Concepts of Calculus Exam 2B Solutions Dr. Rapalje

1. $f(x) = 5x^{-2}$

$f'(x) = -10x^{-3}$

$= -\frac{10}{x^3}$

2. $f(x) = \frac{2x^2}{x^2+1}$

$f'(x) = \frac{(x^2+1)(4x) - 2x^2 \cdot 2x}{(x^2+1)^2}$

$= \frac{4x(x^2+1-x^2)}{(x^2+1)^2} = \frac{4x}{(x^2+1)^2}$

3. $f(x) = x^4 - x^2 + 6x + 4 - 2x^{-1}$

$f'(x) = 4x^3 - 2x + 6 + 2x^{-2}$

$f''(x) = 12x^2 - 2 - 4x^{-3}$

$f'''(x) = 24x + 12x^{-4}$

$f^{IV}(x) = 24 - 48x^{-5}$

4. $f(x) = \frac{x-2}{4x}$

$f'(x) = \frac{4x \cdot 1 - (x-2) \cdot 4}{(4x)^2}$

$= \frac{\cancel{4x} - \cancel{4x} + 8}{16x^2} = \frac{1}{2x^2}$

$f''(x) = -x^{-3}$

$f''(x) = -\frac{1}{x^3} = -\frac{1}{27}$

5. $s(t) = 24t^2 - 2t^3$

$v(t) = 48t - 6t^2$

$a(t) = 48 - 12t$

$V(3) = 48 \cdot 3 - 6 \cdot 3^2 = 90$

$a(3) = 48 - 12 \cdot 3 = 12$

or CALCULATOR

$V(3) = \text{der1}(24x^2 - 2x^3, x, 3) = 90$

$a(3) = \text{der2}(24x^2 - 2x^3, x, 3) = 12$

6. $f(x) = (x^2+1)^{10}$

$f'(x) = 10(x^2+1)^9 \cdot 2x$

$= 20x(x^2+1)^9$

$f''(x) = 20x \cdot 9(x^2+1)^8 \cdot 2x + (x^2+1)^9 \cdot 20$

$= 360x^2(x^2+1)^8 + 20(x^2+1)^9$

$= 20(x^2+1)^8 \boxed{18x^2 + x^2 + 1}$

$= 20(x^2+1)^8(19x^2+1)$

7a) Critical value

1. Function is defined
2. Deriv is 0 or undefined.

b) Point of inflection

1. Function is defined
2. 2nd deriv = 0 or undef.
3. Concavity changes.

8. $f(x) = x^3 - 12x^2 - 60x + 36$

$f'(x) = 3x^2 - 24x - 60$

$= 3(x^2 - 8x - 20)$

$3(x-10)(x+2) = 0$

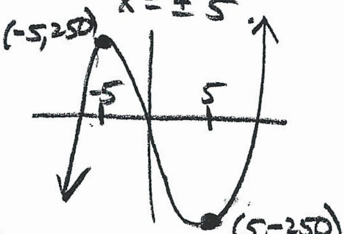
$x = 10 \quad x = -2$

9. $f(x) = x^3 - 75x$

$f'(x) = 3x^2 - 75 = 0$

$3(x^2 - 25) = 0$

$x = \pm 5$



x	-5	5
f	250	-250

f' + + 0 - - - - 0 + + + +

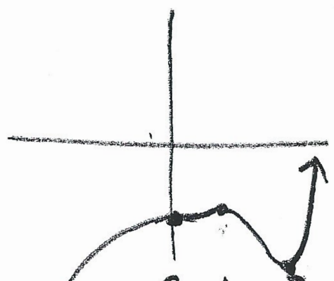
10. $f(x) = x^5 - 5x^4 + 5x^3 - 23$

$f'(x) = 5x^4 - 20x^3 + 15x^2 = 0$

$5x^2(x^2 - 4x + 3) = 0$

$5x^2(x-3)(x-1) = 0$

$x = 0 \quad x = 3 \quad x = 1$

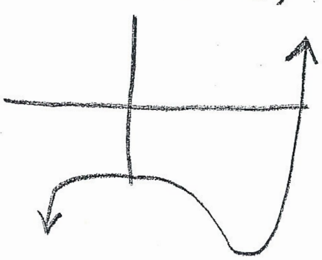


x	0	1	3
f	-23	-22	-50

f' + + + 0 + 0 - 0 +

WINDOW: x: -2, 5

y: -55, 50



11. $f(x) = x^4 - 8x^3 + 18x^2 + 2$

$f'(x) = 4x^3 - 24x^2 + 36x$

$f''(x) = 12x^2 - 48x + 36$

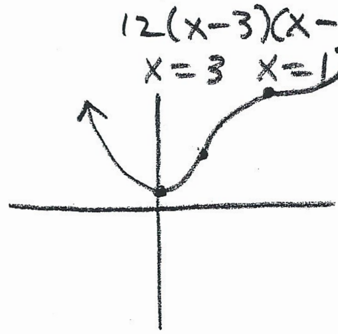
$f'(x) = 4x(x^2 - 6x + 9)$

$x = 0 \quad x = 3$

$f''(x) = 12(x^2 - 4x + 3)$

$12(x-3)(x-1)$

$x = 3 \quad x = 1$



x	0	1	3
f	2	13	25

f' - - - 0 + + 0 +

f'' + + + + 0 - 0 +

Critical points: (0,2) (3,29)

Point of inflection: (1,13) (3,29)

12. $f'(x) = 3x^2 - 12x + 9 = 0$

$= 3(x^2 - 4x + 3) = 0$

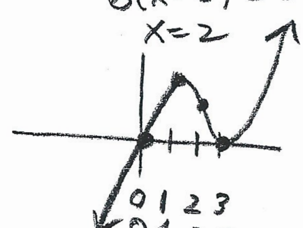
$(x-3)(x-1) = 0$

$x = 3 \quad x = 1$

$f''(x) = 6x - 12 = 0$

$6(x-2) = 0$

$x = 2$



x	0	1	2	3
f	0	4	2	0

f' + + + + 0 - 0 + + +

f'' - - - - - 0 + + + +

b) $x = 1, x = 3$ c) $(2,2)$

d) Down: $(-\infty, 2)$ Up: $(2, \infty)$

13. $f(x) = x^3 - 12x$

$f'(x) = 3x^2 - 12 = 0$

$3(x^2 - 4) = 0$

$x = 2,\ x = -2$

$f(-3) = 9$

$f(3) = -9$

$f(2) = -16$ Minimum

$f(-2) = 16$ Maximum

14 a) Yes.

b) The function must be continuous. It must be a closed interval.

15. $E(x) = -.015x^2 + 1.14x + 8.3$

$E'(x) = -.03x + 1.14 = 0$

$\frac{1.14}{.03} = \frac{.03x}{.03}$

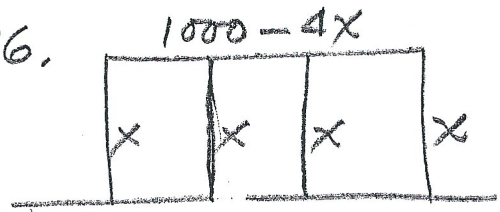
$38 = x$

$E(38) = 29.96$

$E(20) = 25.1$

$E(60) = 22.7$

16.



Area = L·W

$= x(1000 - 4x)$

$= 1000x - 4x^2$

$A' = 1000 - 8x = 0$

$x = \frac{1000}{8} = 125$ yd

Each enclosure is

125 yd × $\frac{500}{3}$ yd.