

Show all work on separate paper. Turn in ALL worksheets.

1. $\int (24x^2 - 8x + 12 + x^{-3}) dx$

2. $\int \left(6\sqrt{x} - 5 + \frac{6}{\sqrt[3]{x}} \right) dx$

3. $\int \left(e^{3x} - \frac{1}{e^{3x}} \right) dx$

4. $\int \left(\frac{1}{x^2} + \frac{1}{x} + 1 \right) dx$

5. $\int \frac{3x^3 - 5x^2 + 6x - 7}{x} dx$

6. Given $\int_0^2 (x^3 + 4e^{-2x}) dx$

a) Find the exact value using calculus.

b) Find the decimal approximation (using the calculator!)

7. Find the area under the curve $f(x) = 3x^2 - 4x + 5$ from $x = 2$ to $x = 5$.
Set up the integral and solve using calculus. Check with the calculator.

8. Find the area between the curves $y = 12x - 3x^2$ and $y = 6x - 24$.

9. Find the average value of the function $f(x) = e^{\frac{1}{2}x}$ on $[0, 2]$.
Give the exact value.

In 10 – 13, find each integral.

10. $\int (x^3 + 5)^5 x^2 dx$

11. $\int e^{x^2} x dx$

12. $\int \frac{x^3 dx}{x^4 + 4}$

13. $\int \frac{(\ln x)^3}{x} dx$

14. Evaluate: $\int_0^3 x\sqrt{x^2 + 9} dx$

a) using calculus (exact value)

b) using calculator (decimal approx)

c) $\int_0^4 x\sqrt{x^2 + 9} dx$ either method.

15. World consumption of aluminum is running at the **rate** of $72 e^{0.06t}$ million tons per year where t is the number of years since year 2000. Find a formula for the total amount of aluminum consumed within t years of 2000. If this rate continues, how long will it take to exhaust the known resources of 8500 million tons?

MAC 2233 EXAM 4D Solutions

1. $\int (24x^2 - 8x + 12 + x^{-3}) dx$
 $= 8x^3 - 4x^2 + 12x + \frac{x^{-2}}{-2} + C$
 $= 8x^3 - 4x^2 + 12x - \frac{1}{2x^2} + C$

2. $\int (6\sqrt{x} - 5 + \frac{6}{\sqrt[3]{x}}) dx$
 $= \int (6x^{1/2} - 5 + 6x^{-1/3}) dx$
 $= 6 \cdot \frac{2}{3} x^{3/2} - 5x + 6 \cdot \frac{3}{2} x^{2/3} + C$
 $= 4x^{3/2} - 5x + 9x^{2/3} + C$

3. $\int (e^{3x} - \frac{1}{e^{3x}}) dx$
 $= \int (e^{3x} - e^{-3x}) dx$
 $= \frac{e^{3x}}{3} - \frac{e^{-3x}}{-3} + C$
 $= \frac{1}{3}(e^{3x} + e^{-3x}) + C$

4. $\int (\frac{1}{x^2} + \frac{1}{x} + 1) dx$
 $= \int (x^{-2} + x^{-1} + 1) dx$
 $= \frac{x^{-1}}{-1} + \ln|x| + x + C$
 $= -\frac{1}{x} + \ln|x| + x + C$

5. $\int \frac{3x^3 - 5x^2 + 6x - 7}{x} dx$
 $= \int (3x^2 - 5x + 6 - \frac{7}{x}) dx$
 $= x^3 - \frac{5}{2}x^2 + 6x - 7\ln|x| + C$

6. a) $\int_0^2 (x^3 + 4e^{-2x}) dx$
 $= \frac{x^4}{4} + \frac{4e^{-2x}}{-2} \Big|_0^2$
 $= (\frac{16}{4} - 2e^{-4}) - (0 - 2e^0)$
 $= 4 - 2e^{-4} + 2$

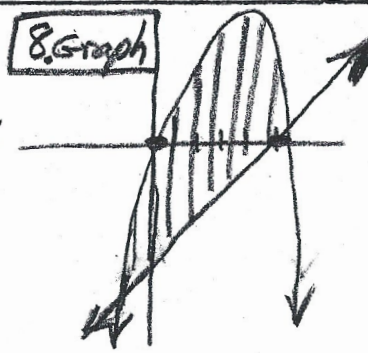
7. $\int_2^5 (3x^2 - 4x + 5) dx$
 $= x^3 - 2x^2 + 5x \Big|_2^5$
 $= (5^3 - 2(5^2) + 5 \cdot 5) - (2^3 - 2 \cdot 2^2 + 5 \cdot 2)$
 $= (125 - 50 + 25) - (8 - 8 + 10)$
 $= 100 - 10 = 90$

8. Upper Lower
 $12x - 3x^2 = 6x - 24$
 $0 = 3x^2 - 6x - 24$
 $0 = 3(x^2 - 2x - 8)$
 $0 = 3(x-4)(x+2)$
 $x=4 \quad x=-2$

$= 4 - 2e^{-4} + 2$
 $= 6 - 2e^{-4}$ EXACT VALUE
 b) ≈ 5.96 (2 ways to calculate!)

9. $A_v = \frac{1}{b-a} \int_a^b f(x) dx$
 $= \frac{1}{2-0} \int_0^2 e^{1/2x} dx$
 $= \frac{1}{2} \cdot \frac{2}{1} e^{1/2x} \Big|_0^2$
 $= e^1 - e^0 = e - 1$
 (≈ 1.72) EXACT!

$\int_{-2}^4 [(12x - 3x^2) - (6x - 24)] dx$
 $= \int_{-2}^4 (3x^2 + 6x + 24) dx$
 $= -x^3 + 3x^2 + 24x \Big|_{-2}^4$
 $= -(4^3) + 3(4^2) + 24(4) - (-(-2)^3 + 3(-2)^2 + 24(-2))$
 $= -64 + 48 + 96 - (8 + 12 - 48)$
 $= 80 - (-28) = 108$



10. $\int (x^3 + 5)^5 x^2 dx$ Let $u = x^3 + 5$
 $du = 3x^2 dx$
 $\frac{du}{3} = x^2 dx$
 $= \int u^5 \frac{du}{3}$
 $= \frac{1}{3} \cdot \frac{u^6}{6} + C$
 $= \frac{1}{18} (x^3 + 5)^6 + C$

11. $\int e^{x^2} x dx$ Let $u = x^2$
 $du = 2x dx$
 $\frac{du}{2} = x dx$
 $= \int e^u \frac{du}{2}$
 $= \frac{1}{2} e^u + C$
 $= \frac{1}{2} e^{x^2} + C$

$$12. \int \frac{x^3 dx}{x^4 + 4} \quad \text{Let } u = x^4 + 4$$

$$du = 4x^3 dx$$

$$\frac{du}{4} = x^3 dx$$

$$= \int \frac{\frac{du}{4}}{u}$$

$$= \frac{1}{4} \int \frac{du}{u} = \frac{1}{4} \ln u + C$$

$$= \frac{1}{4} \ln(x^4 + 4) + C$$

$$14a) \int_0^3 x \sqrt{x^2 + 9} dx \quad \text{Let } u = x^2 + 9$$

$$du = 2x dx$$

$$\frac{du}{2} = x dx$$

$$= \int \sqrt{u} \cdot \frac{du}{2}$$

$$= \frac{1}{2} \cdot \frac{2}{3} u^{3/2}$$

$$= \frac{1}{3} (x^2 + 9)^{3/2} \Big|_0^3$$

$$= \frac{1}{3} (18^{3/2} - 9^{3/2})$$

$$= \frac{1}{3} (18\sqrt{18} - 27)$$

$$= \frac{1}{3} (18 \cdot 3\sqrt{2} - 27)$$

$$= \frac{1}{3} (18\sqrt{2} - 9)$$

EXACT VALUE

b) Calculator (in 2 ways!)

$$\approx 16.46$$

c) $\int_0^4 x \sqrt{x^2 + 9} dx$

(Same integration!)

$$= \frac{1}{3} (x^2 + 9)^{3/2} \Big|_0^4$$

$$= \frac{1}{3} (25^{3/2} - 9^{3/2}) = \frac{1}{3} (125 - 27) = \frac{1}{3} (98)$$

EXACT VALUE

$$= 32.\bar{6}$$

$$13. \int \frac{(\ln x)^3}{x} dx \quad \text{Let } u = \ln x$$

$$du = \frac{1}{x} dx$$

$$= \int u^3 du$$

$$= \frac{u^4}{4} + C = \frac{(\ln x)^4}{4} + C$$

Let y = # million tons consumed

$$15. \text{Rate} = \frac{dy}{dt}$$

$$\frac{dy}{dt} = 72e^{0.06t}$$

$$y = \int 72e^{0.06t} dt$$

$$y = \frac{72}{0.06} e^{0.06t} + C$$

$$y = 1200e^{0.06t} + C$$

At $t=0$ in year 2000,
 y = # million tons consumed = 0

$$0 = 1200e^0 + C$$

$$0 = 1200 + C$$

$$C = -1200$$

$$y = 1200e^{0.06t} - 1200.$$

When will $y = 8500$ (all gone!)?

$$8500 = 1200e^{0.06t} - 1200$$

$$\frac{9700}{1200} = \frac{1200e^{0.06t}}{1200}$$

$$\frac{97}{12} = e^{0.06t} \rightarrow \frac{\ln(\frac{97}{12})}{0.06} = \frac{0.06t}{0.06}$$

$$\ln\left(\frac{97}{12}\right) = \ln e^{0.06t} \rightarrow t = \frac{\ln(\frac{97}{12})}{.06}$$

$$t \approx 34.83 \text{ years}$$