MAC 2233 EXAM 4B **Dr. Rapalje** **NAME** ______________________________
Chapter 5

Show all work on separate paper. Turn in ALL worksheets.

1. $\int (4x^3 - 6x^2 + 6x - 12 + x^{-1} + x^{-2})\, dx$

2. a) $\int \frac{1}{x^2}\, dx$ b) $\int \frac{1}{\sqrt{x}}\, dx$ 3. $\int (e^{3x} + \frac{1}{e^{3x}})\, dx$

4. a) $\int \frac{dx}{3x}$ b) $\int 24e^{-\frac{2x}{3}}\, dx$

5. Given $\int_0^1 (6x^2 - 4e^2)\, dx$
   a) Find the exact value using calculus.
   b) Find the decimal approximation (using the calculator!)

6. Find the area under the curve $f(x) = 9 - 3\sqrt{x}$ from $x = 0$ to $x = 9$.

7. Find the area between the curves $y = 3x - x^2$ and $y = 4 - 2x$.

8. Find the average value of the function $f(x) = \sqrt[3]{x}$ on [0, 8].

In 9 – 12, find each integral.

9. $\int \frac{x^4 dx}{x^5 + 4}$ 10. $\int e^{-x^3} x^2 dx$ 11. $\int (x^4+4)^5\, x^3 dx$ 12. $\int \frac{(\ln x)^2}{x} dx$

13. Evaluate: $\int_0^3 \sqrt{x^2 + 9}\, dx$ a) using calculus b) using calculator.

14. A company's marginal cost function is $0.015\, x^2 - 2x + 80$, where x denotes the number of units produced in one day. The company has fixed costs of $1000 per day. Find the cost function.

15. The rate of change of the temperature of water in an ice cube tray is given by $-12\, e^{-0.2\, t}$ degrees Fahrenheit per hour after t hours. The temperature of the tap water is 70 degrees.
    a) Find a formula for the temperature of the water after t hours.
    b) How long will it take the water to freeze (reach 32 degrees)?

MAC 2233 EXAM 4B Solutions

Dr. Rapalje

1. $\int(4x^3-6x^2+6x-12+x^{-1}+x^{-2})\,dx$

$= x^4-2x^3+3x^2-12x+\ln x-x^{-1}+c$

2a) $\int \frac{1}{x^2}dx$

$=\int x^{-2}dx$

$=-x^{-1}+c$

$=-\frac{1}{x}+c$

b) $\int \frac{1}{\sqrt{x}}dx$

$=\int x^{-1/2}dx$

$=\frac{2x^{1/2}}{1}+c$

3. $\int(e^{3x}+e^{-3x})\,dx$

$=\frac{1}{3}e^{3x}-\frac{1}{3}e^{-3x}+c$

4a) $\frac{1}{3}\int\frac{dx}{x}$

$=\frac{1}{3}\ln x+c$

b) $\int 24e^{-\frac{2}{3}x}dx$

$24\cdot -\frac{3}{2}e^{-2/3x}+c$

$-36e^{-2/3x}+c$

5a) $\int_0^1(6x^2-4e^{2x})\,dx$

$=2x^3-2e^{2x}\Big|_0^1$

$=(2-2e^2)-(0-2)$

$=4-2e^2\approx -10.778$

b) fnInt $(6x^2-4e^{(2x)}, x, 0, 1)$

also -10.778

6. $\int_0^9(9-3\sqrt{x})\,dx$

$=9x-3\frac{2}{3}x^{3/2}\Big|_0^9$

$=81-2\cdot 27-0$

$=81-54$

$=27$

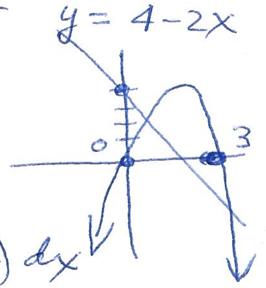
7. $y=3x-x^2 \quad y=4-2x$

$3x-x^2=4-2x$

$0=x^2-5x+4$

$0=(x-4)(x-1)$

$x=4 \quad x=1$



$\int_1^4(\text{Upper}-\text{Lower})\,dx$

$\int_1^4[(3x-x^2)-(4-2x)]\,dx$

$\int_1^4(-x^2+5x-4)\,dx = 4.5$ (Calculator!)

8. Av $f(x)=\frac{1}{b-a}\int_a^b f(x)\,dx$

$=\frac{1}{8-0}\int_0^8 x^{1/3}dx$

$=\frac{1}{8}\frac{3}{4}x^{4/3}\Big|_0^8$

$=\frac{3}{32}(8^{4/3}-0)$

$=\frac{3}{32}\cdot 16=\frac{3}{2}$

9. $\int\frac{x^4}{x^5+4}dx$ Let $u=x^5+4$

$du=5x^4dx$

$\frac{du}{5}=x^4dx$

$\int\frac{\frac{du}{5}}{u}$

$=\frac{1}{5}\int\frac{du}{u}$

$=\frac{1}{5}\ln u$

$=\frac{1}{5}\ln(x^5+4)+c$

10. $\int e^{-x^3}x^2dx$ Let $u=-x^3$

$du=-3x^2dx$

$\frac{du}{-3}=x^2dx$

$=\int e^u\frac{du}{-3}$

$=-\frac{1}{3}e^u+c$

$=-\frac{1}{3}e^{-x^3}+c$

11. $\int(x^4+4)^5x^3dx$ Let $u=x^4+4$

$du=4x^3dx$

$\frac{du}{4}=x^3dx$

$\int u^5\frac{du}{4}$

$\frac{1}{4}\cdot\frac{u^6}{6}+c$

$\frac{1}{24}(x^4+4)^6+c$

12. $\int\frac{(\ln x)^2}{x}dx$ Let $u=\ln x$

$du=\frac{1}{x}dx$

$\int u^2du$

$\frac{u^3}{3}+c$

$\frac{(\ln x)^3}{3}+c$

13a) $\int_0^3\sqrt{x^2+9}\,x\,dx$ Let $u=x^2+9$

$du=2x\,dx$

$\int u^{1/2}\frac{du}{2}$

$\frac{1}{2}\frac{2u^{3/2}}{3}+c$

$\frac{1}{3}(x^2+9)^{3/2}\Big|_0^3$

$\frac{1}{3}\left[18^{3/2}-9^{3/2}\right]$

≈ 16.4558

b) fnInt ≈ 16.4558

14. $MC=.015x^2-2x+80$

$C(x)=\int(.015x^2-2x+80)\,dx$

$=.005x^3-x^2+80x+C$

$C(0)=0-0+0+C=1000$

$C(x)=.005x^3-x^2+80x+1000$

15. $T'(t)=-12e^{-0.2t}$

$T(t)=\int -12e^{-0.2t}dt$

$=\frac{-12e^{-0.2t}}{-.2}+c$

$=60e^{-0.2t}+c$

$T(0)=60\cdot 1+c=70$

$c=10$

a) $T(t)=60e^{-0.2t}+10$

b) Find t when $T(t)=32°$

$32=60e^{-.2t}+10$

$22=60e^{-.2t}$

$t=\frac{\ln\frac{22}{60}}{-.2}\approx 5.02$ hrs