

Show all work on this test or on separate paper. Turn in ALL worksheets.

1. Evaluate: 4^{-2} .
 - a. -16
 - b. $\frac{1}{8}$
 - c. $\frac{1}{16}$
 - d. -8

2. Give the domain for the function $f(x) = \frac{3}{x + 5}$.
 - a. $\{x \mid x \neq -5\}$
 - b. $\{x \mid x > -5\}$
 - c. $\{x \mid x \geq 0\}$
 - d. \mathcal{R}

3. Give the domain for the function $f(x) = \sqrt{1 - x}$.
 - a. $\{x \mid x \geq 1\}$
 - b. $\{x \mid x \leq 1\}$
 - c. \mathcal{R}
 - d. $\{x \mid x \leq -1\}$

4. A company purchased a PC for \$4000. After a useful life of 3 years, the scrap value is \$500.
 - a. Find the linear equation that relates the value V in dollars to the time t in years.
 - b. Graph the equation.
 - c. Find the value of the PC after one year. (Round to the nearest dollar.)

5. The total revenue a business received from the sale of x calculators is given by the function $R(x) = 11x + \frac{100}{x}$ dollars. What is the total revenue from the sale of 50 calculators?
 - a. \$650
 - b. \$448
 - c. \$552
 - d. \$220

6. Solve the equation by factoring or by using the quadratic formula $50 - 15x = -x^2$.

7. Graph $f(x) = \begin{cases} 4 - 2x & \text{if } x < 3 \\ x - 7 & \text{if } x \geq 3 \end{cases}$
8. Graph $f(x) = \begin{cases} 7 - x & \text{if } x \leq 5 \\ 2x - 8 & \text{if } x > 5 \end{cases}$
9. For the function $f(x) = 5x^2 + 8x - 2$, find and simplify $\frac{f(x+h) - f(x)}{h}$.
10. An airport parking garage charges \$4 for the first four hours or less and \$2 per hour for the rest of the day, with a maximum rental time of 24 hours.
- Write a two-part function that defines the cost, C , for the 24 hour period.
 - How much will it cost to park for three hours?
 - How much will it cost to park for eight hours?
 - How much will it cost to park for 24 hours?
11. Evaluate $\lim_{x \rightarrow 7} \frac{x^2 - 2x - 35}{x - 7}$
- 7
 - 12
 - undefined
 - 10
12. If $f(x) = \begin{cases} x & \text{if } x \leq 1 \\ 2x & \text{if } x > 1 \end{cases}$, is $f(x)$ continuous at $x = 1$?
- No, since $f(1)$ does not exist
 - No, since $\lim_{x \rightarrow 1} f(x)$ does not exist
 - Yes
 - No, since $f(x)$ is a piecewise linear function.

13. If $f(x) = \begin{cases} 2x - 1 & \text{if } x \leq 2 \\ x + 1 & \text{if } x > 2 \end{cases}$, is $f(x)$ continuous at $x = 2$?
- No, since $f(x)$ is a piecewise linear function.
 - No, since $f(2)$ does not exist
 - No, since $\lim_{x \rightarrow 2} f(x)$ does not exist
 - Yes

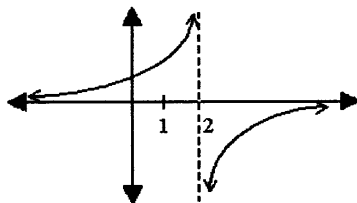
14. Evaluate $\lim_{x \rightarrow 25} \frac{\sqrt{x} - 5}{x - 25}$

15. For the function $f(x) = \begin{cases} 7 - x & \text{if } x < 6 \\ 2x - 7 & \text{if } x \geq 6 \end{cases}$, find

- $\lim_{x \rightarrow 6^-} f(x)$
- $\lim_{x \rightarrow 6^+} f(x)$
- $\lim_{x \rightarrow 6} f(x)$

16. For the function

- find $\lim_{x \rightarrow 2^-} f(x)$
- find $\lim_{x \rightarrow 2^+} f(x)$
- find $\lim_{x \rightarrow 2} f(x)$



17. The total cost function for the sale of backpacks is $C(x) = 450 + 3x + 0.02x^2$, where x is the number of backpacks and $C(x)$ is in dollars. Use the definition of the derivative to find the marginal cost of 144 backpacks.
- \$8.76
 - \$9.01
 - \$1,296.72
 - \$5.88
18. For the function $f(x) = x^2 - 2x + 6$, use the definition of the derivative to find $f'(x)$ by finding
- $f(x + \Delta x)$
 - $f(x + \Delta x) - f(x)$
 - $\frac{f(x + \Delta x) - f(x)}{\Delta x}$
 - $\lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$
19. The derivative of the function $f(x) = x^2 - x + 4$ is $f'(x) = 2x - 1$. Use this result to find the equation of the line tangent to the graph of f at the point $(3, 10)$.
20. Find the derivative of the function $f(x) = 4x^4 - 5x^3 + 2x - 3$.
- $16x^3 - 15x^2 + 2$
 - $16x^3 - 15x^2 + 2x - 3$
 - $4x^4 - 5x^3 + 2x$
 - $4x^3 - 5x^2 + 2$
21. Find the derivative of the function $f(x) = -x - \frac{1}{x}$.
- $\frac{x^2 - 1}{x^2}$
 - $-1 - x$
 - $-1 + x$
 - $\frac{x^2 - 1}{x^2}$

22. If $f(x) = x - \frac{1}{x} + \frac{1}{x^2}$, then $f'(1) =$

- a. $\frac{1}{2}$
- b. $\frac{1}{3}$
- c. 0
- d. 1

23. What is the slope of the line tangent to the graph of $y = \frac{1}{3}x^3 + 7x$ at the point (6, 93)?

- a. $\frac{22}{3}$
- b. 43
- c. 36
- d. 93

One of the acts in a circus is a human cannonball. The height of the human cannonball above the ground is $s(t) = -16t^2 + 80t$, where height is measured in feet and time in seconds.

24. How fast is the human cannonball rising at the end of 1 second?
- a. 64 feet/second
 - b. 16 feet/second
 - c. 48 feet/second
 - d. 72 feet/second

The number of sales of oil-burning furnaces is given by the formula $n = -x^2 + 30x + 10$, where x is the number of months the furnace distributor has been in business.

25. The number of furnaces sold in the sixth month of business was
- a. 190
 - b. 18
 - c. 154
 - d. 42

Counted in hundreds, the number of weeds n in a lawn x weeks after a weed killer was applied is given by $n(x) = -x^2 + 2x + 3$.

26. If the revenue function is $R(x) = 4.3x - 0.003x^2$ and the cost function is $C(x) = 3.1x + 4276$, then the number of units that makes the marginal revenue equal to the marginal cost is
- a. 200
 - b. 257
 - c. 225
 - d. 215

The revenue from the sale of x electric pencil sharpeners is $R(x) = 23x + 0.03x^2$ dollars.

27. The revenue from the sale of 45 sharpeners is
- a. \$1095.75
 - b. \$589.75
 - c. \$1035.00
 - d. \$3060.00

A company that produces and sells cellular telephones determines that the cost of producing x telephones is $C(x) = 0.45x^2 - 15x + 250$ dollars and that the revenue from the sale of x telephones is $R(x) = 0.35x^2 - 7x$ dollars.

28. The marginal profit function is $MP(x) =$
- a. $8 - 0.2x$
 - b. $8x - 0.1x^2 - 2500$
 - c. $0.8x^2 - 23x + 2500$
 - d. $1.6x - 23$

A hammock maker estimates that its hammocks can be priced at $p = 75 - 0.85x$ dollars each, where x is the number sold. The cost to produce x hammocks is $300 + 45x - 0.35x^2$.

29. What is the marginal profit when 5 hammocks are sold?
- a. \$70.75
 - b. \$83.75
 - c. \$25.00
 - d. \$30.50

30. Find the derivative of the function $f(x) = 6\sqrt[3]{x^5} - \frac{1}{x^2}$.

31. Find $\frac{d}{dx}\left[\frac{x^7}{7} - \frac{x^5}{5} - \frac{x^3}{3} - x\right]$

32. If $y = \frac{5}{4x^2}$, then

- $y' =$
- the slope of the tangent to the graph of y at $x = -1$ is
- the y -coordinate of the point on the graph where $x = -1$ is
- the equation of the line tangent to the graph of y at $x = -1$ is

Counted in hundreds, the number of weeds n in a lawn x weeks after a weed killer was applied is given by $n(x) = -x^2 + 2x + 3$.

33. If $P(x) = 135x + 0.3x^2 - 0.006x^3$, what is the marginal profit function?

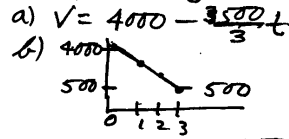
MAC 2233 EXAM 1A Solutions.

1. $4^{-2} = \frac{1}{4^2} = \frac{1}{16}$ (C)

2. $f(x) = \frac{3}{x+5}$
 D: $x+5 \neq 0$
 $x \neq -5$ (A)

3. $f(x) = \sqrt{1-x}$
 D: $1-x \geq 0$
 $-x \geq -1$
 $x \leq 1$ (B)

4. $\text{depr.} = \frac{3500}{3}$ per yr.

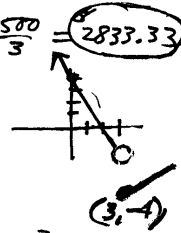


c) $V(1) = 4000 - \frac{3500}{3} = 2833.33$

5. $R(x) = 11x + \frac{100}{x}$
 $R(50) = 11 \cdot 50 + \frac{100}{50} = 550 + 2 = 552$ (C)

6. $50 - 15x = -x^2$
 $x^2 - 15x + 50 = 0$
 $(x-5)(x-10) = 0$
 $x=5$ or $x=10$

7. $f(x) = \begin{cases} 4-2x & \text{if } x < 3 \\ x-7 & \text{if } x \geq 3 \end{cases}$
 $x < 3$ $9m + 4$ $m = -2$
 $x \geq 3$ $(3, -4)$ $m = 1$



8. $f(x) = \begin{cases} 7-x & \text{if } x \leq 5 \\ 2x-8 & \text{if } x > 5 \end{cases}$



9. $f(x) = 5x^2 + 8x - 2$
 $\frac{f(x+h) - f(x)}{h} = \frac{5(x+h)^2 + 8(x+h) - 2 - (5x^2 + 8x - 2)}{h}$
 $= \frac{5x^2 + 10xh + 5h^2 + 8x + 8h - 2 - 5x^2 - 8x + 2}{h}$
 $= \frac{10xh + 5h^2 + 8h}{h} = h(10x + 5h + 8)$

10. $C = \begin{cases} 4 & \text{if } x \leq 4 \\ 4 + 2(x-4) & 4 < x \leq 24 \end{cases}$
 a) $C = \begin{cases} 4 & \text{if } x \leq 4 \\ 2x - 4 & \text{if } 4 < x \leq 24 \end{cases}$

(A) (4)

c) $C(8) = 4 + 2 \cdot 4 = 12$

d) $C(24) = 4 + 2(20) = 44$

11. $\lim_{x \rightarrow 7} \frac{x^2 - 2x - 35}{x-7} = \frac{0}{0}$

$\lim_{x \rightarrow 7} \frac{(x-7)(x+5)}{x-7} = 7+5 = 12$ (B)

12. $f(x) = \begin{cases} x & \text{if } x \leq 1 \\ 2x & \text{if } x > 1 \end{cases}$

$\lim_{x \rightarrow 1^-} = 1$
 $\lim_{x \rightarrow 1^+} = 2$ (B)

$\lim_{x \rightarrow 1} f(x)$ DNE.

13. $f(x) = \begin{cases} 2x-1 & \text{if } x \leq 2 \\ x+1 & \text{if } x > 2 \end{cases}$

$\lim_{x \rightarrow 2^-} = 4-1 = 3$ $f(2) = 3$

$\lim_{x \rightarrow 2^+} = 2+1 = 3$ (D) contin.

14. $\lim_{x \rightarrow 25} \frac{\sqrt{x}-5}{x-25} = \frac{0}{0}$

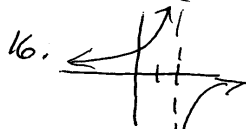
$\lim_{x \rightarrow 25} \frac{(\sqrt{x}-5)(\sqrt{x}+5)}{(x-25)(\sqrt{x}+5)}$
 $= \lim_{x \rightarrow 25} \frac{x-25}{(x-25)(\sqrt{x}+5)} = \frac{1}{10}$

15. $f(x) = \begin{cases} 7-x & \text{if } x < 6 \\ 2x-7 & \text{if } x \geq 6 \end{cases}$

a) $\lim_{x \rightarrow 6^-} = 7-6 = 1$

b) $\lim_{x \rightarrow 6^+} = 2(6)-7 = 5$

c) $\lim_{x \rightarrow 6}$ Does not exist!



a) $\lim_{x \rightarrow 2^-} = +\infty$

b) $\lim_{x \rightarrow 2^+} = -\infty$

c) $\lim_{x \rightarrow 2}$ DNE

17. $C(x) = 450 + 3x + 0.02x^2$

Marg. cost
 $= C'(x) = 3 + 0.04x$

$C'(144) = 3 + 0.04(144) = 3 + 5.76 = 8.76$ (A)

18. $f(x) = x^2 - 2x + 6$

i) $f(x+h) = (x+h)^2 - 2(x+h) + 6$

$= x^2 + 2xh + h^2 - 2x - 2h + 6$

ii) $f(x+h) - f(x) = x^2 + 2xh + h^2 - 2x - 2h + 6 - x^2 - 2x + 6 = 2xh + h^2 - 2h$

18c) $\frac{f(x+h) - f(x)}{h} = \frac{2xh + h^2 - 2h}{h} = h(2x + h - 2)$

d) $\lim_{h \rightarrow 0} = 2x + 2$

19. $f(x) = x^2 - x + 4$
 $f'(x) = 2x - 1$
 $f'(3) = 2(3) - 1 = 5$
 $m = 5$ at $(3, 10)$
 $y = mx + b$
 $10 = 5(3) + b$
 $b = -5$ $y = 5x - 5$

3. $y = \frac{1}{3}x^3 + 7x$
 $y' = x^2 + 7$ at $(6, 43)$
 $= 6^2 + 7$
 $= 36 + 7 = 43$ (B)

26. $R(x) = 4.3x - 0.003x^2$
 $\frac{dR}{dx} = 4.3 - 0.006x = \text{marg Rev.}$
 $C(x) = 3.1x + 4276$
 $\frac{dC}{dx} = 3.1 = \text{marg cost.}$
 $4.3 - 0.006x = 3.1$
 $1.2 = 0.006x$
 $x = \frac{1.2}{0.006} = 200$ (A)

9. Profit = $R - C$ $R = P \cdot x$
 $P = x(75 - 0.85x) - (300 + 45x - 0.35x^2)$
 $P = 75x - 0.85x^2 - 300 - 45x + 0.35x^2$
 $P = -0.50x^2 + 30x - 300$
 $\frac{dP}{dx} = -1x + 30$
 $\frac{dP(5)}{dx} = -5 + 30$
 $= 25$ (C)

32. $y = \frac{5}{4x^2} = \frac{5}{4}x^{-2}$
a) $y' = -\frac{5}{2}x^{-3} = \frac{-5}{2x^3}$
b) $y'(-1) = \frac{-5}{-2} = \frac{5}{2}$

20. $f(x) = 4x^4 - 5x^3 + 2x - 3$
 $f'(x) = 16x^3 - 15x^2 + 2$ (A)

21. $f(x) = -x - x^{-1}$
 $f'(x) = -1 + x^{-2}$
 $= -1 + \frac{1}{x^2}$
 $= \frac{-x^2 + 1}{x^2}$
 $= -\frac{(x^2 - 1)}{x^2}$ (A)

22. $f(x) = x - x^{-1} + x^{-2}$
 $f'(x) = 1 + x^{-2} - 2x^{-3}$
 $f'(1) = 1 + 1 - 2 = 0$ (C)

24. $s(t) = -16t^2 + 80t$
 $s'(t) = v(t) = -32t + 80$
 $v(1) = -32 + 80$
 $= 48 \text{ ft/sec}$ (C)

25. $n = -x^2 + 30x + 10$
 $n(6) = -36 + 30(6) + 10$
 $= -36 + 190$
 $= 154$ (C)

27. $R(x) = 23x + 0.03x^2$
 $R(45) = 23(45) + .03(45)^2$
 $= 1095.75$ (A)

28. $P(x) = R(x) - C(x)$
 $= (0.35x^2 - 7x) - (0.45x^2 - 15x + 250)$
 $= 0.35x^2 - 7x - 0.45x^2 + 15x - 250$
 $= -0.10x^2 + 8x - 250$
Marg. Profit = $\frac{dP}{dx} = -0.2x + 8$ (A)

30. $f(x) = 6x^{2/3} - x^{-2}$
 $f'(x) = 10x^{-1/3} + 2x^{-3}$
 $= 10x^{2/3} + \frac{2}{x^3}$

31. $\frac{d}{dx} \left[\frac{x^7}{7} - \frac{x^5}{5} - \frac{x^3}{3} - x \right]$
 $= x^6 - x^4 - x^2 - 1$

c) $y(-1) = \frac{5}{4}$
d) $y = mx + b$
 $4 \cdot \frac{5}{4} = \frac{5}{2}(-1) + b$
 $5 = -10 + 4b$
 $b = \frac{15}{4}$
 $y = \frac{5}{2}x + \frac{15}{4}$

33. $P(x) = 135x + .3x^2 - .008x^3$
 $\frac{dP}{dx} = 135 + .6x - .024x^2$