MAC 2233 EXAM 2D Dr. Rapalje NAME _____
Sections 2.4 – 3.3

Show all work on separate paper. Turn in this test and ALL worksheets.

1. If $f(x)=\frac{12}{x^2}$, find $f'(x)$ and $f'(-2)$.

2. If $f(x)=\frac{x^2}{x^2+4}$, find $f'(x)$. (Give answer in factored form!)

3. If $f(x)=x^6-x^3+4x-5+\frac{2}{x}+\frac{1}{3x^3}$, find $f'(x)$, $f''(x)$, $f'''(x)$, $f^{(4)}(x)$.

4. If $f(x)=(x^2-4)^5$, find $f''(x)$ and $f''(2)$.

5. It costs a book publisher \$12 to produce each book, and fixed costs are \$1500.
   a) Find the company's cost function for producing x books.
   b) Find the company's average cost function.
   c) Find the marginal average cost function.
   d) Find the marginal average cost function at x = 100. What does this mean?

6. A bullet is fired straight up from the ground, its height t seconds after it is fired will be $s(t)=-16t^2+1280t$ feet.
   a) Find the velocity function.
   b) Find the time when the bullet will be at its maximum height.
   c) Find the maximum height the bullet will reach.
   d) When will the bullet hit the ground?
   e) Find the velocity of the bullet when it strikes the ground.

7a) What is a critical value of a function f?
 b) What is a point of inflection of a function f?

8. Find all critical values of $f(x)=(x^2+6x-7)^2$.

9. Given a function $f(x)=x^4-4x^3-8x^2+64$, find the **first derivative**, make a sign diagram for the derivative, find and plot all critical points, and sketch the graph.

10. Given a function $f(x) = x^4 - 20x^3$, find the first derivative and second derivatives, find and plot all critical points and points of inflection, make a sign diagram for the first and second derivatives, and sketch the graph.

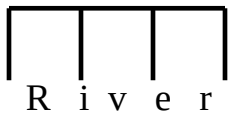
11a) List all "possible" absolute extreme values (maximum and minimum values) of $f(x) = x^3 - 5x^2$ on the interval [-1,6].

b) Find THE maximum and minimum values for the given function on the given interval.

c) According to the authors of the textbook, "to find the absolute extreme values, we need to consider only _____ and _____."

12. A bicycle shop finds that it costs $70 to manufacture each bicycle, and fixed costs are $100 per day. The price function is p(x) = 270-10x where p is the total price at which x bikes will be sold.
   a) Find the cost function c(x) at which x bikes can be produced.
   b) Find the revenue function R(x).
   c) Find the profit function P(x).
   d) How many bikes should be produced each day to maximize profit?
   e) At what price should the bikes be sold to obtain the maximum profit?
   f) What is the maximum profit?

13. A farmer wants to make three identical enclosures along a straight river, as in the diagram below. If he has 1200 feet of fence, and if the sides along the river need no fence, what should be the dimensions if the total area is to be maximized?



MAC 2233 EXAM 2D Solutions — Dr. Rapalje

1. $f(x) = 12x^{-2}$

(8) $f'(x) = -24x^{-3} = \frac{-24}{x^3}$

$f'(2) = \frac{-24}{8} = -3$

2. $f(x) = \frac{x^2}{x^2+4}$

(6) $f'(x) = \frac{(x^2+4)(2x) - x^2(2x)}{(x^2+4)^2}$

$= \frac{2x^3+8x-2x^3}{(x^2+4)^2}$

$= \frac{8x}{(x^2+4)^2}$

3. $f(x) = x^6 - x^3 + 4x - 5 + 2x^{-1} + \frac{1}{3}x^{-3}$

(8) $f'(x) = 6x^5 - 3x^2 + 4 - 2x^{-2} - x^{-4}$

$f''(x) = 30x^4 - 6x + 4x^{-3} + 4x^{-5}$

$f'''(x) = 120x^3 - 6 - 12x^{-4} - 20x^{-6}$

$f^{IV}(x) = 360x^2 + 48x^{-5} + 120x^{-7}$

4. $f(x) = (x^2-4)^5$

$f'(x) = 5(x^2-4)^4 \cdot 2x$

(8) $f'(x) = 10x(x^2-4)^4$

$f''(x) = 10x \cdot 4(x^2-4)^3(2x) + (x^2-4)^4 \cdot 10$

$= 10(x^2-4)^3(8x^2 + x^2 - 4)$

$f''(x) = 10(x^2-4)^3(9x^2-4)$

$f''(2) = 0$

5a) $C(x) = 12x + 1500$

b) $AVC(x) = \frac{12x+1500}{x} = 12 + 1500x^{-1}$

(8) c) $AVC'(x) = -1500x^{-2}$

d) $AVC'(100) = -1500\frac{1}{100^2} = -.15$.

For each additional book produced after the 100th book, the average cost to produce the books decreases by 15¢.

6. $s(t) = -16t^2 + 1280t$

a) $v(t) = -32t + 1280$

b) $v(t) = 0$ at max height

$-32t + 1280 = 0$

(10) $1280 = 32t$

$t = 40$ sec.

c) $s(40) = -16(40)^2 + 1280(40)$

$s(40) = 25600$ ft

d) $s(t) = -16t^2 + 1280t = 0$

$-16t(t-80) = 0$

Ground: $t = 0$ $t = 80$ sec.

e) $V(80) = -32(80) + 1280$

$= -1280$ ft/sec

(Same as initial velocity!)

7a) Critical value of f is a value of x within the domain of f at which $f'(x) = 0$ or $f'(x) = \infty$ (undefined)

(6) b) Point of inflection is a point where the function is defined, $f''(x) = 0$ or ∞, and the concavity changes.

8. $f(x) = (x^2+6x-7)^2$

(6) $f'(x) = 2(x^2+6x-7)(2x+6)$

$= 2(x+7)(x-1)2(x+3)$

$x = -7$ $x = 1$, $x = -3$

9. $f(x) = x^4 - 4x^3 - 8x^2 + 64$

$f'(x) = 4x^3 - 12x^2 - 16x = 0$

(8) $4x(x^2-3x-4) = 0$

$4x(x-4)(x+1) = 0$

$x = 0$ $x = 4$ $x = -1$

x	-1	0	4
f	61	64	-64
f'	$-$	$0+$	$0-$	$0+$

10. $f(x) = x^4 - 20x^3$

$f'(x) = 4x^3 - 60x^2 = 0$

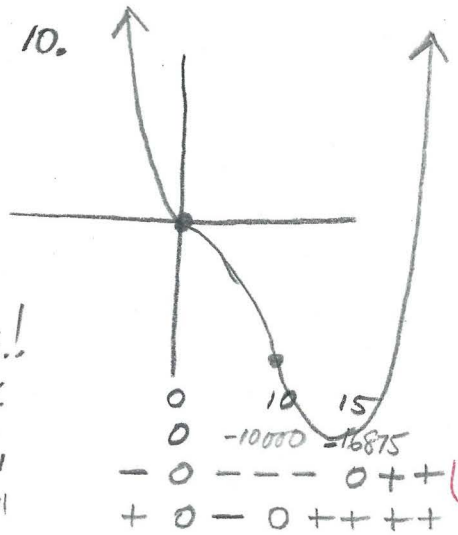
(8) $4x^2(x-15) = 0$

$x = 0$ $x = 15$ c.n.

$f''(x) = 12x^2 - 120x = 0$

$12x(x-10) = 0$ These are p.i.!

$x = 0$ $x = 10$ possible p.i.



x	0	10	15
f	0	-10000	-16875
f'	$-$	$0-$	$--$	$0+$
f''	$+$	$0-$	$0+$	$++$

(10)

11. $f(x) = x^3 - 5x^2$ $[-1, 6]$

(8) $f'(x) = 3x^2 - 10x = 0$

a) $x(3x-10) = 0$

$x = 0$ $x = \frac{10}{3}$ $x = -1$, $x = 6$

b) $f(-1) = -6$

$f(6) = 36$ Maximum

$f(0) = 0$

$f(\frac{10}{3}) = \frac{1000}{27} - \frac{500}{9} = \frac{-500}{27}$ Minimum

11c) Critical numbers and end points.

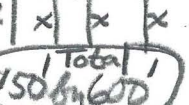
13. $A = x(1200-4x)$

(8) $= 1200x - 4x^2$

$A' = 1200 - 8x = 0$

$1200 = 8x$

$x = 150$ ft each



12a) $C(x) = 70x + 100$

b) $R(x) = \text{Price} \times \#\text{ Bikes}$

$= (270 - 10x) \cdot x$

(10) c) $P(x) = R(x) - C(x)$

$= 270x - 10x^2 - 70x - 100$

$= -10x^2 + 200x - 100$

d) $P'(x) = -20x + 200 = 0$

$200 = 20x$

$10 = x$

e) $P(x) = 270 - 10x = \$170$ (Price)

f) $P(x) = -10x^2 + 200x - 100 = \$900$ (Profit)