MAC 2233 EXAM 1D **R^2** **NAME** ______________________________

Show all work on separate paper. Turn in ALL worksheets.

(Problems are 5 points each, unless multiple parts—2 or 3 each part)

1. Find the domain and range for $f(x) = \sqrt{x-4} + 6$.

   [Hint: Use a graphing calculator to find the range!]

2. Solve for x (explain or describe your method).
   $2x^3 - 8x^2 = 10x$.

3. If $f(x) = \dfrac{x}{x+4}$ and $g(x) = x-4$, find f(g(x)) and g(f(x)).
   (Give answers in the form of a single fraction!)

4. If $f(x) = x^2$, find $\dfrac{f(x+h) - f(x)}{h}$, $h \neq 0$.

5. At a depth d feet underwater, the water pressure is p(d) = 0.45d +15 pounds per square inch. Find the pressure at the bottom of an 8 foot pool and also at the maximum ocean depth of 35,000 feet.

6. Find the equation of the line (in form y=mx+b) passing through (4,3) and (2,-5).

7. Given: $f(x) = \begin{cases} 2x - 4 & \text{if } x \leq 4 \\ -x+4 & \text{if } x > 4 \end{cases}$, find
   a) f(0) b) f(4) c) f(6) d) f(-2).

8. Given: $f(x) = \begin{cases} 2x - 4 & \text{if } x \leq 4 \\ -2x+4 & \text{if } x > 4 \end{cases}$
   a) $\lim_{x \to 4^-} f(x)$ b) $\lim_{x \to 4^+} f(x)$ c) $\lim_{x \to 4} f(x)$
   d) Sketch the graph.

9. Find $\lim_{x \to 5} \dfrac{2x^2 - 10x}{x-5}$.

10. Find $\lim_{x \to 2} \dfrac{x^3 - 4x}{x^2 - 5x + 6}$.

11. Given: $f(x) = \frac{|x|}{x}$

a) $\lim_{x \to 0^-} f(x)$ b) $\lim_{x \to 0^+} f(x)$ c) $\lim_{x \to 0} f(x)$

d) Sketch the graph.

In 12–13, find f ′(x) using the limit definition of the derivative, $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$.

12. $f(x) = x^2 + 2x - 5$.

13. $f(x) = \frac{2}{x}$

14. Find f ′(x) for $f(x) = \frac{2}{x}$ by the “shortcut” method (i.e., the power rule).

15. Find f ′(x) for $f(x) = x^3 - 12x^2 - 4x + 6$ by the “shortcut” method.

16. If $f(x) = \frac{30}{\sqrt[3]{x}}$, find f(64), f ′($x$), and f ′(64)

In 17 – 20, the cost function for a company that produces x units per week is given by C(x) = 120x + 4800, and the revenue is given by R(x) = $-2x^2 + 400x$.

17. Find an equation for profit P(x).

18. Find the company’s break even points (where profit = 0).

19. Find the company’s marginal revenue and marginal profit functions.

20. Find the number of units that should be produced in order to maximize profit and find the maximum profit.

CONCEPTS of CALCULUS EXAM 1D Solutions

1. $f(x) = \sqrt{x-4} + 6$

(upper half of parabola)

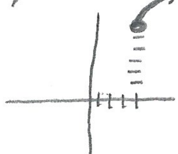
$V(4,6)$

D: $x - 4 \geq 0$

$x \geq 4$

$[4, \infty)$

R: $[6, \infty)$



2. $2x^3 - 8x^2 = 10x$

$2x^3 - 8x^2 - 10x = 0$

$2x(x^2 - 4x - 5) = 0$

$2x(x-5)(x+1) = 0$

$x = 0 \quad x = 5 \quad x = -1$

or Calculator:

TI83/84: Polysmlt

TI85/86: [2nd] [POLY]

3. $f(x) = \dfrac{(x)}{(x)+4} \qquad g(x) = x - 4$

$f[g(x)] = \dfrac{(x-4)}{(x-4)+4} = \dfrac{x-4}{x}$

$g[f(x)] = \dfrac{x}{x+4} - 4 \cdot \dfrac{x+4}{x+4}$

$= \dfrac{x - 4x - 16}{x+4}$

$= \dfrac{-3x - 16}{x+4}$

4. $f(x) = x^2 \qquad f(x+h) = (x+h)^2$

$\dfrac{f(x+h) - f(x)}{h} = \dfrac{(x+h)^2 - x^2}{h}$

$= \dfrac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h}$

$= \dfrac{\cancel{h}(2x + h)}{\cancel{h}} = 2x + h$

5. $p(d) = 0.45d + 15$

$p(8) = (0.45)(8) + 15$

$= 18.6$ psi

$p(35{,}000) = (0.45)(35000) + 15$

$= 15765$ psi

6. $(4,3) \quad (2,-5)$

$m = \dfrac{-5-3}{2-4} = \dfrac{-8}{-2} = 4$

$y = mx + b$

$3 = (4)(4) + b$

$-13 = b$

$y = 4x - 13$

7. $f(x) = \begin{cases} 2x - 4 & \text{if } x \leq 4 \\ -x + 4 & \text{if } x > 4 \end{cases}$

a) $f(0) = 2(0) - 4 = -4$

b) $f(4) = 2(4) - 4 = 4$

c) $f(6) = -6 + 4 = -2$

d) $f(-2) = 2(-2) - 4 = -8$

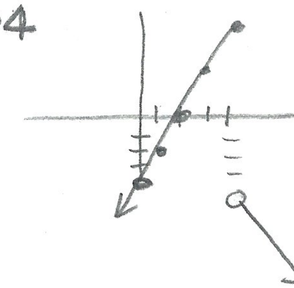
8. $f(x) = \begin{cases} 2x - 4 & \text{if } x \leq 4 \\ -2x + 4 & \text{if } x > 4 \end{cases}$

a) $\lim\limits_{x \to 4^-} = 2(4) - 4 = 4$

b) $\lim\limits_{x \to 4^+} = (-2)(4) + 4 = -4$

c) $\lim\limits_{x \to 4} f(x) = \text{DNE}$

d)



9. $\lim\limits_{x \to 5} \dfrac{2x^2 - 10x}{x - 5}$

$= \lim\limits_{x \to 5} \dfrac{2x\cancel{(x-5)}}{\cancel{x-5}}$

$= \lim\limits_{x \to 5} 2x = 10$

10. $\lim\limits_{x \to 2} \dfrac{x^3 - 4x}{x^2 - 5x + 6}$

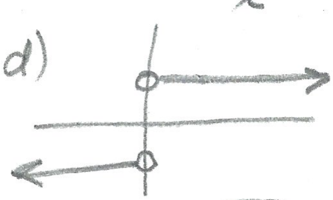
$= \lim\limits_{x \to 2} \dfrac{x(x^2 - 4)}{x^2 - 5x + 6}$

$= \lim\limits_{x \to 2} \dfrac{x\cancel{(x-2)}(x+2)}{\cancel{(x-2)}(x-3)}$

$= \dfrac{2(4)}{-1} = -8$

11. $f(x) = \dfrac{|x|}{x}$

d)



a) $\lim\limits_{x \to 0^-} f(x) = -1$

b) $\lim\limits_{x \to 0^+} f(x) = 1$

c) $\lim\limits_{x \to 0} f(x) = \text{DNE}$

12. $f(x) = x^2 + 2x - 5$

$f(x+h) = (x+h)^2 + 2(x+h) - 5$

$\dfrac{f(x+h) - f(x)}{h} = \dfrac{x^2 + 2xh + h^2 + 2x + 2h - 5 - (x^2 + 2x - 5)}{h}$

$= \dfrac{\cancel{x^2} + 2xh + h^2 + \cancel{2x} + 2h \cancel{- 5} - \cancel{x^2} - \cancel{2x} \cancel{+ 5}}{h}$

$= \dfrac{\cancel{h}(2x + h + 2)}{\cancel{h}}$

$\lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \lim\limits_{h \to 0} (2x + h + 2) = 2x + 2$

13. $f(x) = \frac{2}{x}$ $\quad f(x+h) = \frac{2}{x+h}$

$$\frac{f(x+h)-f(x)}{h} = \frac{1}{h}\left[\frac{2}{x+h} - \frac{2}{x}\right]$$

$$= \frac{1}{h}\left[\frac{2x - 2(x+h)}{x(x+h)}\right]$$

$$= \frac{1}{h}\left[\frac{\cancel{2x}-\cancel{2x}-2h}{x(x+h)}\right]$$

$$= \frac{1}{\cancel{h}}\left[\frac{-2\cancel{h}}{x(x+h)}\right]$$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{-2}{x(x+\cancel{h})} = \frac{-2}{x^2}$$

14. $f(x) = \frac{2}{x} = 2x^{-1}$

$f'(x) = -2x^{-2}$

$= -2 \cdot \frac{1}{x^2}$

$= \frac{-2}{x^2}$

15. $f(x) = x^3 - 12x^2 - 4x + 6$

$f'(x) = 3x^2 - 24x - 4$

16. $f(x) = \frac{30}{\sqrt[3]{x}}$ $\quad f(x) = 30x^{-1/3}$

$f'(x) = -10x^{-4/3}$

$f(64) = \frac{30}{\sqrt[3]{64}}$

$= \frac{30}{4} = \frac{15}{2}$

$f'(64) = -10 \cdot 64^{-4/3}$

$= -10 \cdot 4^{-4}$

$= -10 \frac{1}{256}$

$\approx -.039 = \frac{-5}{128}$

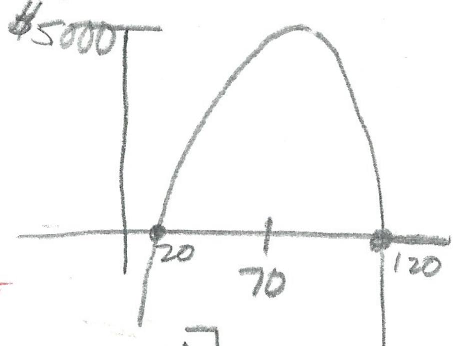
[Or use der1(30÷3√x, x, 64)]

20. Also Graph, fmax

$y1 = -2x^2 + 280x - 4800$

$x = [0, 150]$

$y = [0, 6000]$



17. $P(x) = R(x) - C(x)$

$= -2x^2 + 400x - (120x + 4800)$

$= -2x^2 + 400x - 120x - 4800$

$P(x) = -2x^2 + 280x - 4800$ $\quad MP = -4x + 280$

19. $Rev = -2x^2 + 400x$

$MR = -4x + 400$

$Cost = 120x + 4800$

$MC = 120$

18. Breakeven $\Rightarrow P(x) = 0$

$-2x^2 + 280x - 4800 = 0$

$-2(x^2 - 140x + 2400) = 0$

$-2(x-20)(x-120) = 0$

$x = 20$ or $x = 120$

[Or use Polysmlt or 2nd POLY]

20. Maximum at $x = \frac{-b}{2a}$ or halfway between x intercepts.

$x = 70$ units.

$P(70) = -2(70)^2 + 280(70) - 4800$