MS 133 EXAM 3B (TURN IN ALL SCRATCH) NAME _____

PROVE 3 of the 4 identities working from one side to the other.

(8 ea)

1. $\sec x - \cos x = \tan x \sin x$
2. $\tan^2 x - \sin^2 x = \tan^2 x \sin^2 x$
3. $\cot x = \dfrac{1 + \cos 2x}{\sin 2x}$
4. $\sin x = \tan\frac{x}{2}\cos x + \tan\frac{x}{2}$

} SHOW WORK ON SEPARATE PAPER.

Solve the following equation for $0 \le x < 2\pi$. Use Table III if necessary.

(8 ea)

5. $2\sin^2 x + \sin x = 1$
6. $\tan 2x - 3\sec^2 2x = -5$
7. $2\sin^2 x - 3\cos x = 0$

} SHOW WORK ON SEPARATE PAPER.

Short answer.

(2 ea)

8. $2\sin 3x \cos 3x =$ _____.
9. $2 - 4\sin^2 x =$ _____.
10. $\dfrac{4\tan 2x}{1 - \tan^2 2x} =$ _____.
11. $\cos 3x \cos 2x + \sin 3x \sin 2x =$ _____
12. $4 - 4\sin^2 4x =$ _____.
13. $8\sin^2 8x + 8\cos^2 8x =$ _____

(8) 14. If $\cos\theta = -3/5$, find a. $\sin\frac{\theta}{2} =$ _____.
b. $\cos\frac{\theta}{2} =$ _____.
θ in QIII.

(6) 15. If the terminal side of θ contains $(5, -12)$, then find:
$\sin\theta =$ ____, $\cos\theta =$ ____, $\tan\theta =$ ____, $\cot\theta =$ ____, $\sec\theta =$ ____, $\csc\theta =$ ____

16. Use table III.

(3 ea)

a) $\sin 2°20' =$ _____.
b) $\sec 82°50' =$ _____.
c) $\tan 110°50' =$ _____.
d) $\cot 1030° =$ _____.
e) $\csc 540° =$ _____
f) $\cos 223°22' =$ _____.

(8) 17. A wheel with diameter 10 inches is turning at 3 rev/sec. Find the linear velocity of a point on the outside of the wheel.

MS 133 EXAM 3B Solutions

1. $\sec x - \cos x = \frac{1}{\cos x} - \frac{\cos^2 x}{\cos x}$
$= \frac{1-\cos^2 x}{\cos x}$
$= \frac{\sin^2 x}{\cos x}$
$= \tan x \sin x$

2. $\tan^2 x - \sin^2 x = \frac{\sin^2 x}{\cos^2 x} - \frac{\sin^2 x \cos^2 x}{\cos^2 x}$
$= \frac{\sin^2 x - \sin^2 x \cos^2 x}{\cos^2 x}$
$= \frac{\sin^2 x (1-\cos^2 x)}{\cos^2 x}$
$= \tan^2 x \cdot \sin^2 x$

3. $\frac{1+\cos 2x}{\sin 2x} = \frac{1+2\cos^2 x - 1}{2\sin x \cos x}$
$= \frac{2\cos^2 x}{2\sin x \cos x}$
$= \frac{\cos x}{\sin x}$
$= \cot x$

4. $\tan\frac{x}{2}\cos x + \tan\frac{x}{2} = \left(\tan\frac{x}{2}\right)(\cos x + 1)$
$= \frac{\sin x}{\cos x + 1} \cdot (\cos x + 1)$
$= \sin x$

5. $2\sin^2 x + \sin x - 1 = 0$
$(2\sin x - 1)(\sin x + 1) = 0$
$2\sin x - 1 = 0$ $\quad \sin x = -1$
$\sin x = \frac{1}{2}$ $\quad \boxed{x = \frac{3\pi}{2}}$
$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}}$

6. $\tan 2x - 3\sec^2 2x = -5$
$\tan 2x - 3(1 + \tan^2 2x) = -5$
$\tan 2x - 3 - 3\tan^2 2x = -5$
$3\tan^2 2x - \tan 2x - 2 = 0$
$(3\tan 2x + 2)(\tan 2x - 1) = 0$
$\tan 2x = -\frac{2}{3}$ $\quad \tan 2x = 1$
$\tan 2x = -.6667$
$2x = 33^\circ 40'$ QII, IV.
$2x = 146^\circ 20', 326^\circ 20'$
$+360 \quad +360$
$506^\circ 20' \quad 686^\circ 20'$
$\boxed{x = 73^\circ 10', 163^\circ 10', 253^\circ 10', 343^\circ 10'}$

$2x = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}$
$\boxed{x = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}}$

7. $2\sin^2 x - 3\cos x = 0$
$2 - 2\cos^2 x - 3\cos x = 0$
$2\cos^2 x + 3\cos x - 2 = 0$
$(2\cos x - 1)(\cos x + 2) = 0$
$\cos x = \frac{1}{2}$ $\quad \cos x = -2$
$\boxed{x = \frac{\pi}{3}, \frac{5\pi}{3}}$ $\quad \phi$

8. $2\sin 3x \cos 3x = \boxed{\sin 6x}$

9. $2 - 4\sin^2 x = 2(1 - 2\sin^2 x)$
$= \boxed{2\cos 2x}$

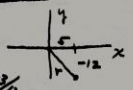
10. $\frac{4\tan 2x}{1-\tan^2 2x} = 2\left(\frac{2\tan 2x}{1-\tan^2 2x}\right)$
$= \boxed{2\tan 4x}$

11. $\cos 3x \cos 2x + \sin 3x \sin 2x$
$= \cos(3x - 2x) = \boxed{\cos x}$

12. $4 - 4\sin^2 4x = 4(1 - \sin^2 4x)$
$= \boxed{4\cos^2 4x}$

13. $8\sin^2 8x + 8\cos^2 8x$
$= 8(\sin^2 8x + \cos^2 8x) = \boxed{8}$

14. $\cos\theta = -\frac{3}{5}$
θ in QIII, means $\frac{\theta}{2}$ in QII.
$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$
$= +\sqrt{\frac{1+\frac{3}{5}}{2}}$, QII.
$= \sqrt{\frac{\frac{8}{5}}{2}} =$
$= \sqrt{\frac{4}{5}} = \boxed{\frac{2}{\sqrt{5}}}$
$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$
$= -\sqrt{\frac{1-\frac{3}{5}}{2}}$, QII
$= -\sqrt{\frac{\frac{2}{5}}{2}} = \boxed{-\frac{1}{\sqrt{5}}}$

15. $(5, -12)$
$r = \sqrt{5^2 + (-12)^2} = 13$


$\sin\theta = \frac{-12}{13}$ $\quad \csc\theta = -\frac{13}{12}$
$\cos\theta = \frac{5}{13}$ $\quad \sec\theta = \frac{13}{5}$
$\tan\theta = \frac{-12}{5}$ $\quad \cot\theta = -\frac{5}{12}$

16. a) .0407 d) -.8391
b) 8.016 e) $\pm\infty$ (No value)
c) -2.628 f) -.7270

17. $r = 5$ in.
$\omega = 3$ rev/sec $= 6\pi$ rad/sec.
$V = r\omega = 5\text{ in} \cdot 6\pi$ rad/sec
$= \boxed{30\pi \text{ in/sec}}$