MAC 2233 EXAM 2C Dr. Rapalje NAME ____________________________
Sections 2.4 – 3.3

Show all work on separate paper. Turn in this test and ALL worksheets.

1. If $f(x) = 12\sqrt[3]{x}$, find f ′ (x) and f ′(-8).

2. If $f(x) = \dfrac{x^6 - 1}{x^6 + 1}$, find f ′ (x) . (Give answer in factored form!)

3. If $f(x) = x^4 - x^2 + 4 - \dfrac{2}{x} + \dfrac{1}{2x^3}$, find f ′ ($x$) , f ″ ($x$), f ‴ ($x$), f $^{(4)}$ (x).

4. If $f(x) = (2x-4)^5$, find f ″ (x) and f ″ (3).

5. A company's cost function is $C(x) = \sqrt{4x^2 + 900}$ where c is the number of units produced. Find the marginal cost function and evaluate it at x = 20.

6. A bullet is fired straight up from the ground, its height t seconds after it is fired will be $s(t) = -16t^2 + 128t$ feet for $0 \le t \le 80$.
   a) Find the velocity function.
   b) Find the time when the bullet will be at its maximum height.
   c) Find the maximum height the bullet will reach.

7. If $f(x) = x^3(3x+7)^4$, find f ′ (x). Give answer in factored form.

8a) What is a critical value of a function f?
 b) What is a point of inflection of a function f?

9. Find all critical values of $f(x) = x^4 + 4x^3 - 20x^2 - 12$.

10. Given a function $f(x) = x^4 - 4x^3 + 15$, find the **first derivative**, make a sign diagram for the derivative, find and plot all critical points, and sketch the graph.

11. Given a function $f(x)=\frac{x^2}{x-3}$, find the **first derivative**, make a sign diagram for the derivative, plot all critical points, and sketch the graph.

12. Given a function $f(x)=x^4+8x^3+18x^2+8$, find the first derivative and second derivatives, find and plot all critical points and points of inflection, make a sign diagram for the first and second derivatives, and sketch the graph.

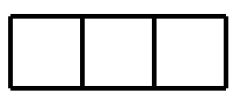
13a) List all "possible" absolute extreme values (maximum and minimum values) of $f(x)=6x^2-x^3$ on the interval [-1,7].

b) Find THE maximum and minimum values for the given function on the given interval.

c) According to the authors of the textbook, "to find the absolute extreme values, we need to consider only __________ and _________." [Hint: two categories of points from 13a) and 13b).]

14. If the amount of a drug in a person's blood after t hours is given by $f(t)=\frac{t}{t^2+9}$, when will the drug concentration be the greatest?

15. A homeowner wants to enclose three adjacent rectangular pens of equal size, as in the diagram below. What is the largest total area that can be enclosed (and the dimensions of each pen) using only 240 feet of fence?



MAC 2233 EXAM 2C Solutions Dr. Rapalje

1. $f(x) = 12\sqrt[3]{x}$

$f(x) = 12x^{1/3}$

$f'(x) = 4x^{-2/3}$ or $\frac{4}{x^{2/3}}$

$f'(-8) = 4(-8)^{-2/3}$

$= 4(\sqrt[3]{-8})^{-2} = 4 \cdot \frac{1}{4} = 1$

2. $f(x) = \frac{x^6-1}{x^6+1}$

$f'(x) = \frac{(x^6+1)\cdot 6x^5 - (x^6-1)6x^5}{(x^6+1)^2}$

$= \frac{6x^5[\cancel{x^6}+1-\cancel{x^6}+1]}{(x^6+1)^2}$

$= \frac{12x^5}{(x^6+1)^2}$

3. $f(x) = x^4 - x^2 + 4 - 2x^{-1} + \frac{1}{2}x^{-3}$

$f'(x) = 4x^3 - 2x + 2x^{-2} - \frac{3}{2}x^{-4}$

$f''(x) = 12x^2 - 2 - 4x^{-3} + 6x^{-5}$

$f'''(x) = 24x + 12x^{-4} - 30x^{-6}$

$f^{IV}(x) = 24 - 48x^{-5} + 180x^{-7}$

4. $f(x) = (2x-4)^5$

$f'(x) = 5(2x-4)^4 \cdot 2$

$f'(x) = 10(2x-4)^4$

$f''(x) = 40(2x-4)^3 \cdot 2$

$f''(x) = 80(2x-4)^3$

$f''(3) = 80(6-4)^3$

$f''(3) = 80 \cdot 2^3 = 640$

5. $C(x) = \sqrt{4x^2+900}$

$MC(x) = C'(x)$

$= \frac{1}{2}(4x^2+900)^{-1/2} \cdot 8x$

$= \frac{4x}{\sqrt{4x^2+900}}$

$MC(20) = \frac{4 \cdot 20}{\sqrt{4 \cdot 400 + 900}}$

$= \frac{80}{\sqrt{2500}} = \frac{80}{50} = \frac{8}{5}$

6. $s(t) = -16t^2 + 128t$

a) $v(t) = -32t + 128$

b) Max height when $v(t) = 0$

$-32t + 128 = 0$

$128 = 32t$

$t = 4$ sec.

c) Max height is

$s(4) = -16 \cdot 4^2 + 128 \cdot 4$

$= -256 + 512 = 256$ ft

7. $f(x) = x^3(3x+7)^4$

$f'(x) = x^3 \cdot 4(3x+7)^3 \cdot 3 + (3x+7)^4 \cdot 3x^2$

$f'(x) = 12x^3(3x+7)^3 + 3x^2(3x+7)^4$

$= 3x^2(3x+7)^3[4x + (3x+7)]$

$= 3x^2(3x+7)^3 \cdot (7x+7)$

$= 21x^2(x+1)(3x+7)^3$

8a) A critical value of f is a value of x in the domain of f where $f'(x) = 0$ or is undefined.

b) A point of inflection is a value of x in the domain of f where $f''(x)$ is zero or undefined and the concavity changes.

9. $f(x) = x^4 + 4x^3 - 20x^2 - 12$

$f'(x) = 4x^3 + 12x^2 - 40x = 0$

$4x(x^2 + 3x - 10) = 0$

$4x(x+5)(x-2) = 0$

$x=0 \quad x=-5 \quad x=2$

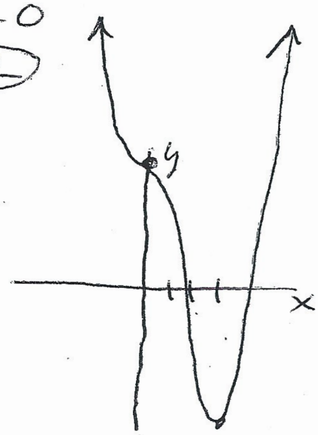
10. $f(x) = x^4 - 4x^3 + 15$

$f'(x) = 4x^3 - 12x^2 = 0$

$4x^2(x-3) = 0$

Critical Points: $x=0 \quad x=3$

$f(0) = 15 \quad f(3) = -12$



x	0		3
f	15		-12
f'	$-$ 0	$-$	0 +

11. $f(x) = \frac{x^2}{x-3}$

$f'(x) = \frac{(x-3)2x - x^2 \cdot 1}{(x-3)^2}$

$= \frac{2x^2 - 6x - x^2}{(x-3)^2}$

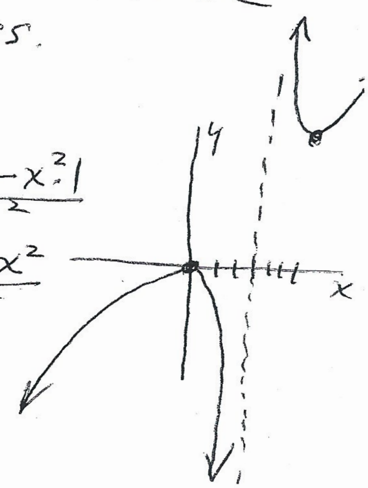
$= \frac{x^2 - 6x}{(x-3)^2}$

$= \frac{x(x-6)}{(x-3)^2}$

Critical Points
$x=0 \quad x=6$

$f(0)=0 \quad f(6)=12$

$x=3$ Asymptote
Not a critical pt!



x	0	3	6
f	0	undef	12
f'	+ 0 $-$	undef	$-$ 0 +

12. $f(x) = x^4 + 8x^3 + 18x^2 + 8$

$f'(x) = 4x^3 + 24x^2 + 36x = 0$

$4x(x^2 + 6x + 9) = 0$

$4x(x+3)^2 = 0$

$x = 0 \quad x = -3$ Critical Pts.

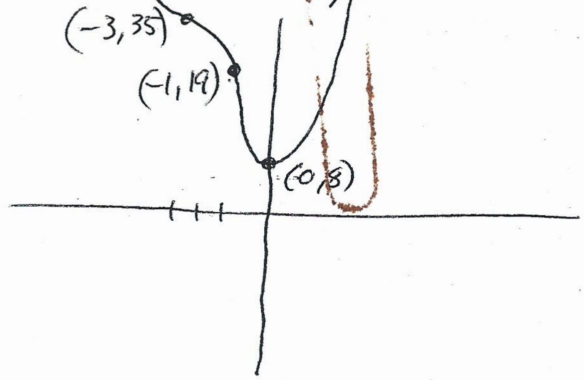
$f''(x) = 12x^2 + 48x + 36 = 0$

$12(x^2 + 4x + 3) = 0$

$12(x+3)(x+1) = 0$

Possible Pts of Infl: $x = -3 \quad x = -1$

(If concavity changes!)



x	-3	-1	0
f	35	19	8
f'	$-\ 0\ -$		$0\ +$
f''	$+\ 0$	$-\ 0$	$+$

13. $f(x) = -6x^2 - x^3$ on $[-1, 7]$

$f'(x) = -12x - 3x^2$

$-3x(4 + x) = 0$

$x = 0 \quad x = -4$ (Not in the interval!)

a) Possible Max or Min

Endpt $x = -1 \quad f(-1) = -5$

Endpt $x = 7 \quad f(7) = -637$

C.N. $x = 0 \quad f(0) = 0$

b) THE MAX $= (0, 0)$

THE MIN $= (7, -637)$

c) endpoints and critical numbers

$f(x) = 6x^2 - x^3$ on $[-1, 7]$

$f'(x) = 12x - 3x^2$

$3x(4 - x) = 0$

$x = 0 \quad x = 4$

a) Possible Max or Min.

Endpt $x = -1 \quad f(-1) = 7$

Endpt $x = 7 \quad f(7) = -49$

C.N. $x = 0 \quad f(0) = 0$

C.N. $x = 4 \quad f(4) = 32$

b) THE MAX $= (4, 32)$

THE MIN $= (7, -49)$

14. $f(t) = \dfrac{t}{t^2 + 9}$

$f'(t) = \dfrac{(t^2+9)\cdot 1 - t \cdot 2t}{(t^2+9)^2}$

$= \dfrac{t^2 + 9 - 2t^2}{(t^2+9)^2}$

$= \dfrac{9 - t^2}{(t^2+9)^2} = 0$

$9 - t^2 = 0$

$9 = t^2$

$t = \pm 3$

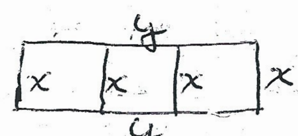
$t = 3$ hrs

$t=0 \qquad t=3$

$f'(t) \quad + \quad 0 \quad -$

Incr. MAX Decreas.

15.



$P = 240$

$240 = 4x + 2y$

$120 = 2x + y$

$y = 120 - 2x$

$A = xy$

$A = x(120 - 2x)$

$A = 120x - 2x^2$

$A' = 120 - 4x = 0$

$120 = 4x$

$x = 30$ ft.

$y = 120 - 2(30) = 60$ ft

MAX Area $= 30' \times 60'$

$= 1800$ sq ft