MAC 3311 CALCULUS I UCF Dr. RAPALJE

Show all work on separate paper. A calculator is allowed.
You may keep this test and a copy of your answers if you wish.
(Show drivers license identification as you leave.)

1. Find the domain
   a) $f(x)=\frac{1}{x^2-3x}$
   (3ea) b) $f(x)=\sqrt{3-x}$
   c) $f(x)=\sqrt{x^2-4}$

2. Find domain and range:
   a) $f(x)=(x-2)^2$
   (4ea) b) $f(x)=\sqrt{4-x^2}$
   c) $f(x)=-\sqrt{2x}$

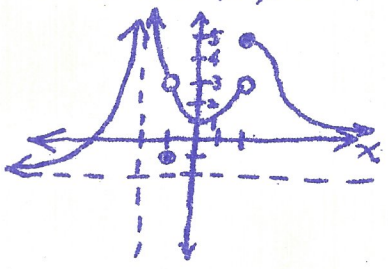
3. Let $f(x)=\frac{3x+2}{x}$, $g(x)=\frac{5}{x}$
   a) $f\circ g=$
   (3ea) b) $g\circ f=$
   c) $f\circ f=$
   } and simplify.

In 4–13, find the limits using algebraic process. Show work.

4. $\lim_{x\to 2}\frac{x^4-16}{x-2}$
5. $\lim_{x\to+\infty}\frac{3x+5}{-2x+1}$
6. $\lim_{x\to-\infty}\frac{\sqrt{2x^2-3}}{3x-2}$
7. $\lim_{x\to 9}\frac{9-x}{3-\sqrt{x}}$

(3ea)
8. $\lim_{x\to+\infty}\cos\left(\frac{1}{x}\right)$
9. $\lim_{x\to+\infty}\cos(x)$
10. $\lim_{x\to 2^-}\frac{x}{x^2-4}$
11. $\lim_{x\to 0}\frac{x}{\sin 2x}$
12. $\lim_{\theta\to 0}\frac{\theta^2}{1-\cos^2\theta}$
13. $\lim_{\theta\to 0}\frac{\theta}{\cos\theta}$

14. Given the graph of $f(x)$



   a) $\lim_{x\to -1} f(x)$
   e) $\lim_{x\to -2} f(x)$
   (2ea) b) $\lim_{x\to 2} f(x)$
   f) $\lim_{x\to -\infty} f(x)$
   c) $\lim_{x\to 2^-} f(x)$
   g) $f(2)=$
   d) $\lim_{x\to 2^+} f(x)$
   h) $f(-1)=$
   (3) i) Discuss continuity.

15. (3) Given $f(x)$ sketch the graph and answer questions:

$$f(x)=\begin{cases}-4x & x<0\\ 2 & 0\le x<4\\ \frac{1}{4}x^2-2 & x\ge 4\end{cases}$$

   (2ea) a) $f(0)=$
   b) $f(4)=$
   c) $\lim_{x\to 0}=$
   d) $\lim_{x\to 4}$
   (3) e) Discuss continuity.

16. State the definition of a limit (in terms of δ and ϵ)
(3) 17. Given $\epsilon=0.01$, find δ to prove the limit $\lim_{x\to 3}3x-5=4$
18. Show by δ-ϵ definition that $\lim_{x\to 5}x^2=25$ (assume that $\delta\le 1$)
(Find δ in terms of ϵ)

CALCULUS I EXAM 1 Solutions UCF (Not guaranteed — 9/7/89) Dr. RAPALJE

1a) $f(x) = \frac{1}{x(x-3)}$

D: all $x \neq 0, 3$

b) $f(x) = \sqrt{3-x}$

D: $3-x \geq 0$

$-x \geq -3$

$x \leq 3$

$(-\infty, 3]$

c) $f(x) = \sqrt{x^2-4}$

D: $x^2-4 \geq 0$

Extremes.

$(-\infty, -2] \cup [2, \infty)$

also $|x| \geq 2$

2a) $f(x) = (x-2)^2$

D: all reals.

R: $y \geq 0$

b) $f(x) = \sqrt{4-x^2}$

D: $4-x^2 \geq 0$

$x^2 - 4 \leq 0$

D: $[-2, 2]$

R: $y \geq 0$

also $y \leq 2$

R: $[0, 2]$

c) $f(x) = -\sqrt{2x}$

D: $x \geq 0$ $[0, \infty)$

R: $y \leq 0$ $(-\infty, 0]$

3. $f(x) = \frac{3x+2}{x}$ $g(x) = \frac{5}{x}$

a) $f[g(x)] = \frac{3(\frac{5}{x})+2}{(\frac{5}{x})} = \frac{15+2x}{5}$

b) $g[f(x)] = \frac{5}{\frac{3x+2}{x}} = \frac{5x}{3x+2}$

c) $f[f(x)] = \frac{3(\frac{3x+2}{x})+2}{(\frac{3x+2}{x})}$

$= \frac{3(3x+2)+2x}{3x+2}$

$= \frac{11x+6}{3x+2}$

4. $\lim_{x \to 2} \frac{x^4-16}{x-2}$

$= \frac{(x^2-4)(x^2+4)}{x-2}$

$= \lim_{x \to 2} \frac{(x-2)(x+2)(x^2+4)}{(x-2)}$

$= (4)(4+4) = 32$

5. $\lim_{x \to +\infty} \frac{\frac{1}{x}3x+5}{-2x+1}$

$= \lim_{x \to +\infty} \frac{3+5/x}{-2+1/x} = -\frac{3}{2}$

6. $\lim_{x \to -\infty} \frac{\sqrt{2x^2}-3}{3x-2}$

$= \lim_{x \to -\infty} \frac{|x|\sqrt{2}-3/x^2}{x(3-3/x)}$

$= -1 \cdot \frac{\sqrt{2}}{3} = -\frac{\sqrt{2}}{3}$

7. $\lim_{x \to 9} \frac{9-x}{(3-\sqrt{x})(3+\sqrt{x})}$

$= \lim_{x \to 9} \frac{(9-x)(3+\sqrt{x})}{9-x} = 6$

8. $\lim_{x \to +\infty} \cos\frac{1}{x}$

$= \cos 0 = 1$

9. $\lim_{x \to +\infty} \cos x$

$=$ DNE (Oscillates)

10. $\lim_{x \to 2^-} \frac{x}{x^2-4} = \frac{2}{0^-}$

$= -\infty$

11. $\lim_{x \to 0} \frac{2x}{\sin 2x} = \frac{1}{2}$

12. $\lim_{\theta \to 0} \frac{\theta^2(1+\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$

$= \lim_{\theta \to 0} \frac{\theta^2(1+\cos\theta)}{1-\cos^2\theta}$

$= \lim_{\theta \to 0} \frac{\theta^2}{\sin^2\theta}(1+\cos\theta)$

$= 1^2 \cdot (1+1) = 2$

13. $\lim_{\theta \to 0} \frac{\theta}{\cos\theta} = \frac{0}{1}$

$= 0$

14a) 3 e) $+\infty$ DNE

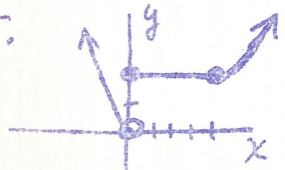
b) DNE f) -2

c) 3 g) 5

d) 5 h) -1

i) Discontinuous at $x = -2, -1, +2$

15.



a) 2 c) DNE

b) 2 d) 2

e) Discontinuous at $x = 0$.

16. Given any $\epsilon > 0$, there exists a $\delta > 0$, such that $|f(x) - L| < \epsilon$ whenever $0 < |x-a| < \delta$

$\lim_{x \to a} f(x) = L$

17. $\lim_{x \to 3} 3x-5 = 4$

$|3x-5-4| < .01$ when $0 < |x-3| < \delta$

$|3x-9| < .01$

$3|x-3| < .01$

$|x-3| < \frac{1}{300}$ Let $\delta = \frac{1}{300}$

18. $\lim_{x \to 5} x^2 = 25$

Assume $\delta \leq 1$

$|x^2-25| < \epsilon$ when $0 < |x-5| < \delta$

$|x-5||x+5| < \epsilon$ $\quad |x-5| < 1$

$|x-5| < \frac{\epsilon}{|x+5|}$ $\quad -1 < x-5 < 1$

$4 < x < 6$

To guarantee the smallest δ, use the maximum value of x in the interval $(4, 6)$

Therefore $\frac{\epsilon}{|x+5|} < \frac{\epsilon}{11}$. Let $\delta = \frac{\epsilon}{11}$