MAC 2233 EXAM 3 UCF Dr. Rapalje NAME ______________________

Show all work as necessary. Simplify all answers completely.

1. $\int \left(\frac{x^3}{3} - \frac{3}{x^3}\right) dx$

2. $\int_1^8 \left(\sqrt[3]{x} - \frac{1}{\sqrt[3]{x}}\right) dx$

3. If $\frac{dy}{dx} = x^2 - x$ and $y(3) = 4$, then find $y$.

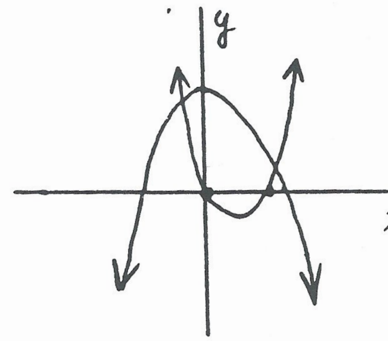
4. If $\frac{dr}{dq} = 15 - \frac{1}{15}q$ is a marginal revenue function, find the demand function.

5. $\int x\sqrt{x^2+9}\, dx$

6. $\int \frac{2x^3+3x}{(x^4+3x^2+7)^4}\, dx$

7. Find the area bounded by the x axis and the curve $y = 4 - x^2$.

8. Find the area between the curves $y = x^2 - 2x$ and $y = 12 - x^2$.




9. A manufacturer's marginal cost function is given by $\frac{dc}{dq} = .3q^2 - 6q + 40$. Find the increase in cost c, in dollars if production increases from 5 to 20 units.

10. For the demand equation $p = 100 - q^2$ and the supply equation $p = 2q + 20$,
a) find the point of equilibrium,
b) find the consumers' surplus,
c) find the producers' surplus.
(A graph sketch will be helpful)

FREE SPIN — Given the area bounded by $y - x = 6$, $y = x^3$, and $2y + x = 0$, draw the graph, label intersections, and:
a) Set up to find the area using vertical rectangles.
b) Set up to find the area using horizontal rectangles.

MAC 3233 Exam 3 Solutions UCF Dr. Rapalje

1. $\int\left(\frac{x^3}{3} - \frac{3}{x^3}\right)dx$

$= \int\left(\frac{1}{3}x^3 - 3x^{-3}\right)dx$

$= \frac{x^4}{12} + \frac{3}{2}x^{-2} + C$

$= \frac{x^4}{12} + \frac{3}{2x^2} + C$

2. $\int_1^8 x^{1/3} - x^{-1/3}\,dx$

$= \frac{3x^{4/3}}{4} - \frac{3x^{2/3}}{2}\Big|_1^8$

$= \left(\frac{3}{4}\cdot 8^{4/3} - \frac{3}{2}8^{2/3}\right) - \left(\frac{3}{4} - \frac{3}{2}\right)$

$= \frac{3}{4}(\sqrt[3]{8})^4 - \frac{3}{2}(\sqrt[3]{8})^2 - \left(-\frac{3}{4}\right)$

$= \frac{3}{4}\cdot 16 - \frac{3}{2}\cdot 4 + \frac{3}{4}$

$= 12 - 6 + \frac{3}{4} = 6\frac{3}{4}$ or $\frac{27}{4}$

3. $\frac{dy}{dx} = x^2 - x$

$y = \frac{x^3}{3} - \frac{x^2}{2} + C$

$4 = \frac{27}{3} - \frac{9}{2} + C$

$C = -\frac{1}{2}$

$y = \frac{x^3}{3} - \frac{x^2}{2} - \frac{1}{2}$

4. $\frac{dr}{dq} = 15 - \frac{1}{15}q$

$r = 15q - \frac{1}{30}q^2 + C$

$pq = 15q - \frac{1}{30}q^2 + C$

$p = 15 - \frac{q}{30} + \frac{C}{q}$

5. $\int x\sqrt{x^2+9}\,dx$ Let $u = x^2+9$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$= \int u^{1/2}\frac{du}{2}$

$= \frac{1}{2}\frac{2}{3}u^{3/2} + C$

$= \frac{1}{3}(x^2+9)^{3/2} + C$

6. $\int \frac{2x^3+3x}{(x^4+3x^2+7)^4}\,dx$ Let $u = x^4+3x^2+7$

$du = (4x^3+6x)dx$

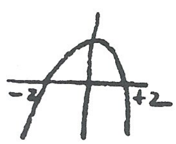
$du = 2(2x^3+3x)dx$

$= \int \frac{du}{2u^4}$

$= \frac{1}{2}\int u^{-4}du = \frac{1}{2}\,\frac{u^{-3}}{-3} + C$

$= -\frac{1}{6}\,\frac{1}{(x^4+3x^2+7)^3} + C$

7. $y = 4 - x^2$

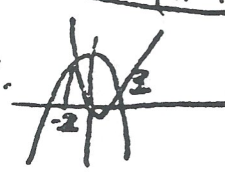



$A = 2\int_0^2 (4-x^2)dx$

$= 2\left[4x - \frac{x^3}{3}\right]_0^2$

$= 2\left[8 - \frac{8}{3}\right] = \frac{32}{3}$

8.




Intersection: $x^2 - 2x = 12 - x^2$

$2x^2 - 2x - 12 = 0$

$2(x^2 - x - 6) = 0$

$2(x-3)(x+2) = 0$

$x = 3 \quad x = -2$

$\int_{-2}^{3}(\text{upper} - \text{lower})dx$

$= \int_{-2}^{3}(12-x^2) - (x^2-2x)\,dx$

$= \int_{-2}^{3}(12 + 2x - 2x^2)dx = 12x + x^2 - \frac{2x^3}{3}\Big|_{-2}^{3}$

$= (36 - 18 + 9) - \left(-24 + \frac{16}{3} + 4\right)$

$= 27 + \frac{44}{3} = \frac{125}{3}$

9. $\frac{dc}{dq} = .3q^2 - 6q + 40$

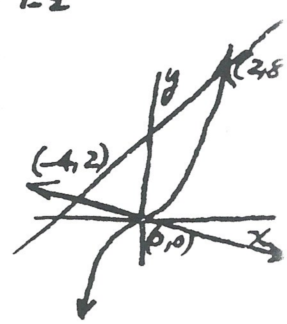
$c = \int(.3q^2 - 6q + 40)dq$

$= .1q^3 - 3q^2 + 40q\Big|_5^{20}$

$= (\$800 - 1200 + 800) - (12.5 - 75 + 200)$

$= \$400 - 137.50 = \$262.50$

FREE SPIN:




INTERSECTIONS:

(0,0) Obvious.

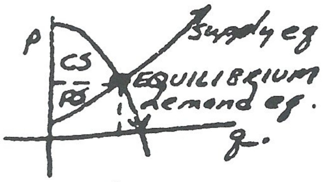
$y - x = 6$
$2y + x = 0$
$3y = 6$
$y = 2$
$x = -4$

$y - x = 6$
$y = x^3$
$x = 2, y = 8$
by trial & error! (Sorry!)

a) $\int_{-4}^{0}\left[(x+6) - \left(-\frac{x}{2}\right)\right]dx + \int_0^2\left[(x+6) - x^3\right]dx$

b) $\int_0^2 \sqrt[3]{y} - (-2y)\,dy + \int_2^8\left[\sqrt[3]{y} - (y-6)\right]dy$

10.




a) $100 - q^2 = 2q + 20$

$q^2 + 2q - 80 = 0$

$(q+10)(q-8) = 0$

$q = 8$

$p = 100 - 64 = 36$

b) $CS = \int(100 - q^2 - 36)dq$

$= 64q - \frac{q^3}{3}\Big|_0^8$

$= 512 - \frac{512}{3} = \frac{1024}{3}$

c) $PS = \int 36 - (2q+20)\,dq$

$= 16q - q^2\Big|_0^8$

$= 128 - 64 = 64$