Show all work on separate paper. Turn in ALL worksheets.

(Problems are 5 points each, unless multiple parts-- 2 each part)

1.   Find the domain and range for $f(x) = \dfrac{16}{x^2 - 4}$.

   [Hint: Use a graphing calculator to find the range!]

2.   Solve for x (explain or describe your method).
   $2x^5 + 8x^4 = 24x^3$.

3.   If $f(x) = \dfrac{x+8}{x-1}$ and $g(x) = \sqrt{x}$, find f(g(x)) and g(f(x)).

4.   If $f(x) = x^2$, find $\dfrac{f(x+h) - f(x)}{h}$, $h \neq 0$.

5.   If $f(x) = \dfrac{1}{x}$, find $\dfrac{f(x+h) - f(x)}{h}$, $h \neq 0$.

6.   Find the equation of the line (in form y=mx+b) passing through (-1,3) and
   (2,-5).

7.   Given: $f(x) = \begin{cases} 2x - 4 & if \ x \leq 2 \\ -x + 4 & if \ x > 2 \end{cases}$, find
   a) f(0)   b) f(2)   c) f(3)   d) f(-2).

8.   Given: $f(x) = \begin{cases} 2x - 4 & if \ x \leq 2 \\ -x + 4 & if \ x > 2 \end{cases}$
   a) $\lim\limits_{x \to 2^-} f(x)$ b) $\lim\limits_{x \to 2^+} f(x)$ c) $\lim\limits_{x \to 2} f(x)$
   d) Sketch the graph.

9.   Find $\lim\limits_{x \to 0} \dfrac{x^2 - x}{x^2 + x}$.

10.   Find $\lim\limits_{x \to 5} \dfrac{x^3 - 25x}{x^2 - 6x + 5}$.

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11.   Given: $f(x) = \dfrac{|x|}{x}$

  a) $\lim\limits_{x \to 0^-} f(x)$ b) $\lim\limits_{x \to 0^+} f(x)$ c) $\lim\limits_{x \to 0} f(x)$

  d) Sketch the graph.

In 12–13, find f '(x) using the limit definition of the derivative, $\lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$.

12. $f(x) = x^2 - 5x$.

13. $f(x) = \dfrac{5}{x}$

14. Find f '(x) for $f(x) = \dfrac{5}{x}$ by the "shortcut" method (i.e., the power rule).

15. Find f '(x) for $f(x) = 6\sqrt{x} + \dfrac{12}{\sqrt[3]{x}}$ by the "shortcut" method.

16. If $f(x) = \dfrac{16}{\sqrt{x}} + 8\sqrt{x}$, find f(16) and f '(16)

In 17 – 20, the cost function for a company that produces x units per week is given by C(x) = 180 x + 16,000, and the revenue is given by R(x) = -2x² + 660x.

17. Find an equation for profit P(x).

18. Find the company's break even points (where profit = 0).

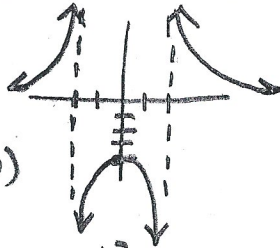19. Find the company's marginal revenue and marginal profit functions.

20. Find the number of units that should be produced in order to maximize profit and find the maximum profit.

MAC 2233 EXAM 1C Solutions

1. $f(x) = \frac{16}{x^2-4}$

D: all $x \neq \pm 2$

R: $(-\infty, -4] \cup (0, \infty)$

2. $2x^5 + 8x^4 = 24x^3$

$2x^5 + 8x^4 - 24x^3 = 0$

$2x^3(x^2 + 4x - 12) = 0$

$2x^3(x+6)(x-2) = 0$

$\boxed{x = 0 \quad x = -6 \quad x = 2}$

Quadratic Eq. Solved by factoring. Calculator methods also apply!

3. $f(x) = \frac{x+8}{x-1}$ $g(x) = \sqrt{x}$

$f[g(x)] = \frac{\sqrt{x} + 8}{\sqrt{x} - 1}$

$g[f(x)] = \sqrt{\frac{x+8}{x-1}}$

4. $f(x) = x^2$ $f(x+h) = (x+h)^2$

$\frac{f(x+h) - f(x)}{h} = \frac{x^2 + 2xh + h^2 - x^2}{h}$

$= \frac{h(2x+h)}{h}$

$= \boxed{2x+h}$

5. $f(x) = \frac{1}{x}$ $f(x+h) = \frac{1}{x+h}$

$\frac{f(x+h) - f(x)}{h} = \left(\frac{1}{x+h} - \frac{1}{x}\right) \div h$

$= \frac{x - x - h}{x(x+h)} \cdot \frac{1}{h}$

$= \frac{-h}{x(x+h)} \cdot \frac{1}{h} = \boxed{\frac{-1}{x(x+h)}}$

6. $(-1, 3)$ $(2, -5)$

$m = \frac{-5-3}{2-(-1)} = \frac{-8}{3}$

$y = mx + b$

$3 = -\frac{8}{3}(-1) + b$

$9 = 8 + 3b$

$\frac{1}{3} = b$ $\boxed{y = -\frac{8}{3}x + \frac{1}{3}}$

7,8. $f(x) = \begin{cases} 2x-4 & \text{if } x \leq 2 \\ -x+4 & \text{if } x > 2 \end{cases}$

7 a) $f(0) = -4$

b) $f(2) = 0$

c) $f(3) = 1$

d) $f(-2) = -8$
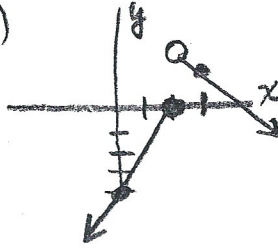
8 a) $\lim\limits_{x \to 2^-} = 0$

b) $\lim\limits_{x \to 2^+} = 2$

c) $\lim\limits_{x \to 2} = DNE$

8d)

9. $\lim\limits_{x \to 0} \frac{x(x-1)}{x(x+1)} = \lim\limits_{x \to 0} \frac{x-1}{x+1}$

$= \boxed{-1}$

10. $\lim\limits_{x \to 5} \frac{x(x-5)(x+5)}{(x-5)(x-1)}$

$= \frac{5 \cdot 10}{4} = \boxed{\frac{25}{2}}$

11. $f(x) = \frac{|x|}{x}$

a) $\lim\limits_{x \to 0^-} = -1$

b) $\lim\limits_{x \to 0^+} = 1$

c) $\lim\limits_{x \to 0} DNE$
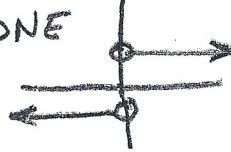
12. $f(x) = x^2 - 5x$

$f(x+h) = (x+h)^2 - 5(x+h)$

$\lim\limits_{h \to 0} = \frac{x^2 + 2xh + h^2 - 5x - 5h - x^2 + 5x}{h}$

$\lim\limits_{h \to 0} \frac{h(2x+h-5)}{h} = 2x + h^0 - 5$

$= \boxed{2x - 5}$

13. $f(x) = \frac{5}{x}$ $f(x+h) = \frac{5}{x+h}$

$\lim\limits_{h \to 0} \left(\frac{5}{x+h} - \frac{5}{x}\right) \div h$

$\lim\limits_{h \to 0} \frac{5x - 5x - 5h}{x(x+h)} \cdot \frac{1}{h}$

$\lim\limits_{h \to 0} \frac{-5h}{x(x+h)} \cdot \frac{1}{h} = \boxed{\frac{-5}{x^2}}$

14. $f(x) = 5x^{-1}$

$f'(x) = -5x^{-2}$

$= \boxed{\frac{-5}{x^2}}$

15. $f(x) = 6x^{1/2} + 12x^{-1/3}$

$\boxed{f'(x) = 3x^{-1/2} - 4x^{-4/3}}$

16. $f(x) = \frac{16}{\sqrt{x}} + 8\sqrt{x} = 16x^{-1/2} + 8x^{1/2}$

$f(16) = \frac{16}{\sqrt{16}} + 8\sqrt{16} = \frac{16}{4} + 8 \cdot 4$

Also use calculators! $= 4 + 32 = \boxed{36}$

$f'(x) = -8x^{-3/2} + 4x^{-1/2}$

$f'(16) = -8 \cdot 16^{-3/2} + 4 \cdot 16^{-1/2}$

$= -8 \cdot \frac{1}{64} + 4 \cdot \frac{1}{4} = 1 - \frac{1}{8} = \boxed{\frac{7}{8}}$

Also calculators = [2nd][calc]

17. $P(x) = R(x) - C(x)$

$= (-2x^2 + 660x) - (180x + 16000)$

$= -2x^2 + 660x - 180x - 16000$

$\boxed{P(x) = -2x^2 + 480x - 16000}$

18. $-2(x^2 - 240x + 8000) = 0$

$(x - 200)(x - 40) = 0$

$\boxed{x = 200 \quad x = 40}$

19. $\boxed{MR(x) = R' = -4x + 660}$

$\boxed{MP(x) = P' = -4x + 480}$

20. $y_1 = -2x^2 + 480x - 16000$ der1, $f(x)$, x, 16 ENTER

Extend window x=0 to 500 or 1000 $\boxed{0.875}$

$\boxed{f_{max} = 120}$

$-2(120)^2 + 480(120) - 16000$

or $x = \frac{-b}{2a}$ $f(120)$