MATH 133 EXAM 4D (Circa 1972)

SHOW ALL WORK (DO ANY 7 PROBLEMS)
+ Extra Credit

In 1-3, prove the identities: 1. work with one side only.
2. state fundamental identities used.
3. make conclusion.

1. $\sin\theta + \cot\theta\cos\theta = \csc\theta$

2. $\tan x = \dfrac{\sin 2x}{1+\cos 2x}$

3. $\dfrac{\sec^2\theta - 1 + \tan^2\theta\cos\theta}{\cos\theta + \cos^2\theta} = \dfrac{\tan^2\theta}{\cos\theta}$

In 4-7, find all values of x on the interval $0 \le x < 2\pi$

4. $2-\sin x = 2\cos^2 x$

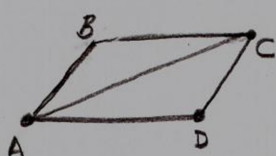
5. $\sin 3x = \frac{1}{2}$

6. $8\sin^4 x + 10\cos^2 x - 7 = 0$

7. $\csc x - 2\sin x = \cot x$

8. Derive: $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$
Hint: Use formulas for $\sin 2\theta$ and $\cos 2\theta$.

9. ABCD is a parallelogram.
$AB = 200'$, $BC = 250'$ $\angle BAD = 78°$
Find AC.



MATH 133 · EXAM 4D SOLUTIONS

1.) $\sin\theta + \cot\theta\cos\theta = \csc\theta$

PROOF: $\sin\theta + \cot\theta\cos\theta = \sin\theta + \dfrac{\cos\theta}{\sin\theta}\cos\theta$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta}$$

$$= \frac{1}{\sin\theta}$$

$$= \csc\theta$$

$\therefore \sin\theta + \cot\theta\cos\theta = \csc\theta$

2.) $\tan x = \dfrac{\sin 2x}{1+\cos 2x}$

PROOF: $\dfrac{\sin 2x}{1+\cos 2x} = \dfrac{2\sin x\cos x}{1+(2\cos^2 x-1)}$

$$= \frac{\cancel{2}\sin x\,\cancel{\cos x}}{\cancel{2}\cos^{\cancel{2}} x}$$

$$= \frac{\sin x}{\cos x}$$

$$= \tan x$$

$\therefore \tan x = \dfrac{\sin 2x}{1+\cos 2x}$

3.) $\dfrac{\sec^2\theta - 1 + \tan^2\theta\cos\theta}{\cos\theta + \cos^2\theta} = \dfrac{\tan^2\theta}{\cos\theta}$

PROOF: $\dfrac{\sec^2\theta - 1 + \tan^2\theta\cos\theta}{\cos\theta + \cos^2\theta} = \dfrac{\tan^2\theta + \tan^2\theta\cos\theta}{\cos\theta(1+\cos\theta)}$

$$= \frac{\tan^2\theta(1+\cos\theta)}{\cos\theta(1+\cos\theta)}$$

$$= \frac{\tan^2\theta}{\cos\theta}$$

$\therefore \dfrac{\sec^2\theta - 1 + \tan^2\theta\cos\theta}{\cos\theta + \cos^2\theta} = \dfrac{\tan^2\theta}{\cos\theta}$

4.) $2 - \sin x = 2\cos^2 x$

$$2 - \sin x = 2(1 - \sin^2 x)$$
$$2\sin^2 x - \sin x = 0$$
$$\sin x(2\sin x - 1) = 0$$
$$\sin x = 0 \quad \text{AND} \quad \sin x = \frac{1}{2}$$
$$x = 0, \pi \quad \text{AND} \quad \frac{\pi}{6}, \frac{5\pi}{6}$$

5.) $\sin 3x = \frac{1}{2}$

$$3x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \frac{25\pi}{6}, \frac{29\pi}{6}$$

DIVIDE BY 3 $x = \frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}, \frac{25\pi}{18}, \frac{29\pi}{18}$

6.) $8\sin^4 x + 10\cos^2 x - 7 = 0$

$$8\sin^4 x + 10(1 - \sin^2 x) - 7 = 0$$
$$8\sin^4 x - 10\sin^2 x + 3 = 0$$
$$(2\sin^2 x - 1)(4\sin^2 x - 3) = 0$$
$$\sin^2 x = \frac{1}{2} \quad \text{AND} \quad \sin^2 x = \frac{3}{4}$$
$$\sin x = \pm\frac{\sqrt{2}}{2} \qquad \sin x = \pm\frac{\sqrt{3}}{2}$$
$$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \quad \text{AND} \quad \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

7.) $\csc x - 2\sin x = \cot x$

CHANGE TO $\frac{1}{\sin x} - 2\sin x = \frac{\cos x}{\sin x}$

$$1 - 2\sin^2 x = \cos x$$
$$1 - 2(1 - \cos^2 x) = \cos x$$
$$1 - 2 + 2\cos^2 x = \cos x$$
$$2\cos^2 x - \cos x - 1 = 0$$
$$(2\cos x + 1)(\cos x - 1) = 0$$
$$\cos x = -\tfrac{1}{2} \quad \cos x = 1$$
$$x = \frac{2\pi}{3}, \frac{4\pi}{3} \quad \cancel{x = 0}$$ REJECT!

8.) $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

$$= \sin(2\theta + \theta)$$
$$= \sin 2\theta \cos\theta + \cos 2\theta \sin\theta$$
$$= 2\sin\theta\cos^2\theta + (1 - 2\sin^2\theta)\sin\theta$$
$$= 2\sin\theta(1 - \sin^2\theta) + \sin\theta - 2\sin^3\theta$$
$$= 3\sin\theta - 4\sin^3\theta$$

EXTRA CREDIT PROBLEM SOLUTION

$$\angle ABC = 180° - 78° = 102°$$
$$d(AC)^2 = d(AB)^2 + d(BC)^2 - 2d(AB)d(BC)\cos 102°$$
$$= 40{,}000 + 62{,}500 + 2(50{,}000)\cos 78°$$
$$= 102{,}500 + 100{,}000(.2079)$$
$$= 102{,}500 + 20{,}790$$
$$= 123{,}290$$
$$d(AC) = \sqrt{123{,}290} \approx 351.1$$
$$\log d(AC) = \frac{1}{2}\log 123{,}290$$
$$= \frac{5.09096}{2}$$
$$= 2.54548$$
$$d(AC) = \text{ANTILOG } 2.54548$$
$$d(AC) = 351.14 \approx 351.1$$

PRACTICE!