MAC 2233 EXAM 3C **Dr. Rapalje** **NAME** ______________________

Sections 3.4 – 4.3

Show all work on separate paper. Turn in ALL worksheets.

1. Use implicit differentiation to find $\frac{dy}{dx}$: $y^3 = 4x^2 - 3y + 6$.

2. Find $\frac{dy}{dx}$ evaluated at $x = 5$, $y = 3$: $x^2 - y^2 = xy + 1$.

3. The hailstone is forming in the clouds so that its radius is growing at the rate of 2 millimeters per minute. How fast is the volume changing when the radius is 6 millimeters? [Hint: $V = \frac{4}{3}\pi r^3$].

4. Find the value of \$20,000 if it is invested for 20 years at 5% interest per year compounded:

   a) annually   b) semiannually

5. Find the value of \$20,000 if it is invested for 20 years at 5% interest per year compounded:

   a) daily (365 days)   b) continuously

6. How much would you have to invest now at 5% interest per year compounded continuously in order to have \$1,000,000 when you retire in

   a) 5 years   b) 30 years?

7. A bank offers 5% interest per year compounded continuously. How long will it take an investment to

   a) double   b) triple

8. Simplify the following expressions

a) $\ln e^{7y}$ b) $\log_5 \frac{1}{125}$ c) $\ln \frac{1}{\sqrt[5]{e}}$ d) $\ln x^2 - 9\ln x$

9. Given $f(x) = x^2 \ln x - x^2$, find

a) $f'(x)$ b) $f'(e)$ c) $f''(x)$ d) $f''(e)$

10. Find $f'(x)$ for each of the following:

a) $f(x) = \ln\left(\frac{e^x}{x^2}\right)$ b) $f(x) = x^3 e^{4x}$

[Hint: Simplify f(x) first!]

11. Given $f(x) = e^{(x^4 + 4x)}$, find

a) $f'(x)$ b) $f''(x)$

12. Manufacturing motorcycles costs $200 each to produce, and that fixed costs are $1500. The price function is p(x) = 600 − 5x, where p is the price in dollars at which exactly x motorbikes will be sold.

a) Find the profit function P(x).
b) How many motorcycles should be produced to maximize profit?
c) Find the maximum profit.

13. Given that $y = e^x$. By taking the *ln* of both sides of the equation, and by using implicit differentiation, show that $y' = e^x$.

14. A computer manufacturer can sell 1500 personal computers per month at a price of $3000 each. The manager estimates that for each $200 price reduction he will sell 300 more each month.

a) If *x* represents the number of $200 price reductions, express the price p and the quantity q as functions of *x*.
b) Find the **price** of the computers and the **quantity** produced that will result in the maximum **revenue**. (Show work and/or explain your method!
c) Find the **maximum revenue**.

MAC 2233 EXAM 3C Solutions

1. $y^3 = 4x^2 - 3y + 6$

$3y^2\frac{dy}{dx} = 8x - 3\frac{dy}{dx}$

$3y^2\frac{dy}{dx} + 3\frac{dy}{dx} = 8x$

$\frac{dy}{dx}(3y^2+3) = 8x$

$\frac{dy}{dx} = \frac{8x}{3(y^2+1)}$

2. $x^2 - y^2 = xy + 1$ Product Rule!

$2x - 2y\frac{dy}{dx} = x\frac{dy}{dx} + y\cdot 1$

$2x - y = 2y\frac{dy}{dx} + x\frac{dy}{dx}$

$2x - y = \frac{dy}{dx}(2y+x)$

$\frac{dy}{dx} = \frac{2x-y}{2y+x}\Big|_{(5,3)}$

$= \frac{2\cdot 5 - 3}{2\cdot 3 + 5} = \frac{7}{11}$

p.259 #50

3. $\frac{dr}{dt} = 2$ mm/min

Find $\frac{dV}{dt}$.

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = \cancel{3}\frac{4}{\cancel{3}}\pi r^2\frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$

$= 4\pi\cdot(6\text{ mm})^2\cdot\frac{2\text{ mm}}{\text{min}}$

$= 288\pi\ \frac{\text{mm}^3}{\text{min.}}$

4. $A = P(1+r)^{nt}$

a) $A = 20{,}000(1+.05)^{20}$

$= \$53{,}065.95$

b) $A = 20{,}000\left(1+\frac{.05}{2}\right)^{40}$

$= \$53{,}701.28$

5a) $A = 20{,}000\left(1+\frac{.05}{365}\right)^{(20\cdot 365)}$

$= \$54{,}361.91$

b) $A = Pe^{rt}$

$= 20{,}000\,e^{(.05\times 20)}$

$= \$54{,}365.64$

6. $P = \frac{A}{e^{rt}}$

a) $P = \frac{1000000}{e^{(.05\times 5)}}$

$P = \$778{,}800.78$

b) $P = \frac{1000000}{e^{(.05\times 30)}}$

$P = \$223{,}130.16$

7. $A = Pe^{rt}$

a) $2P = Pe^{.05t}$

$2 = e^{.05t}$

$\ln 2 = \ln e^{.05t}$

$\frac{\ln 2}{.05} = \frac{.05t}{.05}$

$t = 13.86$ yrs

b) $3P = Pe^{.05t}$

$\ln 3 = .05t$

$t = \frac{\ln 3}{.05}$

$t = 21.97$ yrs

8a) $\ln e^{7y} = 7y$

b) $\log_5\frac{1}{125} = -3$

c) $\ln\frac{1}{\sqrt[5]{e}}$

$= \ln e^{-1/5} = -1/5$

d) $\ln x^2 - 9\ln x$

$= 2\ln x - 9\ln x$

$= -7\ln x$

17 #34.

9. $f(x) = x^2\ln x - x^2$

a) $f'(x) = x^2\cdot\frac{1}{x} + \ln x\cdot(2x) - 2x$

$= x + 2x\ln x - 2x$

$= 2x\ln x - x$

b) $f'(e) = 2e\ln e - e$

$= 2e - e = e$

c) $f''(x) = 2x\cdot\frac{1}{x} + \ln x(2) - 0$

$= 2 + 2\ln x$

d) $f''(e) = 2 + 2\ln e$

$= 2 + 2 = 4$

10a) $f(x) = \ln\left(\frac{e^x}{x^2}\right) = \ln e^x - \ln x^2$

$= x - 2\ln x$

$f'(x) = 1 - 2\cdot\frac{1}{x} = 1 - \frac{2}{x}$ or $\frac{x-2}{x}$

b) $f(x) = x^3e^{4x}$

$f'(x) = x^3\cdot 4e^{4x} + e^{4x}\cdot 3x^2$

$= 4x^3e^{4x} + 3x^2e^{4x}$

or $x^2e^{4x}(4x+3)$

11. $f(x) = e^{(x^4+4x)}$

a) $f'(x) = e^{(x^4+4x)} \cdot (4x^3+4)$

b) $f''(x) =$ Product Rule

$f''(x) = e^{(x^4+4x)} \cdot (12x^2+1) + (4x^3+4)\, e^{(x^4+4x)} \cdot (4x^3+4)$

or $e^{(x^4+4x)} \left[(4x^3+4)^2 + (12x^2+1)\right]$??

12. $C(x) = 200x + 1500$ Rev = Price per bike × No Bikes

a) Profit = Rev − Cost $= (600-5x)\,x$

$= (600-5x)x - (200x+1500)$

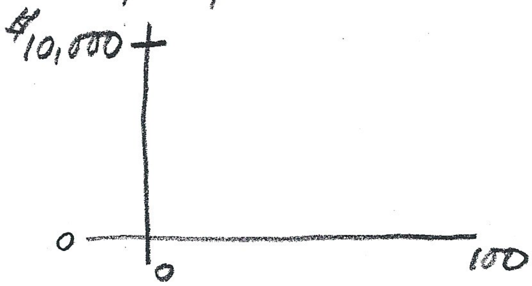
$= 600x - 5x^2 - 200x - 1500$

$= -5x^2 + 400x - 1500$

b) Vertex = maximum profit at $x = -\frac{b}{2a}$ (or other methods!)

$x = \frac{-400}{2(-5)} =$ 40 Bikes

Graphing Calculator:



c) $P(40) = -5x^2 + 400x - 1500$

$= -5 \cdot 40^2 + 400 \cdot 40 - 1500$

$= -8000 + 16000 - 1500$

$=$ \$6500

In class

13. $y = e^x$

$\ln y = \ln e^x$

$\ln y = x$

Take $\frac{d}{dx}$:

$\frac{1}{y}\frac{dy}{dx} = 1$

$y \cdot \frac{1}{y}\frac{dy}{dx} = y \cdot 1$

$\frac{dy}{dx} = y = e^x$

(p 230)

14. Let x = no. of \$200 price reductions.

a) Price = \$3000 − 200x

quantity = 1500 + 300x

b) Revenue = Price × quantity

$R(x) = (3000-200x)(1500+300x)$

$= 4500000 + 900000x - 300000x - 60000x^2$

$= 4500000 + 600000x - 60000x^2$

$R'(x) = 600000 - 120000x = 0$

$600000 = 120000x$

$x = 5$ Price Reductions.

Price = \$2000 Quantity = 3000

c) Revenue = 2000 × 3000 = \$6,000,000