x + \cos^2 x}{\cos x \sin x}$$

$$= \frac{1}{\cos x \sin x}$$

$$= \frac{1}{\cos x} \cdot \frac{1}{\sin x} = \sec x \csc x$$

$$3. \cot x = \frac{1+\cos 2x}{\sin 2x}$$

$$\frac{1+\cos 2x}{\sin 2x} = \frac{1+(2\cos^2 x - 1)}{2\sin x \cos x}$$

$$= \frac{2\cos^2 x}{2\sin x \cos x}$$

$$= \frac{\cos x}{\sin x} = \cot x$$

$$4. \sin 3x = \sin(2x+x)$$

$$= \sin 2x \cos x + \cos 2x \sin x$$

$$= 2\sin x \cos x \cos x + (1-2\sin^2 x)\sin x$$

$$= 2\sin x(1-\sin^2 x) + \sin x - 2\sin^3 x$$

$$= 2\sin x - 2\sin^3 x + \sin x - 2\sin^3 x$$

$$= 3\sin x - 4\sin^3 x$$

$$5. 2\cos^2 x - \cos x = 0$$

$$\cos x(2\cos x - 1) = 0$$

$$\cos x = 0 \quad \cos x = \frac{1}{2}$$

$$\cos x = \pm \frac{1}{2}$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2} \quad x = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4}$$

$$6. 2\sin^4 x + 3\cos^2 x - 2 = 0$$

$$2\sin^4 x + 3(1-\sin^2 x) - 2 = 0$$

$$2\sin^4 x + 3 - 3\sin^2 x - 2 = 0$$

$$2\sin^4 x - 3\sin^2 x + 1 = 0$$

$$(2\sin^2 x - 1)(\sin^2 x - 1) = 0$$

$$\sin x = \pm \frac{1}{\sqrt{2}} \quad \sin x = \pm 1$$

$$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{2}$$

$$7. \tan^2 2x + 3 = 4 \tan 2x$$

$$\tan^2 2x - 4 \tan 2x + 3 = 0$$

$$(\tan 2x - 3)(\tan 2x - 1) = 0$$

$$\tan 2x = 3 \quad \tan 2x = 1$$

$$2x = 71^\circ 30' \quad 2x = \frac{\pi}{4}$$

$$2x = 71^\circ 30', 251^\circ 30', 431^\circ 30', 611^\circ 30'$$

$$x = 35^\circ 45', 125^\circ 45', 215^\circ 45', 305^\circ 45'$$

$$2x = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}$$

$$x = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$$

$$8. \cos^2 2x - \sin^2 2x = \cos 4x$$

$$9. \cos^2 2x + \sin^2 2x = 1$$

$$10. 1 - \cos^2 2x = \sin^2 2x$$

$$11. 4 - 8\cos^2 2x = 4(1 - 2\cos^2 2x)$$


$$= 4\cos 4x$$

$$12. \frac{16 \tan 16x}{1 - \tan^2 16x} = 8 \tan 32x$$

$$13. 16 \sin 16x \cos 16x = 8 \sin 32x$$

$$14. \sin 2x \cos 4x + \sin 4x \cos 2x = \sin(2x+4x) = \sin 6x$$

$$16. (12, -5) \text{ QIII. hypot.} = 13.$$



$$\cos \alpha = \frac{12}{13} \quad \sec \alpha = \frac{13}{12}$$

$$\sin \alpha = -\frac{5}{13} \quad \csc \alpha = -\frac{13}{5}$$

$$\tan \alpha = -\frac{5}{12} \quad \cot \alpha = -\frac{12}{5}$$

$$17. \sin x = \frac{12}{13} \quad \tan x < 0, \text{ so } \cos x < 0.$$

$$\cos x = -\sqrt{1 - \frac{144}{169}} = -\frac{5}{13}$$

$$\cos 2x = 1 - 2\sin^2 x = 1 - 2\left(\frac{144}{169}\right) = \frac{169 - 288}{169} = -\frac{119}{169}$$

$$\sin 2x = 2\sin x \cos x = 2 \cdot \frac{12}{13} \cdot -\frac{5}{13} = -\frac{120}{169}$$

$$18. V = r\omega$$

$$r = 1 \text{ ft.}$$

$$\omega = 1 \text{ rev per hour.} = \frac{2\pi \text{ Rad}}{\text{hr.}}$$

$$V = 1 \text{ ft.} \cdot \frac{2\pi \text{ Rad}}{\text{hr.}} = \frac{2\pi \text{ ft.}}{\text{hr.}}$$

$$15a) \sin 52^\circ 10' = .7898$$

$$b) \sin 100^\circ = \sin 80^\circ = \frac{16}{17}$$

$$c) \cos 100^\circ = -\cos 80^\circ = -\frac{1736}{10000}$$

$$d) \cot 69^\circ 40' = \cot 290^\circ 40'$$

$$= -\cot 69^\circ 40' = -\frac{3772}{10000}$$

$$e) \csc 110^\circ = \csc 70^\circ = \frac{2}{\cos 70^\circ} = \frac{2}{.342} = 5.85$$

$$15f) \sec(39^\circ 30') \text{ QIV} = \text{pos.}$$

$$= 1.213$$

$$g) \arctan(-.5730) = 30^\circ 40'$$

$$h) \arccos(-.9936) \text{ QII.}$$

$$\hat{x} = 6^\circ 30' \quad \hat{x} = 173^\circ 30'$$