MAC 2233 EXAM 1B **R^2** **NAME** ______________________________

Show all work on separate paper. Turn in ALL worksheets.

(Problems are 5 points each, unless multiple parts-- 2 each part)

1. Find the domain and range for $f(x) = \frac{16}{x^2 - 4x}$.

   [Hint: Use a graphing calculator to find the range!]

2. Solve for x (explain or describe your method).
   $2x^3 = 12x^2 - 18x$.

3. Graph: $f(x) = \begin{cases} 8 - 2x & \text{if } x > 2 \\ x + 2 & \text{if } x \le 2 \end{cases}$

4. Given: $f(x) = \begin{cases} 8 - 2x & \text{if } x > 2 \\ x + 2 & \text{if } x \le 2 \end{cases}$
   a) $\lim_{xY\,2^-} f(x)$ b) $\lim_{xY\,2^+} f(x)$ c) $\lim_{xY\,2} f(x)$
   d) Is this graph continuous? Explain your answer.

5. Given: $f(x) = \begin{cases} 2x - 8 & \text{if } x > 2 \\ x - 2 & \text{if } x \le 2 \end{cases}$
   a) $\lim_{xY\,2^-} f(x)$ b) $\lim_{xY\,2^+} f(x)$ c) $\lim_{xY\,2} f(x)$
   d) Is this graph continuous? Explain your answer.

6. If $f(x) = \sqrt{x}$ **and** $g(x) = x^3 + 3x - 6$, find f (g(x)) and g(f(x)).

7. If $f(x) = x^2 - 4x + 5$, find ***f (x + h) – f(x)*** and simplify completely.

8. If $f(x) = x^2 - 4x + 5$, find
   a) $\frac{f(x+h) - f(x)}{h}, \; h \ne 0$ and simplify completely.
   b) $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

9. Find $\lim_{xY\,3} \frac{x^2 - 9}{x^2 - 3x}$.

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10. Find $\lim\limits_{h \to 0} \dfrac{x^2h - xh^2 + h^3}{h}$.

11. Given: $f(x) = \dfrac{|x|}{x}$

a) $\lim\limits_{x \to 0^-} f(x)$ b) $\lim\limits_{x \to 0^+} f(x)$ c) $\lim\limits_{x \to 0} f(x)$

d) Sketch the graph.

In 12–13, find f ′(x) using the limit definition of the derivative, $\lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$.

12. $f(x) = 3x^2 - 5x + 2$.

13. $f(x) = \dfrac{2}{x}$

14. Find f ′(x) for $f(x) = \dfrac{2}{x}$ by the “shortcut” method (i.e., the power rule).

15. Find f ′(x) for $f(x) = 6\sqrt[3]{x} - \dfrac{12}{\sqrt{x}}$ by the “shortcut” method.

16. If $f(x) = \dfrac{54}{\sqrt{x}} + 12\sqrt{x}$, find f ′(3)

In 17 – 20, the cost function for a company that produces x units per week is given by C(x) = 420 x + 72000, and the revenue is given by R(x) = -3x^2 + 1800x.

17. Find an equation for profit P(x).

18. Find the company’s break even points (where profit = 0).

19. Find the company’s marginal revenue and marginal profit functions.

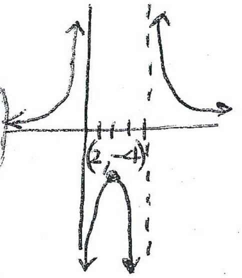
EXTRA CHALLENGE

20. Find the number of units that should be produced in order to maximize profit and the maximum profit.

MAC 2233 Exam 1B Solutions

1. $f(x)=\frac{16}{x(x-4)}$

$\boxed{\begin{array}{l} D: \text{all real } x\neq 0,4 \\ R: (-\infty,-4]\cup(0,\infty) \end{array}}$



2. $2x^3-12x^2+18x=0$

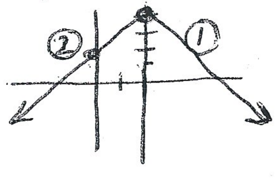
$2x(x^2-6x+9)=0$

$2x(x-3)^2=0$

$\boxed{x=0,\ x=3 \text{ mult } 2}$

OR – Use calculator!

3. $f(x)=\begin{cases} ①\ 8-2x & \text{if } x>2 \\ ②\ x+2 & \text{if } x\leq 2 \end{cases}$



4 a) $\lim\limits_{x\to 2^-} = 2+2 = \boxed{4}$

b) $\lim\limits_{x\to 2^+} = 8-2(2) = \boxed{4}$

c) $\lim\limits_{x\to 2} = \boxed{4}$

d) Graph is $\boxed{\text{continuous}}$ since $\lim\limits_{x\to 2^-} = \lim\limits_{x\to 2^+} = f(2)$

5a) $\lim\limits_{x\to 2^-} = 2-2 = \boxed{0}$

b) $\lim\limits_{x\to 2^+} = 2(2)-8 = \boxed{-4}$

c) $\lim\limits_{x\to 2}$ $\boxed{\text{DNE}}$, since $\lim\limits_{x\to 2^-} \neq \lim\limits_{x\to 2^+}$

d) Graph is $\boxed{\text{NOT continuous}}$ since $\lim\limits_{x\to 2^-} \neq \lim\limits_{x\to 2^+}$.

6. $f(g(x)) = \boxed{\sqrt{x^3+3x-6}}$

$g(f(x)) = (\sqrt{x})^3+3\sqrt{x}-6$

$= \boxed{x^{3/2}+3x^{1/2}-6}$

7. $f(x)=x^3-4x-6$

$f(x+h)-f(x) = (x+h)^2-4(x+h)-6$
$\quad -(x^2-4x-6)$

$= \cancel{x^2}+2xh+h^2-\cancel{4x}-4h-\cancel{6}$
$\quad -\cancel{x^2}+\cancel{4x}+\cancel{6}$

$= \boxed{2xh+h^2-4h}$

8 a) $\frac{2xh+h^2-4h}{h}$

$= \frac{\cancel{h}(2x+h-4)}{\cancel{h}}$

$= \boxed{2x+h-4}$

b) $\lim\limits_{h\to 0} = \boxed{2x-4}$

9. $\lim\limits_{x\to 3} \frac{x^2-9}{x^2-3x}$

$= \lim\limits_{x\to 3} \frac{\cancel{(x-3)}(x+3)}{x\cancel{(x-3)}}$

$= \lim\limits_{x\to 3} \frac{x+3}{x}$

$= \frac{6}{3} = \boxed{2}$

10. $\lim\limits_{h\to 0} \frac{x^2h-xh^2+h^3}{h}$

$= \lim\limits_{h\to 0} \frac{\cancel{h}(x^2-xh+h^2)}{\cancel{h}}$

$= \lim\limits_{h\to 0} (x^2-x\overset{0}{\cancel{h}}+\overset{0}{\cancel{h^2}})$

$= \boxed{x^2}$

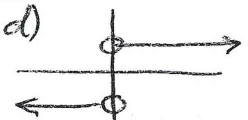
11. $f(x)=\frac{|x|}{x}$

a) $\lim\limits_{x\to 0^-} = \boxed{-1}$

b) $\lim\limits_{x\to 0^+} = \boxed{1}$

c) $\lim\limits_{x\to 0}$ $\boxed{\text{DNE}}$

d)



12. $f(x)=3x^2-5x+2$

$f(x+h)-f(x)$

$=3(x+h)^2-5(x+h)+2$
$\quad -(3x^2-5x+2)$

$= \cancel{3x^2}+6xh+3h^2-\cancel{5x}-5h+\cancel{2}$
$\quad -\cancel{3x^2}+\cancel{5x}-\cancel{2}$

$= 6xh+3h^2-5h$ or $h(6x+3h-5)$

$\lim\limits_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0} \frac{\cancel{h}(6x+3h-5)}{\cancel{h}}$

$\lim\limits_{h\to 0} (6x+3\overset{0}{\cancel{h}}-5) = \boxed{6x-5}$

13. $f(x)=\frac{2}{x}$ $\quad f(x+h)=\frac{2}{x+h}$

$\lim\limits_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$= \lim\limits_{h\to 0} \left(\frac{2}{x+h}-\frac{2}{x}\right)\cdot\frac{1}{h}$

$\lim\limits_{h\to 0} \frac{\cancel{2x}-\cancel{2x}-2h}{x(x+h)}\cdot\frac{1}{h}$

$\lim\limits_{h\to 0} \frac{-2\cancel{h}}{x(x+h)}\cdot\frac{1}{\cancel{h}}$

$\lim\limits_{h\to 0} \frac{-2}{x(x+h)} = \boxed{\frac{-2}{x^2}}$

14. $f(x)=2x^{-1}$

$f'(x)=-2x^{-2}=\boxed{\frac{-2}{x^2}}$

15. $f(x)=6x^{1/3}-12x^{-1/2}$

$f'(x)=\boxed{2x^{-2/3}+6x^{-3/2}}$

16. $f(x)=54x^{-1/2}+12x^{1/2}$

$f'(x)=-27x^{-3/2}+6x^{-1/2}$

$f'(3)=-27\cdot 3^{(-3/2)}+6\cdot 3^{(-1/2)}$

$\approx \boxed{-1.732}$

17. Profit = Revenue − Cost

$P(x)=R(x)-C(x)$

$P(x)=(-3x^2+1800x)-(420x+72000)$

$\boxed{P(x)=-3x^2+1380x-72000}$

18. Breakeven: $C(x)=R(x)$ or $P(x)=0$

(Better yet, use calculator!)

$-3(x^2-460x+24000)=0$

$(x-400)(x-60)=0$

$\boxed{x=400 \quad x=60}$

19. $R(x)=-3x^2+1800x$

$MR(x)=\frac{dR}{dx}=\boxed{-6x+1800}$

$P(x)=-3x^2+1380x-72000$

$MP(x)=\frac{dP}{dx}=\boxed{-6x+1380}$

20. $P(x)=-3x^2+1380x-7200$

Parabola! Max Profit occurs at the vertex of parabola.

20 (Continued)

$P(x)=-3x^2+1380x-72$

$x=-\frac{b}{2a}$

$=\frac{-1380}{2(-3)}=\boxed{230 \text{ units}}$

MAX $P(x)=$
$-3(230)^2+1380(230)$
-7200

$=\boxed{\$86,700}$