MAC 3233 FINAL EXAM (12) UCF ~~Name~~ Dr. Rapalje (12)

Show all work on separate paper. Turn in all work sheets.
CALCULATORS are allowed, but not required.

In 1–4, find the derivatives:

1. $f(x) = x^3 - 8x^2 + 3x - 2$

2. $f(x) = x^4(3x-2)^3$
(Factor answer!)

3. $f(x) = \dfrac{3x-2}{x^2+4}$

4. $f(x) = \dfrac{10x^2}{\sqrt{x^2+9}}$, find $f'(4)$.

5. Find the equation of the tangent line to $y = x^2 - 4x$ at $x = -1$.

6. The cost to produce 10 units of a product is \$30, and the cost to produce 20 units is \$40. Find the <u>linear</u> cost equation (for cost "c" in terms of units "q").

7. A cost equation is $c = q^2 - 50q + 1000$. Find the <u>marginal</u> cost equation.

8. The <u>demand</u> equation is $p = 3q + 8 - \frac{4}{q} + \frac{2}{q^2}$. Find the equation of <u>marginal revenue</u>.

9. Give all asymptotes for $y = \dfrac{2x^2+8}{x^2-3x-4}$

10. Find: a) $\lim\limits_{x \to \infty} \frac{1}{x}$ b) $\lim\limits_{x \to 0} \frac{1}{x}$.

11–12. For $y = -x^5 - 5x^4 + 200$,
a) find all critical points.
b) when is graph increasing?
c) determine rel max & mins.

13. Given the demand function $p = 400 - 8q$, determine when <u>revenue</u> is increasing.

14. For $y = 6x^4 - 8x^3 + 1$
a) determine possible points of inflection.
b) When is the graph concave down?

15–16. For a demand function $p = \dfrac{50}{\sqrt{q}}$ and cost function $c = .5q + 1000$, a) find the revenue and profit equations.
b) For what value of q will profit be maximized?

17. $\int (3x^2 - x + 5)\,dx$

18. If $y' = -x^2 + 2x$ and $y(2) = 1$, find the equation for y.

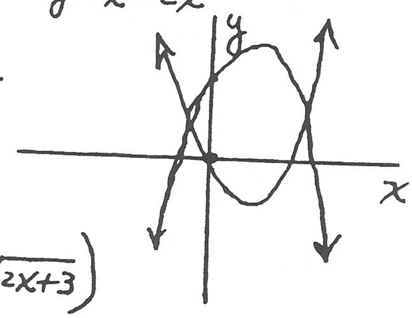
19. If $\frac{dc}{dq} = .09q^2 - 1.2q + 4.5$ is a marginal cost function, with fixed costs 75, then find the cost equation.

20. $\int \frac{x^2}{(x^3+5)^3}$

21. $\int \frac{x^2\,dx}{x^3+5}$

22. $\int_0^1 x^2 \sqrt[3]{7x^3+1}\,dx$

23–24. Find the area between $y = 4x - x^2 + 8$ and $y = x^2 - 2x$



25. Find x: $\log_{10} .01 = x$.

26. Simplify: $\ln e^{4x}$

27. Write as a single log: $\ln 3 + \ln 7 - \ln 5 - 2\ln 2$

28. If $y = \ln\left(x^2\sqrt{2x+3}\right)$ find y'. (Hint: simplify first.)

29. A demand function is given by $q = 100e^{-.04p}$. Find the value of p for which revenue is maximized.

30. $f(x, y) = xe^{xy}$. Find f_y and f_{yx}. Give factored form.

FORMULAS

1. $\frac{d}{dx}(e^x) = e^x \qquad \int e^x\,dx = e^x + c$
2. $\frac{d}{dx}(e^u) = e^u \frac{du}{dx} \qquad \int e^u\,du = e^u + c$
3. $\frac{d}{dx}(a^u) = a^u \ln a \frac{du}{dx} \qquad \int a^u\,du = \frac{a^u}{\ln a} + c$
4. $\frac{d}{dx}(\ln x) = \frac{1}{x} \qquad \int \frac{1}{x}\,dx = \ln|x| + c$
5. $\frac{d}{dx}(\ln u) = \frac{1}{u}\frac{du}{dx} \qquad \int \frac{1}{u}\,du = \ln|u| + c$
6. $\frac{d}{dx}(\log_b u) = \frac{1}{u}\log_b e \frac{du}{dx}$

1. $f(x)=x^3-8x^2+3x-2$

$f'(x)=3x^2-16x+3$

2. $f(x)=x^4(3x-2)^3$

$f'(x)=x^4(3)(3x-2)^2\cdot 3+(3x-2)^3 4x^3$

$=x^3(3x-2)^2[9x+4(3x-2)]$

$=x^3(3x-2)^2(21x-8)$

3. $f(x)=\frac{3x-2}{x^2+4}$

$f'(x)=\frac{(x^2+4)\cdot 3-(3x-2)2x}{(x^2+4)^2}$

$=\frac{3x^2+12-6x^2+4x}{(x^2+4)^2}$

$=\frac{-3x^2+4x+12}{(x^2+4)^2}$

4. $f(x)=\frac{10x^2}{\sqrt{x^2+9}}$

$f'(x)=\frac{\sqrt{x^2+9}(20x)-10x^2\cdot\frac{1}{2}(x^2+9)^{-1/2}2x}{(x^2+9)}$

$f'(4)=\frac{\sqrt{25}\cdot 80-10\cdot 16\cdot\frac{1}{5}\cdot 4}{25}$

$=\frac{400-128}{25}=\frac{272}{25}$

5. $y=x^2-4x$ at $x=-1$

$y'=2x-4$ $\quad y=(-1)^2-4(-1)=1+4=5$

$m=y'(-1)=-2-4=-6$

$y-5=-6(x+1)$

$y=-6x-6+5$

$y=-6x-1$

6. $c=mq+b,\ m=\frac{c_2-c_1}{q_2-q_1}=\frac{40-30}{20-10}=\frac{10}{10}=1$

$\$30=1\cdot 10+b$

$b=20$

$C=q+20$

7. $c=q^2-50q+1000$

$\frac{dc}{dq}=2q-50$

8. $p=3q+8-\frac{4}{q}+\frac{2}{q^2}$

revenue $=pq=3q^2+8q-4+2q^{-1}$

marg. rev $=\frac{dr}{dq}=6q+8-2q^{-2}$

9. $y=\frac{2x^2+8}{x^2-3x-4}$ Asympt.

Vert: $x^2-3x-4=0$

$(x-4)(x+1)=0$

$x=4\quad x=-1$

Horiz: Powers are equal, so $y=\frac{2}{1}$

10a) $\lim_{x\to\infty}\frac{1}{x}=0$

b) $\lim_{x\to 0}\frac{1}{x}=\infty$ or undef.

11–12. $y=-x^5-5x^4+200$

$y'=-5x^4-20x^3=0$

$-5x^3(x+4)=0$

a) $x=0\quad x=-4$

b) $-\ \ +\ \ -$; -4, 0

$(-4,0)$ or $-4<x<0$

c) Rel min at $x=-4$
Rel max at $x=0$

13. $p=400-8q$

rev $=400q-8q^2$

$\frac{dr}{dq}=400-16q>0$

$-16q>-400$

$q<25$

14. $y=6x^4-8x^3+1$

$y'=24x^3-24x^2$

$y''=72x^2-48x=0$

$24x(3x-2)=0$

a) $x=0$, $x=2/3$

b) $24x(3x-2)>0$

0, $2/3$

$(-\infty,0)\cup(2/3,\infty)$

15–16. a) rev $=p\cdot q=\frac{50}{\sqrt{q}}\cdot q=50\sqrt{q}$

Profit $=$ rev $-$ cost

$=50\sqrt{q}-(.5q+1000)$

b) $(\text{Profit})'=25q^{-1/2}-.5=0$

$\frac{25}{\sqrt{q}}=\frac{1}{2}$

$\sqrt{q}=50$

$q=50^2=2500$

17. $\int(3x^2-x+5)dx=x^3-\frac{x^2}{2}+5x+C$

18. $y'=-x^2+2x$

$y=-\frac{x^3}{3}+x^2+C\quad y(2)=1$

$1=-\frac{8}{3}+4+C$

$C=-1/3$

$y=-\frac{x^3}{3}+x^2-\frac{1}{3}$

19. $\frac{dc}{dq}=.09q^2-1.2q+4.5$

cost $=.03q^3-.6q^2+4.5q+C$

If $q=0$, cost $=75$

$75=0+C$

Cost $=.03q^3-.6q^2+4.5q+75$

20. $\int\frac{x^2dx}{(x^3+5)^3}\quad u=x^3+5$

$du=3x^2dx$

$\frac{du}{3}=x^2dx$

$=\frac{1}{3}\int\frac{du}{u^3}$

$=\frac{1}{3}\int u^{-3}du=\frac{1}{3}\frac{u^{-2}}{-2}+C$

$=-\frac{1}{6}\frac{1}{(x^3+5)^2}=-\frac{1}{6(x^3+5)^2}+C$

21. (As in #20, $u=x^3+5$, $du=3x^2dx$)

$\frac{1}{3}\int\frac{du}{u}=\frac{1}{3}\ln u+C$

$=\frac{1}{3}\ln(x^3+5)+C$

22. $\int_0^1 x^2\sqrt[3]{7x^3+1}\,dx$ Let $u=7x^3+1$

$du=21x^2dx$

$\frac{1}{21}\int u^{1/3}du$

$=\frac{1}{21}\frac{3}{4}u^{4/3}=\frac{1}{28}(7x^3+1)^{4/3}\Big|_0^1$

$\frac{1}{28}(8^{4/3}-1^{4/3})=\frac{1}{28}(16-1)=\frac{15}{28}$

23–24. $y = 4x - x^2 + 8$ upper graph.
$y = x^2 - 2x$ Lower graph.

$4x - x^2 + 8 = x^2 - 2x$
$0 = 2x^2 - 6x - 8$
$0 = 2(x^2 - 3x - 4)$
$(x-4)(x+1) = 0$
$x = 4 \quad x = -1$

$$\text{Area} = \int_{-1}^{4} \left[(4x - x^2 + 8) - (x^2 - 2x)\right] dx$$
$$= \int_{-1}^{4} (6x - 2x^2 + 8)\, dx$$
$$= 3x^2 - \frac{2x^3}{3} + 8x \Big|_{-1}^{4}$$
$$= \left(48 - \frac{2}{3} \cdot 64 + 32\right) - \left(3 + \frac{2}{3} - 8\right)$$
$$= 80 - \frac{128}{3} + 5 - \frac{2}{3} = 85 - \frac{130}{3}$$
$$= \frac{255}{3} - \frac{130}{3} = \frac{125}{3} \text{ or } 41.67$$

25. $\log_{10} .01 = x$

$10^x = .01$

$x = -2$

26. $\ln e^{4x} = 4x$

27. $\ln 3 + \ln 7 - \ln 5 - 2 \ln 2$
$= \ln \frac{3 \cdot 7}{5 \cdot 4} = \ln \frac{21}{20}$

28. $y = \ln\left(x^2 \sqrt{2x+3}\right)$
$y = \ln x^2 + \ln \sqrt{2x+3}$
$y = 2 \ln x + \frac{1}{2} \ln(2x+3)$
$y' = 2 \cdot \frac{1}{x} + \frac{1}{\cancel{2}} \cdot \frac{1}{2x+3} \cdot \cancel{2}$
$= \frac{2}{x} + \frac{1}{2x+3}$

29. $q = 100 e^{-.04p}$
revenue $= pq$
$= p \cdot 100 e^{-.04p}$
$(rev)' = 100p\, e^{-.04p}(-.04) + e^{-.04p}\, 100$
$= 100 e^{-.04p}(-.04p + 1) = 0$

$-.04p + 1 = 0$

$.04p = 1$

$4p = 100$

$p = 25$

30. $f(x,y) = x e^{xy}$
$f_y = x^2 e^{xy}$
$f_{yx} = x^2 e^{xy} \cdot y + e^{xy} \cdot 2x$
$= x e^{xy}(xy + 2)$